



APPLICATION OF THE METHODS OF SELECTION OF
GEAR BOX RATIOS TO PASSENGER CARS

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ABSTRACT

Selection of ratios in mechanical transmission of motor vehicles can be done according to various mathematical progressions such as : arithmetic, harmonic and geometric with constant and increasing roots.

In a paper presented by the same authors, a comparison of the methods of selecting the gear ratios has been made. This was done by calculating the wasted power due to their stepped tractive effort - speed characteristics relative to the ideal one with continuous power transmission. It has been concluded that the geometric progression would give the least wasted power. This conclusion was based on calculations considering the data of only one Jeep car.

The objective of this paper is to prove the validity of the above mentioned conclusion. In this regard, same method of comparison was applied to 14 small and medium class different passenger cars having engines of swept volume between 0,9 and 1.6 litre.

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INTRODUCTION

Matching of the vehicle engine and transmission is an important job that a vehicle designer has to fulfil. A good matching depends upon both engine and transmission characteristics.

In a mechanical transmission, the choice of the gear ratios represents the most important factor influencing the degree of matching and consequently the vehicle performance.

Selection of the gear ratios is usually done according to certain mathematical progressions such as; arithmetic, harmonic and geometric with constant or increasing roots.

In a paper published by the authors in May 1985 (1) a comparison of the methods of gear ratios spacing has been done. As a measure of comparison, the unobtainable power due to traction-speed characteristics of mechanical transmission relative to the ideal one has been considered. On the traction-speed curve in Fig. 1, the areas representing the unobtainable or wasted power is shown.

The comparison has been made considering the data of a Jeep car and it has been concluded that the geometric progression would give the least wasted power.

To prove the validity of this conclusion, the present study considers a variety of small and medium class passenger cars.

TECHNICAL DATA OF THE PASSENGER CARS CONSIDERED IN THE ANALYSIS.

The passenger cars considered in the study belong to the small and medium class cars have swept volume of engine ranging between 0.9 and 1.6 litres. The main technical data of these cars are given in Tables 1a, 1b, and the characteristic power and torque curves are shown in Figs. 2,3,4,5.

The engine power curve can be predicted by an equation of the following form :

$$P_e = P_{e \max} \left[A \left(\frac{n_e}{n_N} \right) + B \left(\frac{n_e}{n_N} \right)^2 + C \left(\frac{n_e}{n_N} \right)^3 + D \left(\frac{n_e}{n_N} \right)^4 \right] \quad (1)$$

Where :

- P_e = engine power, HP
- $P_{e \max}$ = engine maximum power, HP
- n_e = engine revolutions, r.p.m.
- n_N = engine revolutions at max. power, r.p.m.
- A, B, C, D = constants.

