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Mathematical Models For Integrated Production Planning As Application in Inventory Control

Dr. Eng. Farouk Mohammed El. Said Rashed

ABSTRACT

In mechanical industry products are usually manufactured by assembling a large number of subcomponents. The production and stocking of such components are often of much greater significance then the assembly and stocking of final products. The problem is set within a multi-criteria framework, and the consequences of the tactical decisions are presented to the decision maker as goal outcomes.

The methodology involves embedding relatively simple models within a larger and more comlex model. The large total model is a simulation model which uses the optimal decisions of the inventory model to determine the strategic concequences for each important criterion. The methodology was applied to an inventory system of a manufacturer of spare parts for cars.

The results of that application are reported in some detail. Of particular interest is the comparison of two inventory models: the economic order quantity model and a stochastic inventory model.

These two models are compared on the basis of their performance in producing desirable strategic consequences.

Associate Professor, Production Engineering Dept., Faculty of Engineering & Technology, Helwan University Cairo.

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INTRODUCTION

Inventory control is a total problems where management is interested in utilizing its resources in the best way. This is the strategic problem. But an inventory control system has many subsystems items to be controlled in the best way ... these are tactical problems. Obviously, strategic and tactical problems are interdependent, and one should not be considered without concern for the other. This fact is often neglected in system today because no relationship has been established between the tactical and strategic decisions. Tactical decisions are made without considering the strategic consequences; but once the tactical decisions have been made, the strategic performance is implicitly determined.

Numerous inventory control models exist which minimize the costs incurred for each item, considering only that item. Based on estimates, of the cost coefficients involved, this gives a solution to the tactical problem of how to control each item. When such models are used in practice, often the resulting strategic solutions are for from satisfactory. The real problem is the selection of proper costs. The cost coefficients are estimates of marginal costs which are difficult to obtain even from historical data, Estimates of future marginal costs will normally require some knowledge about the reseclting outcome levels, but these are strategic consequences, which are not considered in treditional inventory control methods.

CONCEPTUAL FRAMEWORK

Our method for strategic decision-making has been applied to an inventory system consisting of approximately 8200 parts that are used in the production of cars (Naser Car Company). Production of the cars is done in batches, called runs, according to a production schedule. The sales rate for each car fluctuates, and for all practical purposes we can assume these sales rates to be independent. Because the production schedule is changed every month to reflect realized sales and forecast changes. We therefore make the assumption that the usage of the parts are normally distributed and independent.

Currently the parts are purchased on an order point/order quantity basis, and we can use the framework of the strategic planning method to evaluate alternative purchasing rules for the inventory systems. The inventory performance is measured in terms of the following three goals.

1. Average total inventory.
2. Number of replenishment orders per year.
3. Number of backorders at any points in time.

We want to keep the number of backorders as low as possible because when a backorder is active then a production run is missing a part and might therefore be delayed. Only one backorder is created each time an unsuccessful pull is attempted from the stockroom for any number of the same part.

An inventory analysis was undertaken to find the impact of the purchasing rules used on the performance of the inventory system. The following two studies were performed.

1. Comparison between the theorotical performance of the current purchasing rules and purchasing rules derived from two different inventory models.

2. Analysis of the impact of forecast errors on the performance of the inventory system.

Consequence Calculations For The Total Model:
In order to compare the performance of different inventory control policies we need to express the performance in terms of the sames strategic consequences calculated in a unique way. The following definitions and the discussion of those definitions will help to explain the process for determining the the strategic consequences of an action:

Th e compments for consequence calculations can be expressed as follows:

Decision Vector

$$X = X_1 \ldots X_2 \ldots X_n \qquad (1)$$

Strategic Consequences

$$S(X) = \left[s_1(X) \ldots, s_j(X) \ldots s_p(X) \right] \qquad (2)$$

Goal Vector for Sub system i

$$s^i(X_i) = \left[s^i_1(X_i) \ldots s^i_j(X_i) \ldots s^i_p(X_i) \right] \qquad (3)$$

Data Vector

$$V = (V_1 \ldots V_i \ldots V_n) \qquad (4)$$

Consequence Model

$$S(X) = M(X, V) \qquad (5)$$

In the above formulation, each subsystem i, has a decision vector X_i; a data vector V_i; and a goal vector $s^i(X_i)$.

