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MATHEMATICAL OPTIMIZATION VERSUS SIMULATION FOR THE SELECTION OF CUTTING PARAMETERS FOR C.A.M.

BY

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ABSTRACT

This paper investigates the feasibility of applying mathematical programming and heuristic simulation algorithms for the optimal selection of cutting parameters in M/C tools.

Recently, this problem has become of major interest to researchers working in the area of Computer Aided Manufacturing (C.A.M). The concepts and methods developed in this paper are very useful for the production engineers and can be successfully applied for semi-autcmatic and automatic machine tool production centres as well as Numerically Controlled M/C Tools or direct numerically controlled M/C centres (DNC) and FMS.

INTRODUCTION

The optimal choice of cutting parameters had always been an interesting subject for the production engineers. With the rapid development of production systems and the development of Numerically controlled M/C tools to direct numerically controlled M/C tools by centeral computers (DNC) and the advent of DNC with automatic transfer known as Flexible Manufacturing Systems (FMS) [1], the choice of cutting parameters become a continious decision process.

The machining process in the new Machine centers involves multiple machining processes. Thus the choice of cutting parameters becomes a more difficult problem.

In the following study we will consider two possible approaches to solve the problem, the first one is to use mathematical optimization techniques and the second one is to develop a simulation Heuristic Algorithm the properties of optimal solution and

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the special nature of the problem.

The study will focus on the two main machining processes used in the new development M/C centers, namely the turning and Milling operations.

The study is divided into two sections, the first section formulates the various machining problems and develops the concept of multiple-process. In the second section the solution of the machining model by mathematical programming and simulation is discussed.

PROBLEM FORMULATION

The selection of optimal cutting parameters can be solved as a programming problem, the production engineer aims to optimize a given objective and satisfy the relevent constraints in cutting.

The function that the engineer wants to optimize, known as objective function or effectiveness or performance index[2] must be defined and it varies according to the goals, in our study we will take the total cutting (variable) costs. C_t

$$C_t = C_1 \left[T_c + C_2 \cdot \frac{T_c}{T} \right] \qquad \text{.........(1)}$$

C_t - Total cutting cost,
C_1 - Total Machining time cost,
C_2 - Total cost coefficient,

$T_s = \dfrac{L}{F.N}$,

T_c - Cutting time,
F - Feed/rev.,
N - Rev. per Min., $N = \dfrac{12.V_c}{\pi D}$
L - Cutting Length,
V_c - Cutting Velocity (Ft/Min),

. . .

6

T - Tool life = $\dfrac{k}{V^{1/n}.F^{1/n_1}.d^{1/n_2}}$

d - depth of cut,

k, n, n_1, n_2 - Constant For Modified Taylor equation,

T - Tool life (Minutes).

The performance index as defined by (1) - will always take the form:

$$C_{to} = k_{ol} \cdot F^{-1}.V^{-1} + K_{02}.V^{ao}.F^{bo}.d^{co} \qquad ......(2)$$

F,V,d decision variables.

The values of the decision variables in (2) are subjected to various types of constraints:-

- Machine Constraints:

  Such as speed ranges and feed ranges and Max. Power:-

  (Feed range) $F_L \leq F \leq F_u$

  (Speed range) $N_L \leq N$

  (Max. Power) $P \leq P_u$

- Surface Roughness Requirement:

  Specially in Finishing operation.

- Limitation For Max. Cutting Force:

  Due to Tool or W.P. deflection limits.

All the above constraints can generally be stated as:-

$$L(i) \leq K_i.V^{ai}.F^{bi}.d^{ci} \leq U(i) \qquad ......(3)$$

i - indicate constraint (i),

ai,bi,ci = Constants for constraints (i),

L(i),U(i) - Lower and upper bounds for cutting operation constraints.

The Turning Process:-

Using expressions (2) and (3) for the turning process.