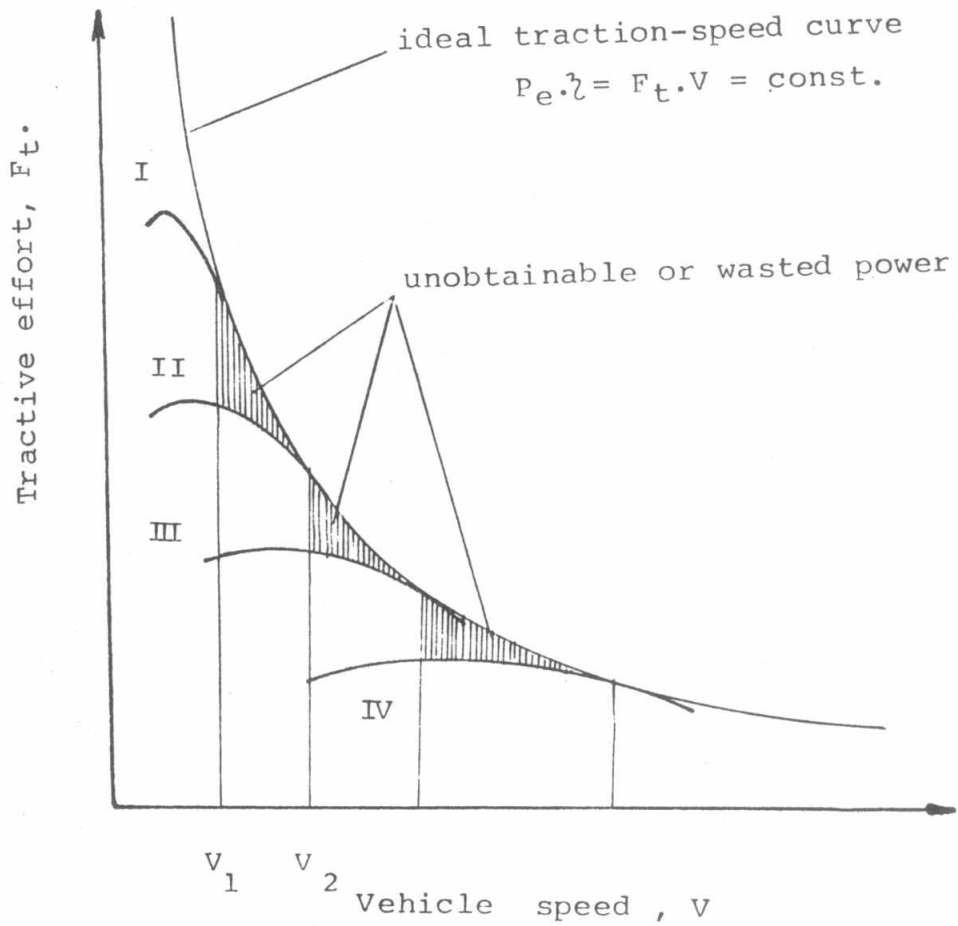


Fig. 1 : Ideal and Stepped Tracttion-speed relationships



Table 1 a,b: Technical data of small and medium class vehicles

Type of Vehicle	Power (HP)		Torque (kg.m) M _e max at (r.p.m.)	max. r.p.m. n _e max	Swept volume (litre)	dyn. radius r _d (m)	final drive ratio i _o	gear box ratios					
	N _e max at (r.p.m.)	at (r.p.m.)						i ₁	i ₂	i ₃	i ₄	i ₅	
a) Small Class													
MAZDA 323	55	6000	8	4000	6000	1.071	0.275	4.39	3.42	1.84	1.29	0.92	-
NISSAN PULZAR	50	6000	7.58	4000	6500	0.988	0.279	4.47	3.33	1.96	1.29	0.9	-
PEUGEOT 205	50.84	5800	7.58	3000	6000	1.188	0.274	4.79	3.88	2.07	1.38	0.94	-
FIAT UNO 45	45	5600	6.8	3000	6000	0.903	0.265	4.07	3.91	2.06	1.34	0.96	-
SEAT IPIZA	42.8	5800	6.8	3000	6000	0.903	0.274	4.79	3.5	1.95	1.32	0.97	-
RENAULT 5	40.8	5750	6.3	3000	6000	0.956	0.274	3.875	3.73	2.05	1.32	0.9	-
OPEL CORSA	40	5200	6.32	3000	5400	0.993	0.274	4.18	3.55	1.96	1.3	0.89	-
b) Medium Class													
HONDA CIVIC	81.83	6000	11.9	3500	6500	1.488	0.275	4.27	2.92	1.76	1.18	0.85	0.71
V.W. JETTA	74.8	5000	12.74	2500	6000	1.595	0.293	3.67	3.46	1.94	1.25	0.91	-
OPEL ASCONA	70.72	5400	12.03	3000	6000	1.598	0.291	3.94	3.42	1.96	1.28	0.89	-
NISSAN SUNNY	70	5200	11.7	3200	5500	1.488	0.279	3.79	3.33	1.96	1.29	0.9	-
FIAT REGATA	74	5500	11.35	3500	6000	1.498	0.279	3.59	4.09	2.24	1.46	1.03	0.83
SEAT MALAGE	82.5	5800	11.5	3500	6000	1.461	0.279	3.74	3.5	1.95	1.32	0.97	0.769
RENAULT 9	58.48	5250	10.194	3000	5500	1.397	0.279	4.08	3.73	2.05	1.32	0.97	0.794

