MATHEMATICAL MODEL OF VEHICLE
STARTING SYSTEM

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ABSTRACT

For starting the Internal Combustion Engine I.C.E. the motor vehicle must be equiped with a starting system which converts the electrical energy into the mecanical energy needed for putting the I.C.E. in function.

This paper deals with the main characteristics of starting system elements. A linear and non linear mathematical models have been analysed for the starting system under investigation .

INTRODUCTION

The Internal Combustion Engine I.C.E. needs an external force to put it in function. This force is exerted using a starting system(Fig.1).

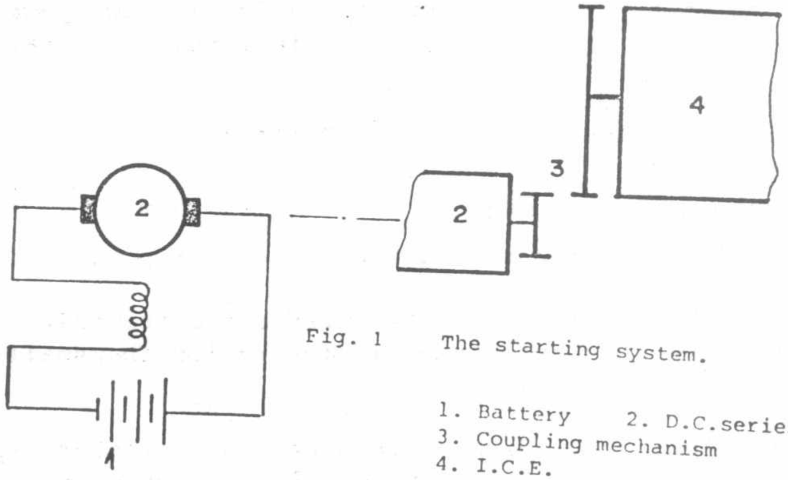

Fig. 1 The starting system.

1. Battery 2. D.C.series motor
3. Coupling mechanism
4. I.C.E.

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The storage battery is used to ensure a high loading current without extreme drop of voltage [1] . The terminal voltage 'V' can be expressed as a function of it's discharge current 'i' as :

$$V = E_B - R_B \cdot i \; ..... \qquad (1)$$

Where : E_B ... the electro-motive force e.m.f. of the storage battery

R_B ... the battery internal resistance.

(Fig.2) shows this dependance for different values of R_B.

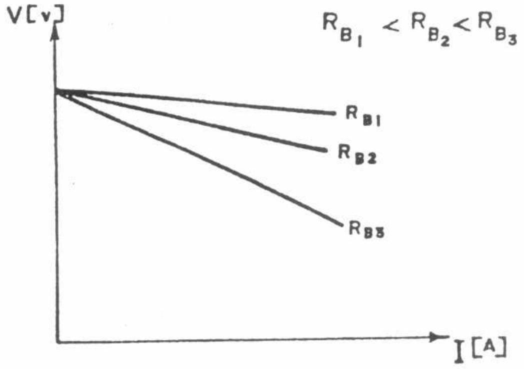


Fig.2 Battery terminal voltage.

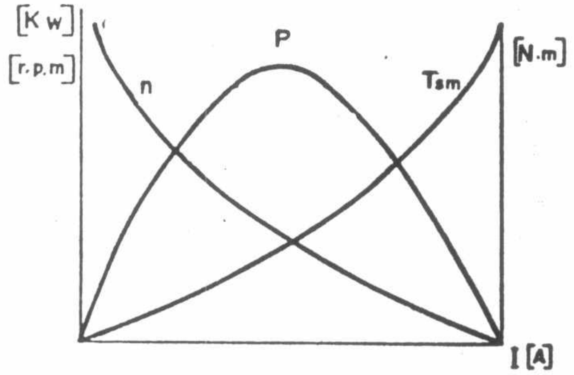


Fig.3 Steady state starting motor characteristics.

The starting motor, D.C. series motor with steady state characteristics shown in (Fig.3), can deliver a high power for a short periods of time. This motor helps the engine to operate under its own power. The mechanical power P needed to rotate the movable parts of I.C.E. is expressed as :

$$P = \frac{T_r \cdot n_s}{9550} \quad KW \; ..... \qquad (2)$$

Where : T_r ... the resisting torque, N.m

n_s ... engine starting revolution r.p.m.

A mesh couple toothed gearing is used to enlarge the transmitted torque from the starting motor. The used transmission ratios is usually about 13 ÷ 17.

The carburation and ignition modern engines need a crankshaft speed of 100 r.p.m. to ensure satisfactory starting [2] , even under the adverse conditions of winter season. Hence, the starting motor must be capable to drive the crankshaft at a minimum speed of 100 r.p.m. and with a driving torque T_{sm} necessary to over come the resisting torque T_r from friction of all movable parts and from the compression. (Fig.4) represents T_{sm} and T_r as a function of the number of revolution n.The Figure shows that n_s must be less than n_c to have reliable function of the starting system.. Considering the worst conditions, that is the resisting torque is constant for a given engine. Generally it is a function of swept volume S, in liters, of any I.C.E.