Performance index:

$$C_{to} = C_{10}.(\frac{L\pi D}{12}).V^{-1}.F^{-1}+C_{20}(t_o\ \frac{L\pi D}{12k})d^{1/n2}.V^{(\frac{1}{n}-1)}.F^{(\frac{1}{n1}-1)}+$$

$$C_{30}(\frac{L\pi D}{12k})\ d^{1/n2}.V^{(\frac{1}{n}-1)}.F^{(\frac{1}{n1}-1)} \quad \ldots\ldots\ldots(4)$$

C_{10} - Machining cost/unit time (L.E/Min),

C_{20} - Labour cost/unit time (L.E/Min),

t_0 - Tool change time (Min).

Constraints:-

1 -	depth of cut	$d \leq U_1$	
2 -	Power	$k_2.V.F^{b2}.d \leq U_2$	
3 -	Force	$k_3\ .F^{b3}.d \leq U_3$	.........(5)
4 -	Feed	$L_4 \leq F \leq U_4$	
5 -	Finish	$\frac{F^2}{8R} \leq U_s$	

(R-Tool radius)

As an example consider the rough cutting by H.S.S. Tool where the constants are[3]:-

$\frac{1}{n} = 6.67$, $\frac{1}{n1} = 2.8$, $\frac{1}{n2} = 0.93$

$K = 4.3772 \times 10^{11}$ $L = 10$ in , $D = 2$ in

$C_{o3} = 2.64$ L.E.

$C_{o2} = 0.107$ L.E./Min.

$C_{o1} = 0.151$ L.E./Min.

$t_o = 2$ Min. , $k_2 = 20$, $k_3 = 143895$

Max.Power = $U_2 = 0.75$ K.W

Max Force = $U_3 = 150$ lb.

$b_2 = b_3 = 0.8$, $U_1 = 0.05$ in , $U_4=0.02$, $L_4=0.002$

This gives the following problem:
Minimize

$$C_{to} = 0.791\ F^{-1}.V^{-1}+6.542 \times 10^{-10}\ d^{0.93}.V^{5.67}.F^{1.80}$$

subject to:-

$$d \leq 0.05$$

$$20\ VF^{0.8}.d \leq 0.75$$

$$143895\ F^{0.8}.d \leq 150 \quad \ldots\ldots\ldots(6)$$

The Milling Process:-

The following variations are met in the Milling process:-

a) D - diameter of Milling cutter is decision variable,

b) S_z - feed per tooth $\frac{F}{Z}$, and Z is a decision variable,

c) B - Width of cutter (or work piece) may, or may not be a decesion variable.

$$T_c = \frac{L}{S_z.Z.N} = (\frac{L\pi}{12}) \frac{D}{S_z.Z.V} \qquad \text{..........(7)}$$

The extended tool life formula is:-

$$V.T^n.d^{n1}.S_z^{n2}.B^{n3}.D^{n4} = K \qquad \text{..........(8)}$$

Using (7,8) we develop the following performance index

$$C_{to} = C_{10}(\frac{L\pi}{12})\ D.v^{1}\ .S_z^{-1}.Z^{-1} + (\frac{C_{20}.t_o.L.\pi}{12\ k})\, v^{\frac{1}{n}-1}.d^{\frac{n_1}{n}}.S_z^{(\frac{n_2}{n}-1)}.B^{\frac{n_3}{n}}.D^{(\frac{n_4}{n}+1)}.Z^{-1} + (\frac{C_{30}}{12k}.L\pi)\, v^{\frac{1}{n}-1}.d^{\frac{n_1}{n}}.S_z^{(\frac{n_2}{n}-1)}.B^{\frac{n_3}{n}}.D^{(\frac{n}{n}+1)}.Z^{-1}$$

and the following constraints:-

depth of cut	: $d \leq U_1$	
Power	: $k_2.d^{a2}.S_z^{b2}.B^{c2}.D^{d2}.Z.V \leq U_2$	
Force	: $k_2.d^{a2}.S_z^{b2}.B^{c2}.D^{d2}.Z \leq U_3$	
Finish	: $\frac{C.S_z^2\ .\ Z^2}{4D} \leq U_4$, $C = 1.5$	..... (9)
Speed range	: $L_5 \leq (\frac{12}{\pi}).v.D^{-1} \leq U_5$	
Feed range	: $L_6 \leq S_z.\ Z \leq U_6$	