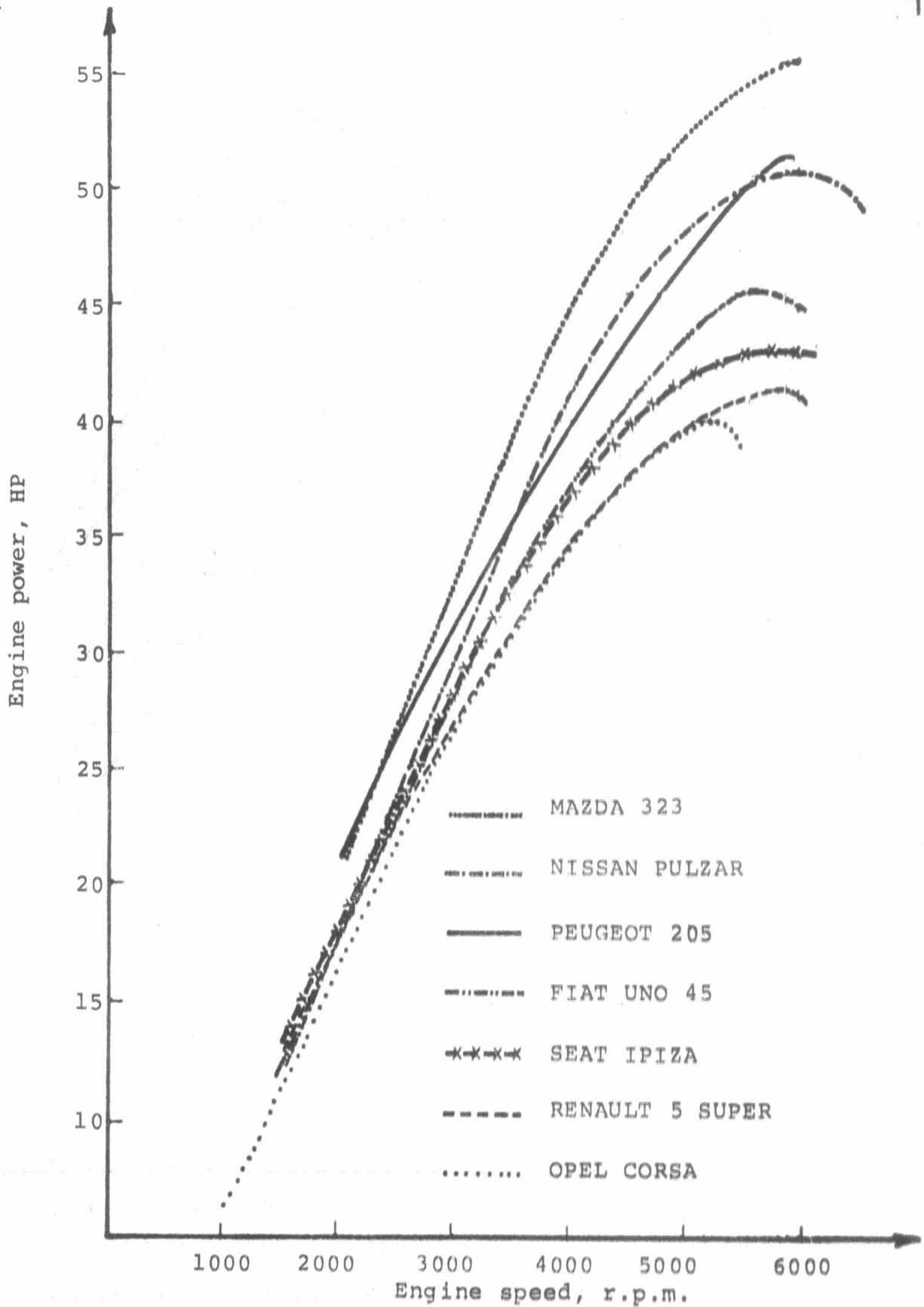


Fig. 2 : Engine power curves for small class passenger cars.

