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FEW IMPROVEMENTS ON THE CHARACTERISTICS OF $\bar{X}$ AND R CONTROL CHARTS

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ABSTRACT

The $\bar{X}$ and R control charts have been used as an effective tool for carrying out process control from the standpoint of statistical quality control. However, the 3σ method is not so powerful in testing arousing a feeling of dissatisfaction particularly when controlling production processes. In other words; when process shift is not so great, the $\bar{X}$,and R control charts does not have a strong power of test and it is sensitive to dispersion, showing the possibility of carrying out wrong control.

Hence, the purpose of this paper is to study the characteristics of process adjustment on the basis of the $\bar{X}$ and R control charts, and we have proposed the run method as a measure to remove these defects of the charts.

1. THE CONTROL CHARACTERISTICS OF THE $\bar{X}$ CHART

When the control characteristics of control charts are studied from the statistical standpoint, stress must be laid on the research on their power of test and average run length.

Measurement, sampling and other errors included in the standard deviation in the control charts are usually treated as negligible. No action is done to correct these errors when controlling given processes. However, they frequently become so large as not to be neglected and make control activities themselves fruitless [2].

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Here, we differentiate " actions to do " and " actions not to do and let σ_p denote process dispersion corresponding to the former and σ_M signify that corresponding to the latter.

In Fig. 1, let D denote the interval between target value μ and action limit ,then we have [5] ;

$$D = \beta \sigma' = \beta \sigma_M / \sqrt{n} \quad (1)$$

and

$$\delta' = \mu' - \mu = \delta \sigma_p \quad (2)$$

In ordinary charts,

$$D = B.\sqrt{(\sigma_p^2 + \sigma_M^2/n)} \qquad \text{and } B = 3.$$

On the other hand ,in a chart drawn up according to eq. (1), the test power P , when only the upper control limit is given , is expressed as follows;

$$P = \Phi(-B + \delta\sqrt{n} / \sqrt{(\sigma_M^2/\sigma_p^2)}) \quad (3)$$

Fig.2 shows some of the results of calculation.

For control schemes, suppose that μ denotes the target value and that action (compensation) is done is proportion to the deviation which results when the $\bar{x}_i$ plotted at the time i is located on or outside the control limit, and also suppose for simplicity that the effects of action appear instantaneously.

Since corrective action is due to $\bar{x}_i$ located outside the limit, the amount of compensation should be larger than required actually. As shown in Fig. 1, the distribution of the $\bar{x}_i$ serving as a standard for compensation constitutes part of N (μ' , σ_M^2).

Therefore ,the population mean μ_i is given by the following equation [4].

$$\mu_i = \mu' + \frac{\frac{\sigma_M}{\sqrt{2\pi n}} \int_{(D-\mu')/(\sigma_M/\sqrt{n})}^{\infty} u \cdot e^{-u^2/2} du}{\sqrt{1/2\pi} \int_{(D-\mu')/(\sigma_M/\sqrt{n})}^{\infty} e^{-u^2/2} du} \quad (4)$$

Using equations (1),(2) and (4) , we get

$$\mu_i = \left[\delta + \frac{\frac{1}{\sqrt{2\pi n}} \sqrt{\sigma_M^2/\sigma_p^2} \cdot e^{-\{B - \delta\sqrt{n/(\sigma_M^2 + \sigma_p^2)}\}^2/2}}{\Phi\{-B + \delta\sqrt{n/(\sigma_M^2/\sigma_p^2)}\}} \right] \sigma_p \quad (5)$$

When the process mean shifts in a step form at the time i=0, we will find it stochastically. Suppose that μ = i* denotes a state of its being corrected by action . Its cahnge at the time i=0 is signified by $\delta_0 \sigma_p$ where the target value μ = 0 .

Denoting ,by P_0 ,the power of test at that time, we obtain;

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$$E(\mu^*) \doteq \delta_o \sigma_p (1-P_o) + \delta_1 \sigma_p P_o$$
$$= [\delta_o(1-P_o) + \delta_1 P_o]\sigma_p \qquad (6)$$

Where $\delta_1 \sigma_p$ is the difference between the process mean at the time i = 1 and the target value.