Consider the case of peipheral Milling with carbide tools[4]

$k = 20.1 \quad n = 0.6 \quad n_1 = 0.1 \quad n_2 = 0.75 \quad n_3 = 0.2 \quad n_4 = -0.2$

$k_2 = 1.262 \quad , a_2 = 1 \quad , b_2 = 0.75, \quad c_2 = 1.1$

$d_2 = -0.2$

$k_3 = 43200$

$U_1 = 1 \quad U_2 = 6 \quad , U_3 = 435 \quad , U_4 = 0.002$

$L = 20$

$$L_6 = 0.02 \,, \quad U_6 = 0.1 \,,$$
$$C_{o1} = 0.3 \,, \quad C_{o2} = 0.2 \,, \quad C_{o3} = 31.4 \,,$$
$$L = 12 \,, \quad t_o = 15 \,.$$

Minimize

$$C_{t_o} = 0.9372\, S_z^{-1}\, V^{-1}\, D_2^{-1} + 5.373\, v^{0.66}\, d^{0.16}\, S_z^{0.25} . B^{0.33}\, D^{0.67}\, z^{-1}$$

Subject to:-

depth of cut :- $d_1 \leq 0.5$

Power :- $1.262\, d\, S_z^{0.75}\, B^{1.1}\, D^{-0.2}\, Z\, V \leq 6$

Force :- $43200\, d\, S_z^{0.75}\, B^{1.1}\, D^{-0.2}\, Z \leq 435$

Finish :- $0.375\, S_z^{2}\, Z^2\, D^{-1} \leq 0.02$

Speed range :- $20 \leq 3.82\, V\, D^{-1} \leq 500$

Feed range :- $0.02 \leq S_z\, Z \leq 0.1$ ....(10)

Multiple Cutting Process:-

The cutting processes discussed in sections (2.1), (2.2) are single machining processes. In the new machining technique where various tools and processes are performed in the machining center, the situation is far more complicated.

The work-piece is machined in the "so called" machining center by several tools and operations, that can be numbered j = 1, 2, ..., M- every cutting tool j will perform sequence of operation Nj, where:

$$\sum_{j=1}^{M} N_j = N_j = N$$

where N - Total No of operations

This means that both the performance index (1) and the contraints must be modified to an overall performance index and overall constraints set.

$$C_t = \sum_{j=1}^{M} C_t(j)$$

$$= \sum^{M} C_1(j) \left[T_?(j) + C_?(j) \frac{T_c(j)}{\ldots} \right] \quad \ldots\ldots (11)$$

$$C_t = \sum_{j=1}^{M} \frac{C_o(j)\ L\pi}{12}\ F^{-1}.v^{-1}.D +$$

$$+ \sum_{j=1}^{M} \left\{ \frac{C_{o3j} L}{12\,k} + \left\{ \frac{C_{o2\,j}.T_{oj}.L\pi}{12\,k} \right\} \left[v_j^{a_{oj}}.d_j^{b_{oj}}.F_j^{C_{oj}}.D^{d_{oj}} \right] \right. \qquad \ldots\ldots\ldots (12)$$

Subject to:-

depth of cut :- $d_j \leq U_i(j)$

Power/Force :- $L_{ij} \leq K_{ij}.v_j^{aij}.d_j^{bij}.F_j^{cij}.D_j^{dij} \leq Uij$