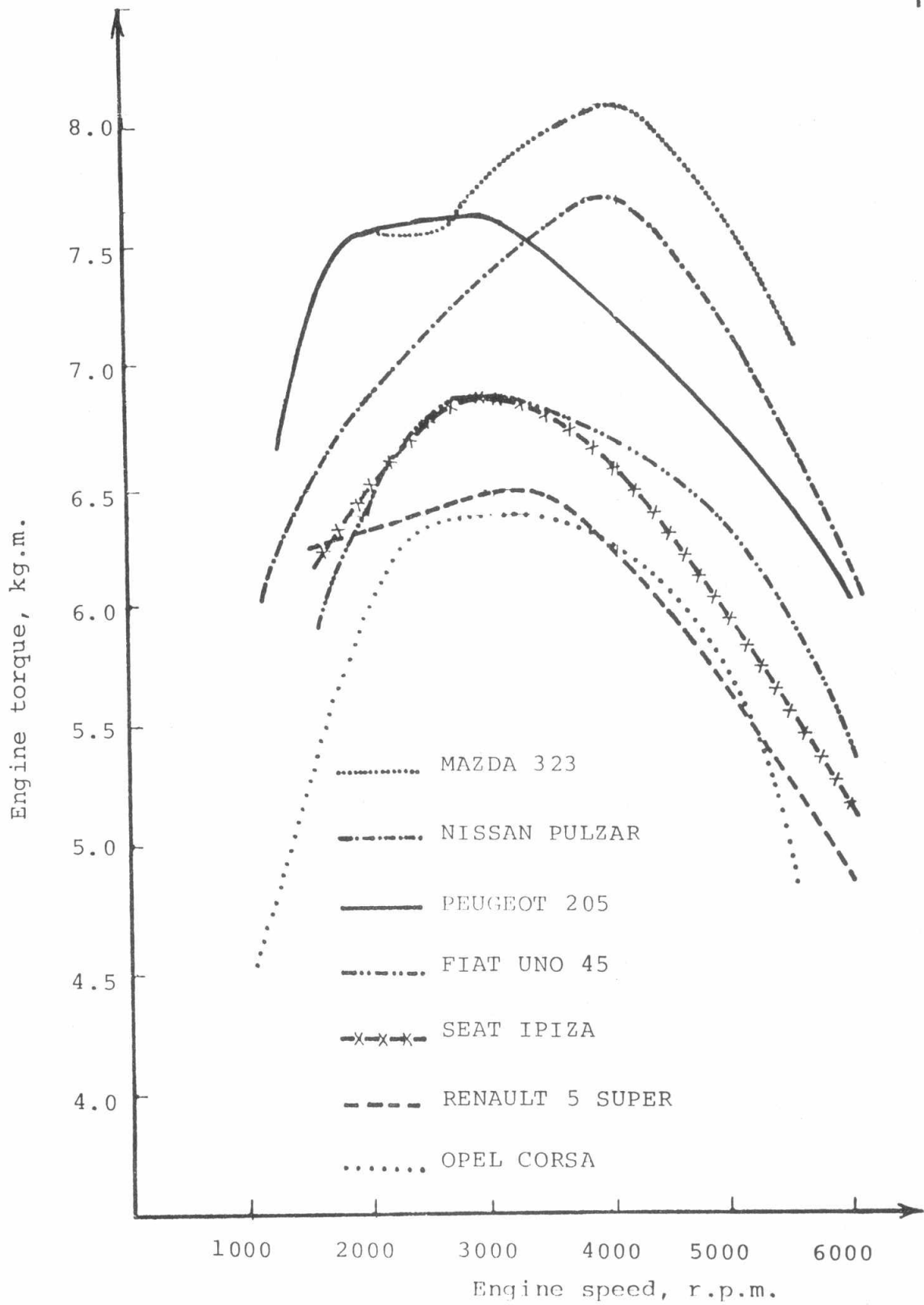


Fig. 3 : Engine torque curves for small class passenger cars.

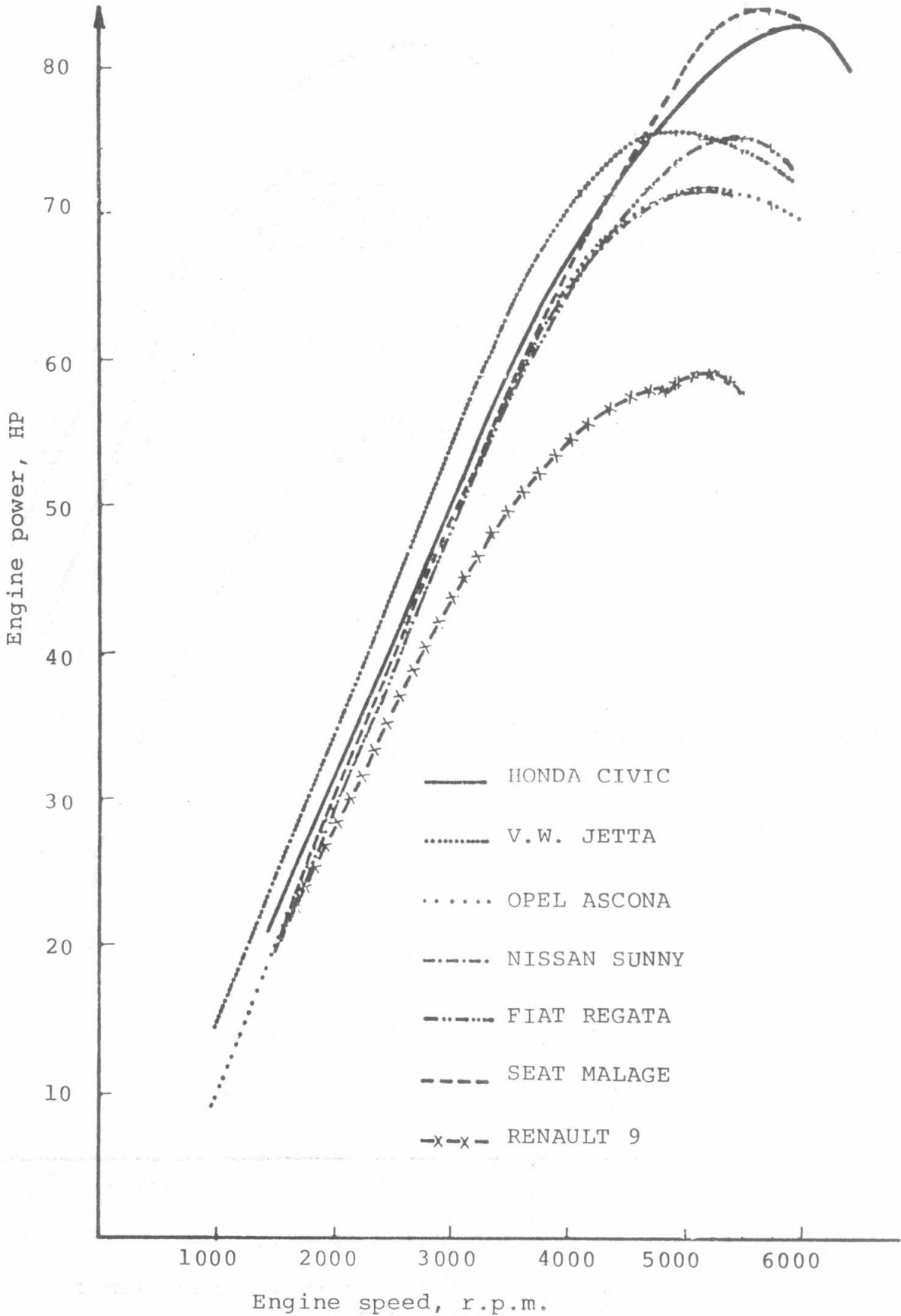


Fig. 4 : Engine power curves for medium class passenger cars.