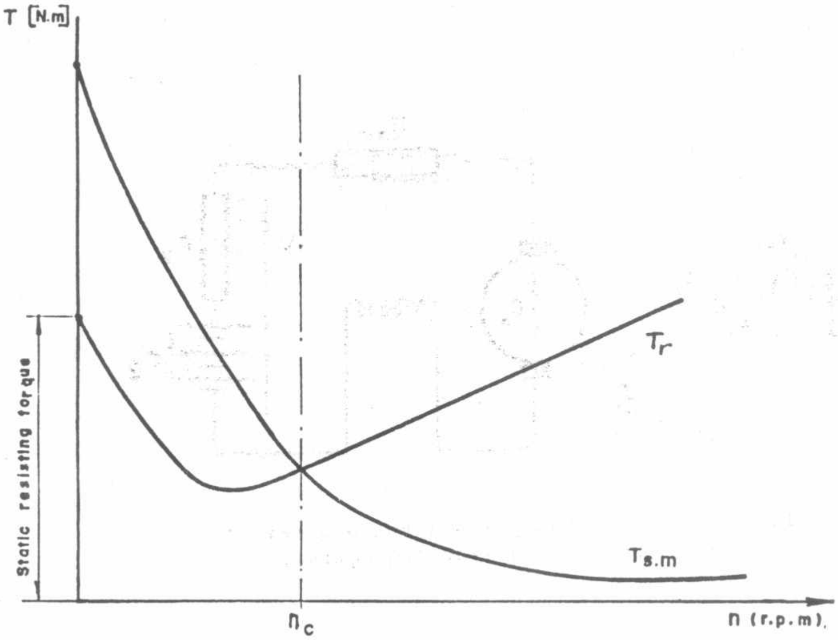


Fig.4 The variation of T_{sm} and T_r with the number of rev.n.

$$T_r = C. S$$

Where : C ... is a constant depend on the engine type.
C = (3.5 - 4.0) for spark ignition engines.
C = (7 - 9) for diesel engines.

As a result of previously briefed explaination of the starting system, it is necessary to formulate a mathematical model to determine the various important features of this system.

ANALYTICAL REPRESENTATION OF STARTING SYSTEM

The mathematical model can be represented by using the following two differential equations(mechanical and electrical one). [3]

$$T_r = T_{sm}(i) - J\frac{dw}{dt} \quad ..... \qquad (3)$$

$$V = e_b(w,i) + R_{sm}.i + L\frac{di}{dt} \quad ..... \qquad (4)$$

Where : J ... the moment of interia of the system N.m.sec^2.
w ... the angular speed .
e_b... the back e.m.f. of the starting motor V
R_{sm}.. the total resistance of the starting motor.
L ... the self inductance of the starting motor windings H

The starting system parameters are clearly shown on the mechanical and electrical schemes (Fig.5). The two differential equations(3)&(4) could be solved using two models (linear and non linear one). The formulation of linear model is based on the following assumption :

- The magnetic flux is linearly dependent on the armature current.
- The armature and field windings self inductance have constant values.
- Iron losses and armature reaction are neglected.

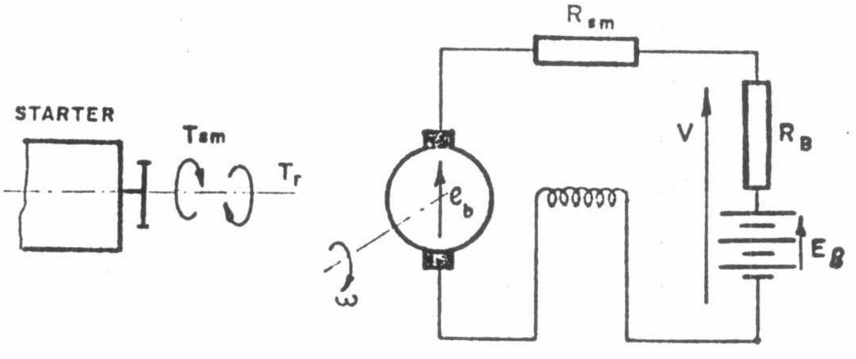


Fig.5 The mechanical and the electrical schemes of a starting system.

Rearange equations(3)&(4) using the mentioned assumptions, then,:

$$J\frac{dw}{dt} = K.i^2 - T_r \quad ..... \qquad (5)$$

$$L\frac{di}{dt} = E_B - R_B.i - K.w.i - R_{sm}.i \quad ..... \qquad (6)$$

For the non linear model also some assumptions are taken into consideration:

- The relation between the air gap flux and current is described by the magnetisation curve(Fig.6). Consequently, the self inductance of the starting motor windings L is not constant and depends on the current.
- Iron losses and armature reaction are neglected.