Finish :- $K_{Fj}.f^2\ .D^{-1} \leq U_{f(j)}$

Feed range :- $L_{F(j)} \leq F_j \leq U_{F(j)}$

Speed range :- $L_{N(j)} \leq \frac{12}{\pi}\ D_j^{-1}.v_j \leq U_{N(j)}$ ....(13)

$j = 1, 2, \ldots\ldots M$

$i = 1, 2, \ldots\ldots R$

Moreover it is possible to include the total process time of the part T_p as constrained.

$$Tp \leq Tu$$

$$Tp = \sum_{j=1}^{M} Tp\ (j)$$

$$Tp = \sum \{Tc\ (j) + \delta_j\ t_{oj}\} \leq Tu \qquad \ldots\ldots\ldots (14)$$

$\delta j=1$ if $To(j) > Tj$

$\delta j=0$ other-size

PROBLEM SOLUTION

Solution By Mathematical Programming:

Until recently the choice of cutting parameters for single cutting operation as given in eq. (4,5,9) was a difficult non-linear programming problem, however the development of geometrical programming for optimization of posynomicals and signonicals offered us a powerful tool to solve the problem.

The algorithm stated in this work originates from the geometric programming technique of Zener[7] and developed by Blau[9] the algorithm has the following steps:-

1 - Enter problem as specified by format (correspondence between problem coef. and format coef.)

2 - Determine Initial Weight

$$Z = \sum_{t=1}^{To} \sigma_{ot} C_{ot} \prod_{n=1}^{N} x_n^{aotn}$$

$$V = /Z/$$

$$B_{nt} = C_{mt} \prod_{n=1}^{N} x_n^{a_{mtn}} \qquad m = 0,1,\ldots\ldots,M$$

3 - Calculate the vector of orthogonality condition.

$$K = \left[\sum_{t=1}^{Tm} \sigma_{mt} \quad a_{mth} \quad B\,mt \right]$$

$$H = \sum_{t=1}^{To} \sigma_{ot} \quad a_{otm} \; Bot$$

4 - Evaluate Initial Multiplin

$$Y = (K^T K^{-1}) \quad K^T H +$$

K^T = Transpose of K

5 - If this is the first Iteration go to step 6, other-wise determine new weight as follows:

$$Y_{NEW} = Y_{OLD} + \Delta Y$$

GOTO STEP 6

6 - CALCULATE MATRIX T

$$T = \sum_{m=1}^{M} \left[\sum_{t=1}^{Tm} \sigma mt \quad amti \quad amtj \quad Bmt \right] Ym$$

$$- \left[\sum_{t=1}^{To} \sigma\,ot \quad amti \quad amtj \quad Bot \right]$$

$$i = 1,\ \ldots\ldots\ N$$

7 - EVALUATE ERROR

$$
ei = \begin{cases}
\sum_{t=1}^{To} \sigma ot \quad aoti \quad Bot - \sum_{m=1}^{M} \left[\sum_{t=1}^{Tm} \sigma mt \quad amti \quad B\ mt \right] \quad Ym & i = 1, \ldots\ldots N \\
\sigma o - \sum_{t=1}^{T} \sigma ot \quad B\ t & i=N+1 \\
1 - \sum_{t=1}^{Tm} \sigma mt \quad Bm & i=N+1, \ldots\ N+1+M
\end{cases}
$$

8 - FORMULTE NEWTON -- RAPHSON MATRIX

j \ i	1 2 .... N	N+1	N+2 .... N+M+1
	n 1 2	O	m 1 2 .... M
1 2 N	T	$\bar{H}$	K
N+1	$\bar{H}^{T}$	$\underline{+1}$	$\bar{O}$
N+2 ⋮ N+M+1	K^{T}	O	O

9 - INVERT MATRIX R

10 - Find VECTOR ADJUSTMENT

$$
R^{-1}\ e^{-} = \begin{bmatrix} \Delta \ln \bar{X} \\ \Delta \ln v \\ \Delta \bar{Y} \end{bmatrix}
$$