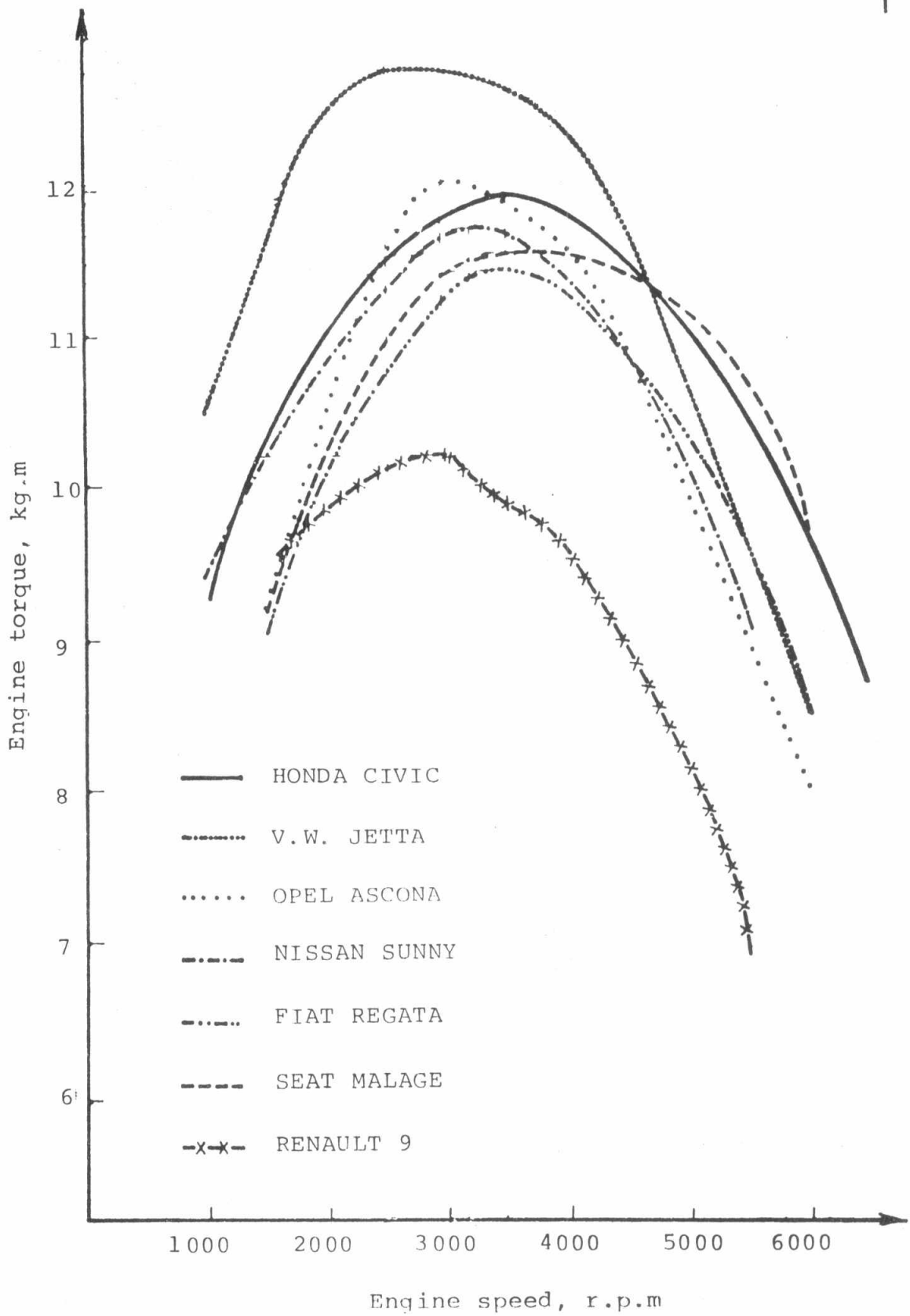


Fig. 5 : Engine torque curves for medium class passenger cars.



The constants A,B,C,D in equation (1) could be determined by taking the values of engine power and the corresponding engine speed at four points widely spaced on the power curve between the engine speed values corresponding to max power and torque.