Thus, we also get;

$$E(\mu_2^*) = \left[\delta_o (1-P_o)^2 + \delta_1 P_o \{(1-P_o)+(1-P_1)\} + \delta_2 P_1 P_2\right] \sigma_p \qquad (7)$$

$$E(\mu_3^*) = \left[\delta_o (1-P_o)^3 + \delta_1 P_1 \left\{(1-P_o)^2 + (1-P_o)(1-P_1) + (1-P_1)^2\right\}\right.$$
$$\left. + \delta_2 P_o P_1 \left\{(1-P_o) + (1-P_1) + (1-P_2)\right\} + \delta_3 P_o P_1 P_2\right] \sigma_p$$

etc...

Suppose that compensation is carried out in proportion to the difference between μ_4 and the limit and that K stands for control gain, we obtain using eq. (8), [1] ;

(a) For $\delta_{i-1} > 0$

$$\delta_i = \delta_{i-1}(1-K) - K\left\{\frac{\frac{1}{\sqrt{2\pi n}} \cdot e^{\{B-\delta_{i-1}\sqrt{n/(\sigma_\mu^2/\sigma_p^2)}\}^2/2}}{P_{i-1}} - \frac{B}{\sqrt{n}}\right\}\sqrt{\frac{\sigma_\mu^2}{\sigma_p^2}} \qquad (9)$$

(b) For $\delta_{i-1} < 0$

$$\delta_i = \delta_{i-1}(1-K) + K\left\{\frac{\frac{1}{\sqrt{2\pi n}} \cdot e^{-\{B-|\delta_{i-1}|\sqrt{n/(\sigma_\mu^2/\sigma_p^2)}\}^2/2}}{P_{i-1}} - \frac{B}{\sqrt{n}}\right\}\sqrt{\frac{\sigma_\mu^2}{\sigma_p^2}} \qquad (10)$$

where

$$P_{i-1} = \Phi\{-B + |\delta_{i-1}|\sqrt{n/(\sigma_\mu^2/\sigma_p^2)}\} \qquad (11)$$

In the case of compensation carried out in proportion to the difference between μ_i and the target value $\mu = 0$,$B/\sqrt{n}$ becomes null.

Examples of $E(\mu_i^*)$ concerning the above two cases are shown in figures 3 and 4 respectively ,where i is equal to the average run length from the time when a step-form shift $\delta_o \sigma_p$ occurs in the process through the time when it reaches $\delta_i \sigma_p$.

A problem facing process control carried out according to the $\bar{X}$ chart is that the chart exerts only a small test power when process shift is on a small scale. But making B smaller than 3 poses a problem of errors of the first kind. To cope with this, appropriate choice of σ_M^2/σ_p^2 will make it possible to provide a necessary power of test.

Another problem concerns the estimation of the shifted process

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mean μ_i. As stated above, bias toward overestimation is unavoidable. Hence, we have mapped out two control schemes one with too much to be controlled. Improvements are possible if K is properly chosen. But checks are needed at later times if K is to be empirically decided on through close analysis of the characteristics of each process.

2. THE CONTROL CHARACTERISTICS OF THE R CHART

The target value of process dispersion is set according to standards, process capability, economic restrictions and other factors. Dispersion control often requires fundamental actions, rather than a problem of setting as in the case of mean control. Therefore, condit ons differ but we shall consider this problem for a while.

Let R_p calculated from σ_p employed as a central line in the chart and suppose that action is done to correct$(R_i - R_p)/R_p$ to 0 at the time when the plotted R_i gets out of the limit and that it is completed at the time $(i+1)$, that is;

$$\frac{R_{i+1} - R_p}{R_p} = (1 - k)\frac{R_i - R_p}{R_p} \qquad (12)$$

where k means effect of corrective action, k=1 in ideal cases and K < 1 when action is inadequate.

Using equation (12), we have;

$$\frac{\sigma_{i+1} - \sigma_p}{\sigma_p} = (1-k)\frac{\sigma_i - \sigma_p}{\sigma_p} \qquad (13)$$

Let $D = \sigma_i / \sigma_p$ we also have ;

$$D_{i+1} - 1 = (1 - k)(D_i - 1) \qquad (14)$$

where k is equivalent in form to control gain in the case of ratio control.