11 - Calculate New Values

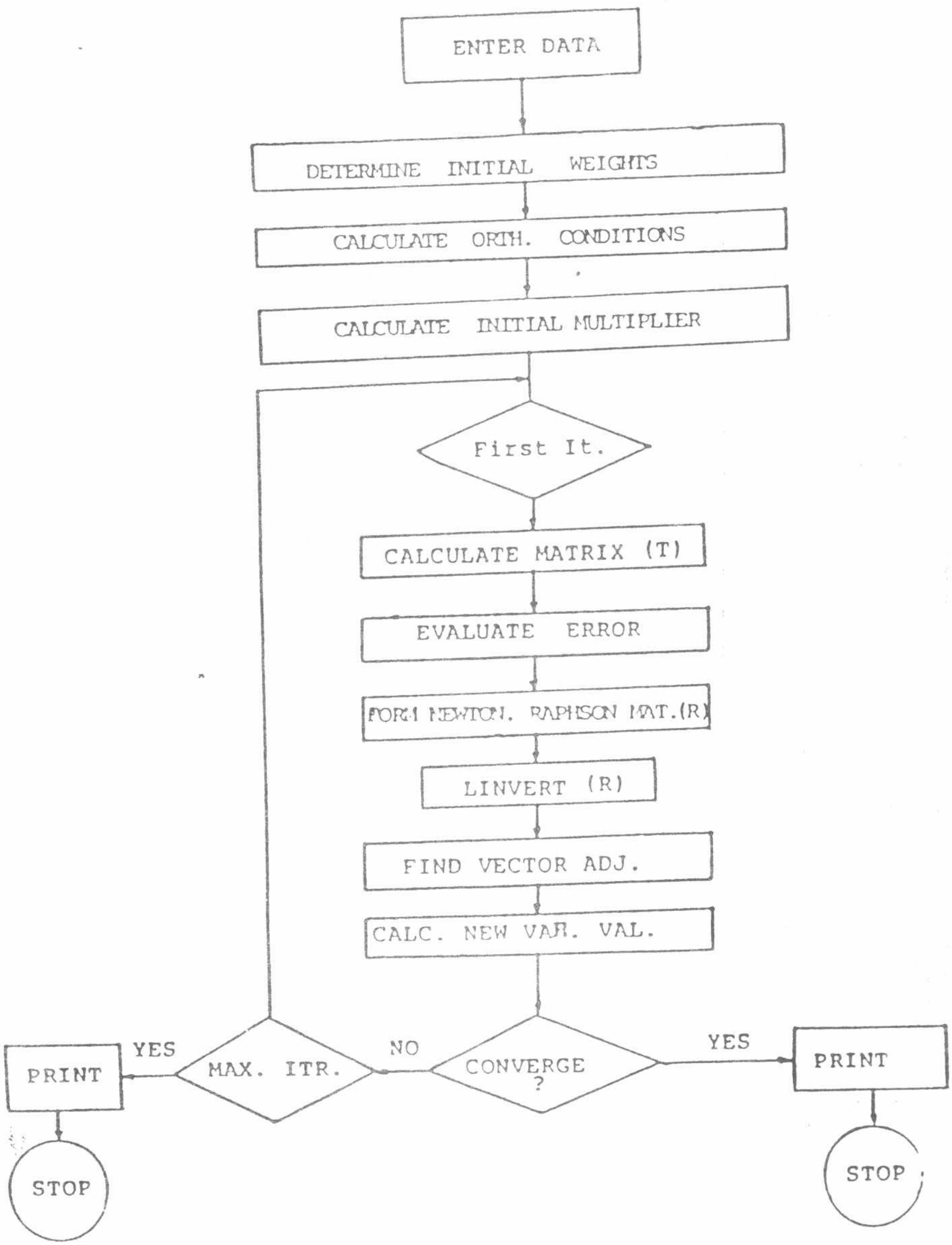
$$\bar{X} = \bar{X} \exp(\Delta \ln \bar{X})$$
$$V = v \exp(\Delta \ln V)$$

12 - HAS SOLUTION CONVERGED TO ACCEPTABLE LEVEL LIMIT ?