A special BASIC computer program (Appendix 1) has been written to calculate the constants A,B,C,D for each of the fourteen power curves.

CALCULATION OF THE GEAR RATIOS ACCORDING TO DIFFERENT METHODS.

The following methods were used to calculate the intermediate ratios while the maximum and minimum are given as vehicle data.

A special BASIC computer program (Appendix 2) has been written to calculate the gear ratios according to arithmetic, harmonic, geometric with constant root and geometric with increasing roots.

The relationships for calculating the individual ratios according to the mentioned progressions are as follows :

Arithmetic :

$$i_1 - i_2 = i_2 - i_3 = \dots = i_{n-1} - i_n = \text{const}$$

Harmonic :

$$1/i_2 - 1/i_1 = 1/i_3 - 1/i_2 = \dots = 1/i_{n-1} - 1/i_n = \text{constant}$$

Geometric with constant root :

$$i_1/i_2 = i_2/i_3 = \dots = i_{n-1}/i_n = \text{constant}$$

Geometric with increasing root :

$$\frac{i_1}{i_2} = q_1, \frac{i_2}{i_3} = q_2, \dots, \frac{i_{n-1}}{i_n} = q_{n-1}$$

and $q_1/q_2 = q_2/q_3 = \dots = q_{n-2}/q_{n-1} = q = \text{constant}$

COMPUTATION OF THE WASTED POWER.

The wasted power represented by the hatched areas in Fig.1 can be computed using the following mathematical integration:

$$\Delta P_e = \sum_{i=1}^n \left[\int_{v_1}^{v_2} F_t \cdot dv - \int_{v_1}^{v_2} F_t \cdot dv \right] \quad (2)$$



Where :

- ΔP_e = the total wasted power, HP
 V_1, V_2 = vehicle speeds as in Fig. 1
 F_t = tractive effort given by the ideal traction characteristics,
 $F_t = P_e \max \cdot \eta \cdot 2700/V$, Newtons
 F'_t = Tractive effort given by the individual traction curves of the mechanical transmission.
 $F'_t = P_e \cdot \eta \cdot 2700/V$
 P_e : given by the equation (1)
 V : given by the equation : $V = \frac{0.377 \cdot n_e \cdot r_d}{i_t}$
 r_d : wheel dynamic radius.
 i_t : total transmission ratio.
 η : total mechanical efficiency.

To facilitate the computation of the wasted power, a special BASIC computer program (Appendix 3) has been written.

RESULTS

The actual power delivered at wheels (areas under the stepped curves in Fig. 1), the power at wheels in case of ideal transmission (area under the hyperbolic curve in Fig. 1) were calculated in five cases for each car. In the first case the actual gear ratios of the car were considered and in the other four cases the gear ratios spaced according to the previously mentioned mathematical progressions have been taken into account. The results of computations are shown in Table 2.

CONCLUSIONS.

- 1- Based on the calculated unobtainable power due to mechanical transmission characteristics relative to the ideal one, the geometric with constant root stands as the best progression for gear ratios spacing.
- 2- The geometric progression with increasing root gave slightly higher wasted power than in case of constant root.
- 3- The arithmetic and harmonic progressions gave approximately 30% higher wasted power relative to the geometric progression.



Table 2 : Unobtainable power for used vehicles with different progressions.

PROGRAM TO CALCULATE THE LOSSES

PROGRESSION	DIFFERENCE
<u>FIAT UNO</u>	
ARITHMATIC PROGRESSION:	19619.99
HARMONIC PROGRESSION:	19636.72
GEOMETRIC PROGRESSION:	16252.81
GEOMET.WITH INCR.ROOT:	16943.54
REAL GEAR RATIOS:	18105.14
<u>OPEL CORSA</u>	
ARITHMATIC PROGRESSION:	19128.95
HARMONIC PROGRESSION:	19829.43
GEOMETRIC PROGRESSION:	15993.47
GEOMET.WITH INCR.ROOT:	16655.7
REAL GEAR RATIOS:	16980.68
<u>MAZDA 323</u>	
ARITHMATIC PROGRESSION:	39909.41
HARMONIC PROGRESSION:	39916.97
GEOMETRIC PROGRESSION:	23459.03
GEOMET.WITH INCR.ROOT:	27500.1
REAL GEAR RATIOS:	35542.79
<u>NISSAN PULZAR</u>	
ARITHMATIC PROGRESSION:	15401.21
HARMONIC PROGRESSION:	15403.79
GEOMETRIC PROGRESSION:	12236.3
GEOMET.WITH INCR.ROOT:	13058.95
REAL GEAR RATIOS:	12917.37
<u>SEAT IPIZA</u>	
ARITHMATIC PROGRESSION:	11034.9
HARMONIC PROGRESSION:	11039.1
GEOMETRIC PROGRESSION:	8740.774
GEOMET.WITH INCR.ROOT:	9387.649
REAL GEAR RATIOS:	10188.74
<u>NISSAN SUNNY</u>	
ARITHMATIC PROGRESSION:	20988.31
HARMONIC PROGRESSION:	20992.19
GEOMETRIC PROGRESSION:	16465.96
GEOMET.WITH INCR.ROOT:	17661.31
REAL GEAR RATIOS:	17446.63
<u>V.W. JETTA</u>	
ARITHMATIC PROGRESSION:	27963.39
HARMONIC PROGRESSION:	27966.99
GEOMETRIC PROGRESSION:	22952.44
GEOMET.WITH INCR.ROOT:	24164.83



Table 2 continued.