As the amount to be controlled depends on the test power of the R chart, the expectation is given as follows [6] ;

(a) For k =1 ,we have a simple equation of

$$E(D_{i+1} - 1) = (1 - k)(1 - P_0)^i \qquad (15)$$

(b) For $k \neq 1$, since$(R_i - R_p)/R_p \neq 0$ even if action is done, we get

$$E(D_1 - 1) = (D_0 - 1)\left[(1 - P_0) + (1-k)P_0\right] \qquad (16)$$

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$$E(D_2-1)=(D_0-1)\left[(1-P_0)^2+(1-k)P_0(1-P_1)+(1-k)P_0(1-P_0)+(1-k)^2 P_0P_1\right] \quad (17)$$

where P_i is a power to $(D_0-1)(1-k)^i$

Suppose that dispersion ratio $\sigma'_P/\sigma_P=D$, $R/\sqrt{\sigma_P^2+\sigma_M^2}=\omega$, the limit interval is B sigma ,the power of test P is given by;

$$P = P_r\left\{\omega \geqslant \frac{(d_2+B\cdot d_3)\sqrt{\sigma_P^2+\sigma_\mu^2}}{\sqrt{\sigma_P^2+\sigma_\mu^2}}\right\}$$

$$= P_r\left\{\omega \geqslant (d_2+B\cdot d_3)\sqrt{\frac{1+\sigma_M^2/\sigma_P^2}{D^2+\sigma_M^2/\sigma_P^2}}\right\} \quad (18)$$

The results are shown in Fig .5.

3.THE RUN METHOD

We shall consider control scheme as follows ; an $\bar{X}$ outside the limit does not mean action , and when the successive $\bar{X}$ is plotted in the same region, action is taken.

If the first kind error is set at 0.0027 to compare with the ordinary 3σ chart,it follows that the limit only needs be set to be 1.79σ . When action is done at the run length 3, the limit is equal to 1.20σ. But the run length 2 will be better for practical purposes.

Fig.6 indicates a power of test in the case in which only the mean shifts while fig.7 shows that in the case of the dispersion only changing. Figures 6 and 7 shows the case of $\sigma_M^2/\sigma_P^2=1$.

Obviously, this method is more sensitive to the mean shift than the ordinary 3σ chart.

For control schemes, action is to be done to the target value μ when $\bar{x}_i$ and $\bar{x}_{i+1}$ appear on or outside the same limit in succession . Although the $\bar{x}$ serving as a standard for control follows the truncated distribution as in the case of item 1 mentioned above,the bias is rather small because B= 1.79, [3].

From the standpoint of only detecting its difference from the ordinary 3σ chart,we shall consider the problem on the assumption that $E\left[(\bar{x}_i+\bar{x}_{i+1})/2\right]=\mu$ to simplify the calculation.

(a) Let $\delta = \sigma_p$ stand for the shift in process mean for i = 1 . Since no action is done for i=1 ,we have;

$$E(\mu_1^*) = \delta_o \sigma_p \tag{19}$$

(b) For i=2 or is larger ,we get;

$$E(\mu_2^*) = \delta_o \sigma_p \left[(1-K) P_o^2 + P_o(1-P_o) + (1-P_o)P_o + (1-P_o)^2 \right] \tag{20}$$

Let P_j denote a power of test for the shift $\delta_o \sigma_p (1-k)^j$ in process mean,then we also have;

$$E(\mu_3^*) = \delta_o \sigma_p \left[(1-K) P_o^2 P_1 + (1-K) P_o^2 (1-P_1) + P_o (1-P_o) P_o + P_o (1-P_o)^2 + (1-K)(1-P_o) P_o^2 + (1-P_o) P_o (1-P_o) + (1-P_o)^2 P_o + (1-P_o^3) \right] \tag{21}$$

etc.....

Fig.8 shows the results of calculation.

Under the run method, overall correction is somewhat faster than in the 3σ chart. However ,as no action is done at the initial plot, the method does not display a fine performance for i=1.

SUMMARY

With respect to the ratio control,using control charts ,of production processes where there are many cases in which, compared with changes in process itself, other changes are hard to be neglected, we have studied the control characteristics of the control limit formula not using the changes in process itself as a standard.

In order to decide on the amount to be controlled, it is necessary to accurately estimate the shifted process mean. However, as sample size is generally small and estimation is liable to be accompanied by a bias, some empirical steps must be taken anyway . Furthermore, statistical checks must be conducted on observed values after control.