YES:- PRINT RESULTS AND STOP
NO :- GO TO STEP 13

13 - MAXIMUM ALLOWABLE INTERATIONS REACHED

YES:- STOP and PRINT RESULTS
NO :- GO TO STEP 5



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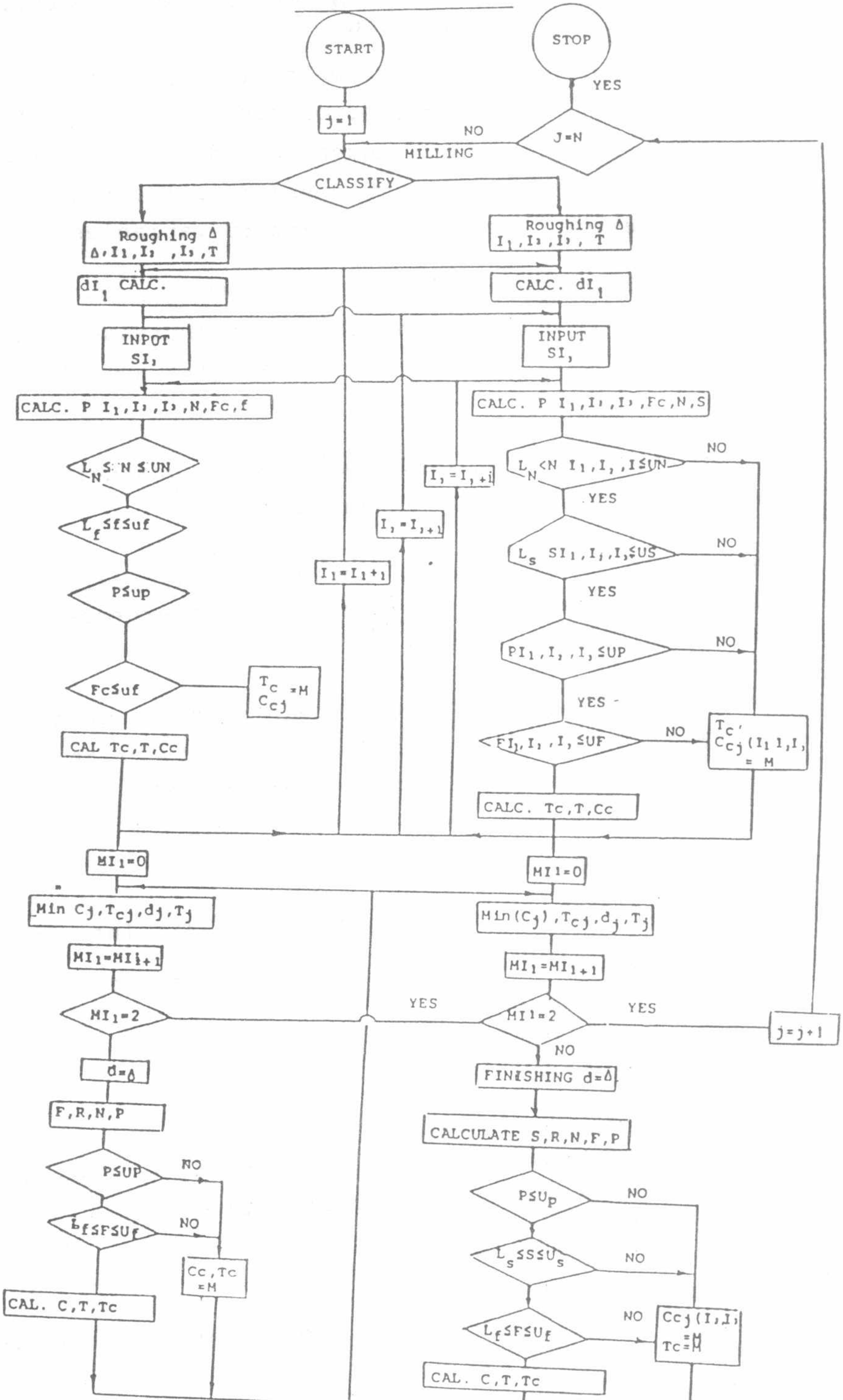
SIMULATION FLOW CHART.