<u>OPEL ASCONA</u>	
ARITHMATIC PROGRESSION:	21287.42
HARMONIC PROGRESSION:	21296.46
GEOMETRIC PROGRESSION:	16439.63
GEOMET.WITH INCR.ROOT:	17565.11
REAL GEAR RATIOS:	17656.23
<u>PEUGEOT 205</u>	
ARITHMATIC PROGRESSION:	27682.99
HARMONIC PROGRESSION:	27686.68
GEOMETRIC PROGRESSION:	24421.4
GEOMET.WITH INCR.ROOT:	25055.17
REAL GEAR RATIOS:	25682.6
<u>RENAULT 5</u>	
ARITHMATIC PROGRESSION:	15689.05
HARMONIC PROGRESSION:	15690.5
GEOMETRIC PROGRESSION:	12650.9
GEOMET.WITH INCR.ROOT:	13233.69
REAL GEAR RATIOS:	13449.47
<u>FIAT REGATA</u>	
ARITHMATIC PROGRESSION:	29711.87
HARMONIC PROGRESSION:	29702.62
GEOMETRIC PROGRESSION:	22108.2
GEOMET.WITH INCR.ROOT:	24962.24
REAL GEAR RATIOS:	26965.1
<u>HONDA CIVIC</u>	
ARITHMATIC PROGRESSION:	25587.15
HARMONIC PROGRESSION:	25577.15
GEOMETRIC PROGRESSION:	20545.52
GEOMET.WITH INCR.ROOT:	23330.8
REAL GEAR RATIOS:	23925.69
<u>SEAT MALAGA</u>	
ARITHMATIC PROGRESSION:	34575.92
HARMONIC PROGRESSION:	34571.35
GEOMETRIC PROGRESSION:	27793.74
GEOMET.WITH INCR.ROOT:	31054.83
REAL GEAR RATIOS:	32506.97
<u>RENAULT 7</u>	
ARITHMATIC PROGRESSION:	18127.82
HARMONIC PROGRESSION:	18131.22
GEOMETRIC PROGRESSION:	12927.15
GEOMET.WITH INCR.ROOT:	14932.5
REAL GEAR RATIOS:	16881.03



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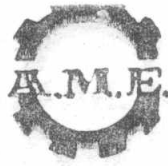


Appendix 1 : Program to calculate constants of power curves.

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5 REM PROGRAM TO CALCULATE CONSTANTS OF POWER CURVES
10 REM USING PROGRAM TO SOLVE SYSTEM OF LINEAR EQUATIONS
11 LPRINT"VALUES OF CONSTANTS OF POWER CURVES"
12 LPRINT"-----"
13 LPRINT
20 N=4
30 DIM A(N,N+1),B(N),X(N)
35 READ T# ' TYPE OF VEHICLE ("END" TO FINISH)
36 IF T#="END" THEN 340
37 READ RMAX,FMAX 'rpm at max power , max power
40 FOR I=1 TO N
45 READ REV,POWER 'one point on the curve
50 FOR J=1 TO N
60 A(I,J)=(REV/RMAX)^J
70 NEXT J
80 B(I)=POWER/FMAX
90 NEXT I
100 FOR I=1 TO N
110 A(I,N+1)=B(I)
120 NEXT I
130 FOR K=1 TO N
140 T=1/A(K,K)
150 FOR J=K TO N+1
160 A(K,J)=A(K,J)*T
170 NEXT J
180 FOR J=1 TO N
190 IF K=J THEN 240
200 T=A(J,K)
210 FOR I=K TO N+1
220 A(J,I)=A(J,I)-T*A(K,I)
230 NEXT I
240 NEXT J
250 NEXT K
260 FOR I=1 TO N
270 X(I)=A(I,N+1)
280 NEXT I
285 LPRINT"TYPE OF VEHICLE:",T#
286 LPRINT"VALUES OF CONSTANTS ="
290 FOR I=1 TO N
300 LPRINT X(I),
310 NEXT I
320 LPRINT : LPRINT
330 GOTO 35
340 END

```



Appendix 2 : Program to calculate speed ratios according
to different progressions.