In this application the goal vector has three components - average inventory level, number of orders; and level of backorders. The data vector has five components - unit price, usage rates, average leadtime, standard deviation of leadtime and the average full size. The decision vector has two components- order point and order quantity. Thus the system vectors X, V and S(X) are really vectors with each component relating to a specific subsystem data as inputs; its output is the vector of strategic consequences for the entire system. Consequences will always be calculated using this total model. However in some cases, these calculations can be tgreatly simplified. In particular under the assumption that the items are independent, the consequences can be calculated as.

$$s_J(X) = \sum_{i=1}^{n} s^i(X_J) \qquad (6)$$

where

$s_1^i(X_i)$ = is the average inventory for part i

$s_2^i(X_i)$ = is the number of orders per year for part i

$s_1^i(X_i)$ = is the number of backorders for part i

Consequence Calculations Using Samples:
In the previous section we outlined a total model for calculating the strategic consequences of an inventory policy. This requires the calculation of tactical decisions and the corresponding consequences for each part. Since the calculation of the tactical decisions for a part involves an oftimization algorithm, the computations can be considerable. Therefore, we perform the consequence calculations for a sample of the parts and extrapolate the result to obtain estimates of the strategic consequences for the total inventory.

The inventory considered here consists of 8,200 parts classified as A, B, C, and D parts. How much each class constitutes of the Egyptian pound usage and the current inventory pound is shown in table (1) Randam samples of 100 A parts, 100 B parts, 100 C parts and 100 D parts were selected for detailed analysis.

First we calculate the three consequence for the 100 A parts. Each of these three consequences is then multiplied by the ratio between the total forecast for all A parts and the forecast for the 100 parts in the sample. In this way, we get estimates of the strategic consequences for the inventory of all A parts. We repeat these calculations for the other three classes and sum the consequences for all four classes.

Table (1)

Class	% of L.E usage	% of inventory L.E.	Number of parts	Sample size
A	87	69	820	100
B	19	26	1280	100
C	6	14	1820	100
D	1	4	4280	100

Data Required:
In addition to reducing the computations involved, sampling greatly reduces the amount pf part data required by the method. For the 400 parts in the samples, the current list prices and leadtimes were retrieved from a data file. The number of monthly pulls and the size of each pull were obtained from a file containing past transactions for the 400 parts covering a six month period. The forecast of the usage for each part is based on the part explosion of scheduled production for six month period. This forecast is revised every month, but for simplicity, a single six month forecast was used.

From the data on monthly pulls, we estimated the average pull size, the mean actual usage and the standard deviation of the usage rate for each part. As is commonly the case in practice the data available to this study does not represent ideal data; but it provides an opportunity to test the usefulness and applicability of the method for strategic planning in inventory control.

EOQ Inventory Model:
The simplest possible mathematical inventory model, based on Economic Order Quantity (EOQ) is included in this study. It is important to note here that the EOQ model is deterministic but it is being applied to a stochastic problem. The only adjustment made is to carry saftey stock. However, the order quantity is not adjusted. The order quantities are determined from Economic Order Quantity (EOQ) formula:

$$q_i = \sqrt{\frac{2\lambda_i}{p_i}\left(\frac{k_2}{k_1}\right)} \qquad (7)$$

$$r_i = L_i + S_i \qquad (8)$$

where for part i

q_i = Order quantity

r_i = Order point

L_i = Average usage during leadtime

σ_i = Standard deviation of usage during leadtime

S_i = Safety stock

λ_i = Average annual usage

P_i = Price

k_1 = Unit inventory carrying cost.

k_2 = Unit ordering cost

Safety stock is often calculated using

$$S_i = C \times \sigma_i \tag{9}$$

where C is a constant determined by a policy decision. Using this safety stock calculation for all parts theoretically implies the same chance of stockouts for all parts during each order cycle.

Table 2 shows how probability of a stockout in each order cycle depends on C.

Table (2)

Probability of Stockouts During Order Cycle

Probability of Stockouts order cycle	C
50%	0
60%	0.5
70%	0.8
80%	1.4
90%	1.7
99%	2.7

In this simple inventory model we do not wish to use the standard deviation from each part. It is known that the demand placed on an inventory system, under some assumptions, is generated by a poisson process. Our situation does not strictly satisfy these assumptions, but we will use the following approximation:

$$\sigma_i = \sqrt{L_i} \qquad (10)$$

Thus we use the following order point calculation:

$$r_i = L_i + C\sqrt{L_i} \qquad (11)$$

Stochastic Inventory Model:

Under the assumptions of normally distributed independent usage rates, this inventory model optimizes the combination of the three consequences used in evaluating the inventory per formance. The only modification made is the translation of expected number of parts on backorder to expected number of backorders. In order to use the continuous review model in a situation where reviews are only made every fifteen days. We must increase the standard deviation of usage during the leadtime, accordingly.