When process shift is not so great,the $\bar{X}$ and R control charts does not have a strong power of test and it is sensitive to

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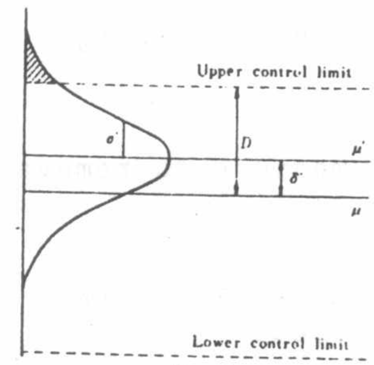


Fig. 1

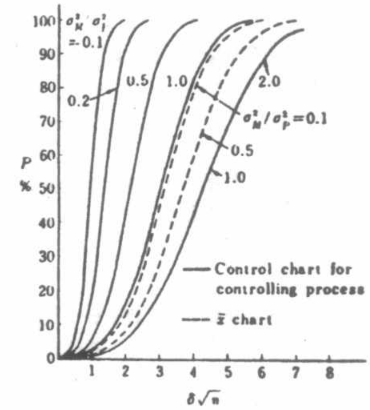


Fig. 2

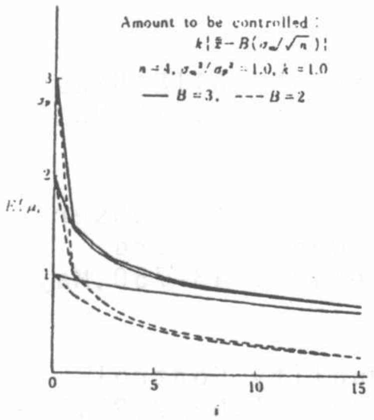


Fig. 3

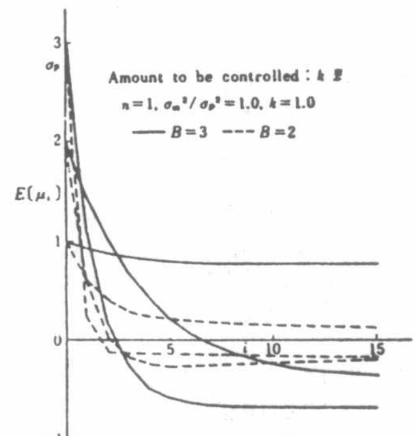


Fig. 4

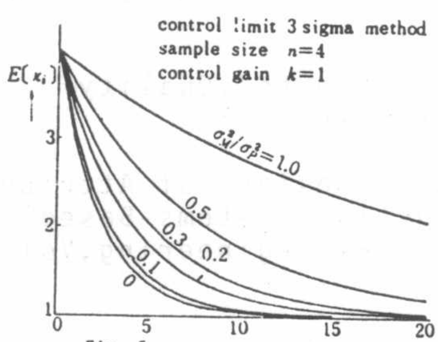


Fig. 5

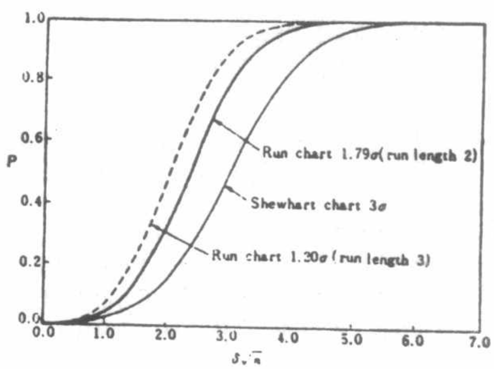
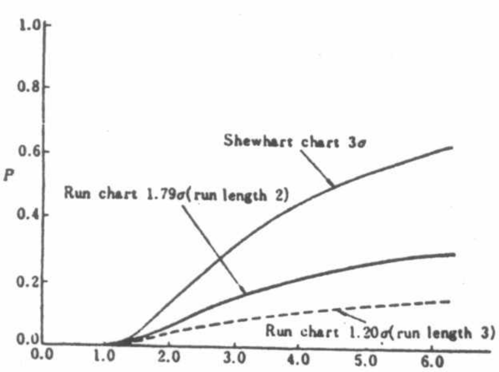
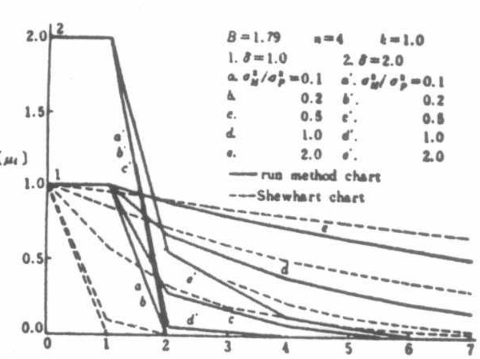


Fig. 6





dispersion, showing the possibility of carrying out wrong control.

Hence, we have proposed the run method as a measure to remove these defects of the control charts.

If it is possible to shorten the sampling intervals , the demerite of the run length 2 will be alleviated.

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