```

30 LPRINT"          CALCULATION OF SPEED RATIOS ACCORDING TO "
40 LPRINT"          DIFFERENT PROGRESSIONS          "
50 LPRINT"          -----"
55 LPRINT
56 LPRINT"FIRST","SECOND","THIRD","FOURTH","FIFTH"
57 LPRINT"SPEED","SPEED","SPEED","SPEED","SPEED"
75 READ T#,FI,LA,N
76 IF T#="END" THEN 540
80 LPRINT
100 LPRINT
110 LPRINT"VEHICLE TYPE:";T#
120 LPRINT"ARITHMETIC PROGRESSION"
130 I(1)=FI
140 I(N)=LA
150 Q=(I(1)-I(N))/(N-1)
160 LPRINT I(1),
170 FOR K=2 TO N-1
180 I(K)=I(1)-(K-1)*Q
190 LPRINT I(K),
200 NEXT K
210 LPRINT I(N)
230 LPRINT"HARMONIC PROGRESSION"
240 LPRINT I(1),
250 Q=(1/I(N)-1/I(1))/(N-1)
260 FOR K=2 TO N-1
270 I(K)=1/I(1)+(K-1)*Q
280 I(K)=1/I(K)
290 LPRINT I(K),
300 NEXT K
310 LPRINT I(N)
330 LPRINT"GEOMETRIC PROGRESSION"
340 LPRINT I(1),
350 Q=(I(1)/I(N))^(1/(N-1))
360 FOR K=2 TO N-1
370 I(K)=I(1)/(Q^(K-1))
380 LPRINT I(K),
390 NEXT K
400 LPRINT I(N)
420 LPRINT "GEOMETRIC WITH INCREASING ROOTS"
430 Q=1.1
440 IF N=5 THEN 470
450 Q1=(I(1)/(I(N)*Q^3))^(1/5)
460 GOTO 480
470 Q1=(I(1)/(I(N)*Q^6))^(1/4)
480 LPRINT I(1),
490 FOR K=2 TO N-1
500 I(K)=I(K-1)/(Q1*Q^(N-K))
510 LPRINT I(K),
520 NEXT K
530 LPRINT I(N)
535 GOTO 75

```



Appendix 3 : Program to calculate the unobtainable power.

```

10 REM PROGRAM TO CALCULATE THE LOSSES
20 LPRINT " PROGRAM TO CALCULATE THE LOSSES"
30 LPRINT " -----"
40 READ T# 'type of vehicle
50 IF T#="END" THEN 480
60 LPRINT:LPRINT"TYPE OF VEHICLE:",T#
70 REM INPUT MAX POWER,REV.AT MAX POWER,DYN.RAD.,
80 REM FINAL DRIVE GEAR RATIO,NO OF SPEEDS
90 READ PMAX,NN,RD,IO,N
95 REM INPUT FIRST AND LAST SPEED RATIOS
96 READ G(1),G(N)
130 READ A,B,C,D 'constants of power curves
140 LPRINT"GREAT AREA","SMALL AREA","DIFFERENCE"
150 FOR J=1 TO 5 'different progressions
160 ON J GOTO 170,190,210,230,235
170 LPRINT"ARITHMATIC PROGRESSION:"
180 GOTO 240
190 LPRINT"HARMONIC PROGRESSION:"
200 GOTO 240
210 LPRINT"GEOMETRIC PROGRESSION:"
220 GOTO 240
230 LPRINT"GEOMETRIC WITH INCREASING ROOT:"
234 GOTO 240
235 LPRINT"REAL GEAR RATIOS:"
240 FOR I= 2 TO N-1
250 READ G(I) 'gear ratios
260 NEXT I
270 SUMDIF = 0
280 SUMF1=0
290 SUMF2=0
291 FOR I=1 TO N
292 V(I)= .377*NN*RD/(IO*G(I))
293 NEXT I
300 K1= 2700*PMAX*.9
310 FOR I=2 TO N 'different gears
320 F1 = K1*(LOG(V(I))-LOG(V(I-1)))
330 K2 = IO*G(I)/(0.377*RD*NN)
340 A1=(PMAX*.9*2700)*(A*K2*V(I)+B*K2^2*(V(I))^2/2
+ C*K2^3*(V(I))^3/3+D*K2^4*(V(I))^4/4)
345 A2=(PMAX*.9*2700)*(A*K2*V(I-1)+B*K2^2*(V(I-1))^2/2+
C*K2^3*(V(I-1))^3/3+D*K2^4*(V(I-1))^4/4)
350 F2 = A1-A2
360 SUMF1=SUMF1+F1
370 SUMF2=SUMF2+F2
380 DIF=F1-F2
390 SUMDIF=SUMDIF+DIF
400 LPRINT F1,F2,DIF
410 NEXT I
420 LPRINT"SUMMATION:"
430 LPRINT SUMF1,SUMF2,SUMDIF
440 LPRINT
450 NEXT J
460 LPRINT:LPRINT

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