Thus:

$S_1^i(X_i)$ = expected average inventory for part i

$$S_1^i(X_i) = P_i\left[\frac{q_i}{2} + r_i - L_i + \frac{B_i(r_i)}{q_i}\right] \qquad (12)$$

S_2^i = expected number of orders per year for part i

$$S_2^i = \lambda_i / q_i \qquad (13)$$

S_3^i = expected number of backorders for part i

$$S_3^i = B_i(r_i) / q_i\, z_i \qquad (14)$$

where:

$B_i(r_i)/q_i$ = is an approximation of the number of parts on backorder;

Z_i = the average pull size for part i.

The criterion function for each scalar optimization problem is

$$W_i = k_1\left[P\left(\frac{q_i}{2} + r_i - L_i + \frac{B_i(r_i)}{q_i}\right) + k_2\left(\frac{\lambda_i}{q_i}\right) + k_3\left(\frac{B_i(r_i)}{q_i, z_i}\right)\right] \qquad (15)$$

where: k = the unit cost per backorder.

The minimization of this criterion function is an optimization problem in two variables, q_i and r_i. Setting the partial derivative of W_i with respect to q_i equal to zero, leads to the following equation for the optimal value of order quantity q_i^* in terms of r_i.

$$q_i^* = \sqrt{\frac{2}{k_1}\left[k_1 B_i(r_i) + k_2\frac{\lambda_i}{p_i} + k_3\frac{B_i(r_i)}{P_i\, z_i}\right]}$$

Substituting this q_i^* back into the expression for W_i results in a criterion function with one variable r_i which is convex and can be minimized by a simple and fast optimization algorithm. The computation time is important because optimal value of q and r must be found for each part and each set of cost ratios k_2/k_1 & k_3/k_1.

Comparing The Inventory Models:
The strategic consequences that were calculated for the total inventory of 8200 parts will next be used to compare of three inventory control techniques just described. At the same time we shall see how the consequences can be presented to the decision maker in a way that facilitates strategic decision making in inventory control. The inventory control techniques are compared in retrospect using knowledge of the forecasted and the actual usage rates for each part. The order points and order quantities are calculated based on the forecast, while the strategic consequences are calculated using the actual usage rate of each part.

The EOQ model defines for each value of the parameter M, a set of inventory policies that are efficient with respect to that model, each corresponding to a ratio k_2/k_1. Figure 1 shows the possible combinations of the strategic conseauences for M=1.5 and M=3.5.

The ratio k_2/k_1 is known for all the points marked on the curves, and through interpolation it is possible to find the value of k_2/k_1 and M for any desired point near the curves.

By using three values of k_2/k_1 and M in the EOQ model, we can find the tactical decisions corresponding to any particular point.

The stochastic inventory model defines a two dimensional surface of efficient points corresponding to different values of the ratios k_2/k_1 & k_3/k_1. In Figure 2 level curves (a,b) of this surface are shown for constant number of backorders.

It is obvious that the three inventory control techniques perform very differently. As we would expect, the stochastic inventory model can always find solutions that dominate those of the EOQ model, and the EOQ model in turn can produce solutions that dominate the consequence of the current buying rules, i. A closer comparison is made for number of orders per year = 13,000 and expected of backorders = 1450.

Technique	Inventory
EOQ	L.E. 714000
Stochastic	L.E. 385000

ii. For inventory level = L.E. 800.000 and expected number of backorders = 1300

Technique	Number of Order per year
EOQ	7500
Stochastic	4200

iii. For inventory level = L.E. 500.000 and number of order per year=8000.

Technique	Expected Number of Backorders
EOQ	1100
Stochastic	830

In Figure 2, the efficient surface for the stochastic inventory model is shown in broken lines when a perfect forecast was used in the calculation of order points and order quantities. In this case the impact of the forecast error can be seen by comparing the broken lines with the solid lines in Figure 2.

CONCLUIONS:

The proposed method of calculating the efficient surface for an inventory system is a tool which enables managers to deal with the strategic decisions for inventory systems. The efficient surface tells the managers about alternative combinations of the three out come - average inventory level, number of orders per year and the level of beckorders. They can relate these outcome to the overall objectives and operating restrictions of the company. The method will not calculate one magic optimal solution, but it will leave the final strategic decision to the managers and thus make it possible to include their experience and intiution. The method will prove particularly useful in adjusting to changes in the enviroment which effect the inventory system such as the supply of capital, leadtimes and cost changes. The method can be used with any existing or future tactical inventory models, and strategic consequence other than those considered here can be included.