The following draws-backs are to be stated for the application of mathematical optimization.

1 - The size of the problem for multiple cutting operation will result in a huge number of constraints and variable. Consider the case of 10 sequences and 6 cutting variables with 5 constraints for each operation, the result will be a system of 50 constraints and 60 variables.

The resultant Matrices T,R will become very large, the error for matrix inversions with increased No. of iterations will affect the computational accuracy.

2 - Another problem known as the degree of difficulty which arises with no of variables exceeding constraints, this is almost the case for our problem.

This means that also the degree of difficulty which increase for Multiple cutting - and convergence will take longer and longer times.

3 - There are many practical constraints in the metal cutting that will further increase the difficulty of the problem. For instance, in the roughing operation the no. of paths must be integer.

$$\frac{\Delta}{d} = I$$

Constraint (14) is another example for δ = Integer.

4 - In some M/CS the speeds and feeds are not stepless so that

$$UF \geq F \geq L_F$$

$$UN \geq \frac{12}{\pi} VD^{-1} \geq L_N$$

Will not ensure feasible solutions.

All the above remarks creates a necessity to develop and easier and more practical algorithm to solve the multiple-cutting problem.

SOLUTION By SIMULATION

The simulation technique developed depend on the following facts:-

1 - The performance index has its optimal

. . .

2 - Performance index is a convex function.

3 - The optimal value of the overall performance index is the sum of optimal value of single cutting problem with given constraints.

The operations were classified as turning or Milling-each operation can be either roughing or finishing.

01) The Number of paths Index $I_1 = 1,2,\ldots\ldots R_1$

02) The Number of feeds Index $I_2 = 1,2,\ldots\ldots R_2$

$$R_2 = \text{No of feeds}$$

If R_2 is not stated - its is entered as

$$R_2 = \bar{R}_2$$

and $F_1 = F\ Min$

$$F_{I_2} = F\ Min + (I2-1)\left(\frac{F\ MAX - F\ MIN}{R_{2-1}}\right)$$

03) The Number of Tool life iterations I_3

T_{Min} Minimum permissible life,

T_{MAX} MAX. permissible life,

$$T_{I_3} \quad T\ MIN + (I_3-1)\ \frac{T\ MAX - T\ MIN}{R_{3-1}}$$

04) J = No of Tools.

$$J = 1,2,\ \ldots\ldots\ldots\ldots\ldots\ldots,\ M$$

4 - CONCLUSION:-

Application of both techniques for problem (6), (10) and the multiple problem of the combined operations gives the following results:-

	G.P				HEURISTIC			
	C_c	F,S	Vc	Comp. Time sec	C_c	F,S	Vc	Comp. Time
Turning	0.6327	0.005	252	6	0.643	0.005	250	4
Milling	2.8090	0.075	131	1.2	2.953	0.075	140	4
Multiple	3.442	-	-	40	3.596	-	-	16

This proves the validity of Heuristic simulation tech. It should be noted that if V_c is incremented (I_3) to more points

The total cost is higher in the simulation Mech. by app. 3% - however the computational time is reduced to 40%. Of course with more processes (this case N = 2 Only). The simulation method will be more practical.

FUTURE WORK

The only performance index we considered was the cost- however other performances may be considered such as maximum productivity, Minimal wear, etc. Also in the cost function the power consumption cost may be included this will certainly improve the total cost equation.

The system configuration with data base and the file constructions and soft-ware is a very interesting subject that we did not yet managed to design as integral system.

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