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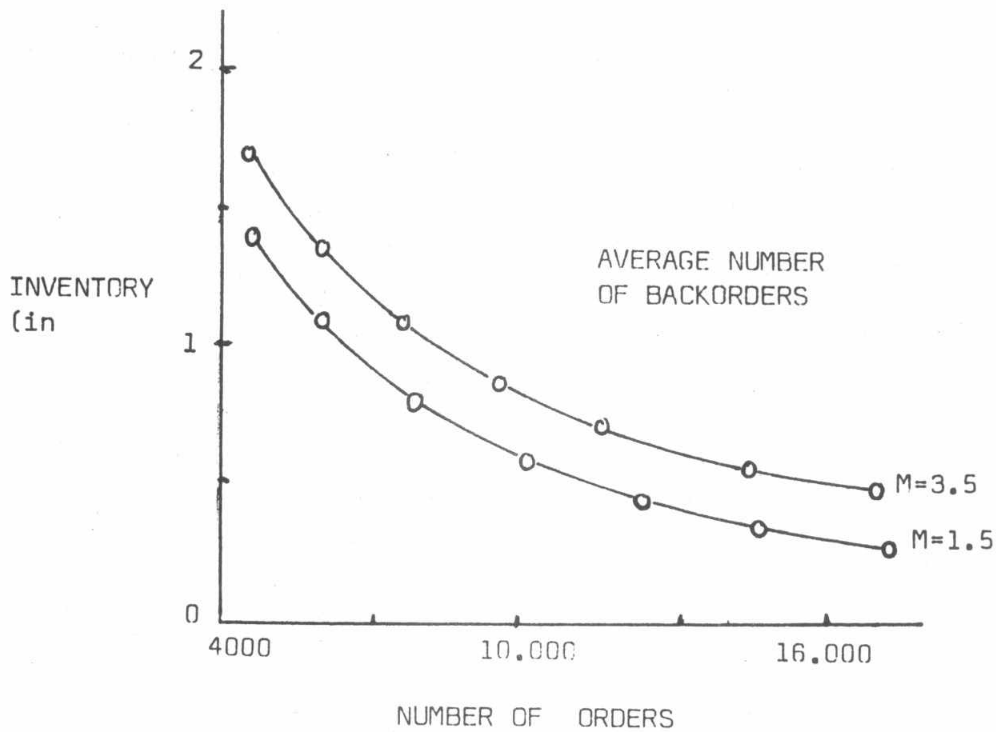

Figure 1. Efficient Surface for EOQ Model.

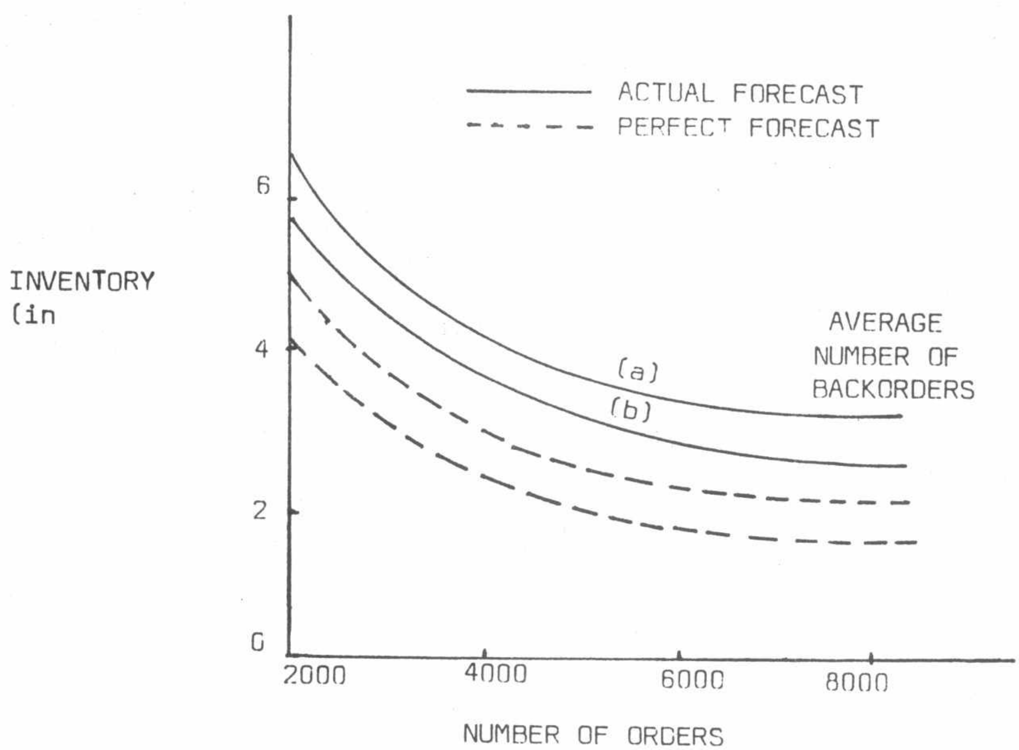

Figure 2. Efficient Surface for Stochastic Model.