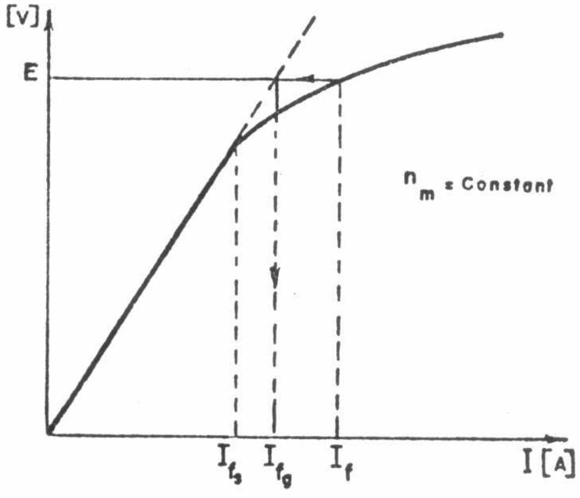


Fig.6 The magnetisation curve.

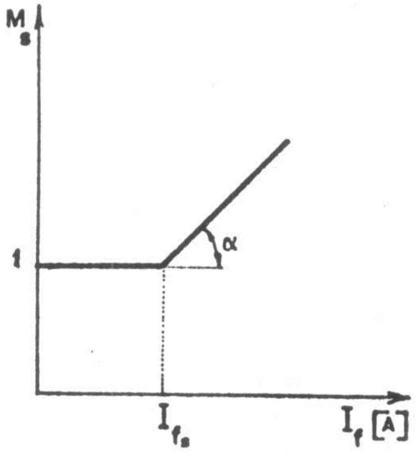


Fig.7 The variation of the saturation factor M_s

It is clear from(Fig.6) that the magnetisation curve departs from linearity at a certain value of the field current due to the saturation. The linear portion is described by the equation:

$$E = R_c \cdot I_f \qquad \text{for } I_f \leqslant I_{fs} \quad \ldots\ldots \quad (7)$$

Where : R_c ... the slope of the magnetisation curve.

At the saturated portion, a saturation factor M_s is defined as :

$$M_s = \frac{I_f}{I_{fg}} = \frac{R_c}{E}\, I_f \quad \ldots\ldots \quad (8)$$

Where : I_{fg}... the field current, on the extention of the linear portion, corresponding to the e.m.f. E for linear dependance.

A simple dependance between M_s and I_f is shown in (Fig.7) which can be expressed as :

$$M_s = 1 + \propto (I_f - I_{fs}) \quad \ldots\ldots \quad (9)$$

Then, $E = I_f . R_c / \; 1 + \propto (I_f - I_{fs}) \quad$ for $I_s > I_{fs}$ ..... (10)

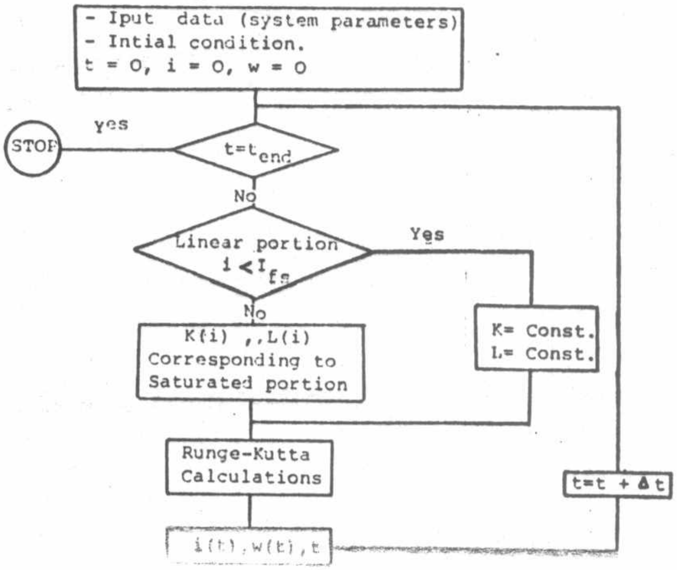
From the above analysis, the magnetisation constant K and self inductance L are given for linear and non linear portions in the following table :

portion	$K \left[\frac{V}{A.\ rad/sec}\right]$	$L\ [H]$
Linear $0 < i \leqslant I_{fs}$	$\frac{60 . R_c}{2\pi\, n_m}$ = constant	$\frac{2\pi a\ K\ N}{z\ p}$ = constant
Non linear (saturated) $i > I_{fs}$	$\frac{60 . R_c}{2\pi\, n_m} \left[1 + \alpha (i - I_{fs})\right]$	$\frac{2\pi a\ K\ N}{z\ p} \cdot \frac{1 - \alpha I_{fs}}{1 + \alpha (i - I_{fs})}$

N,a,z,p are the construction parameters of the starting motor.

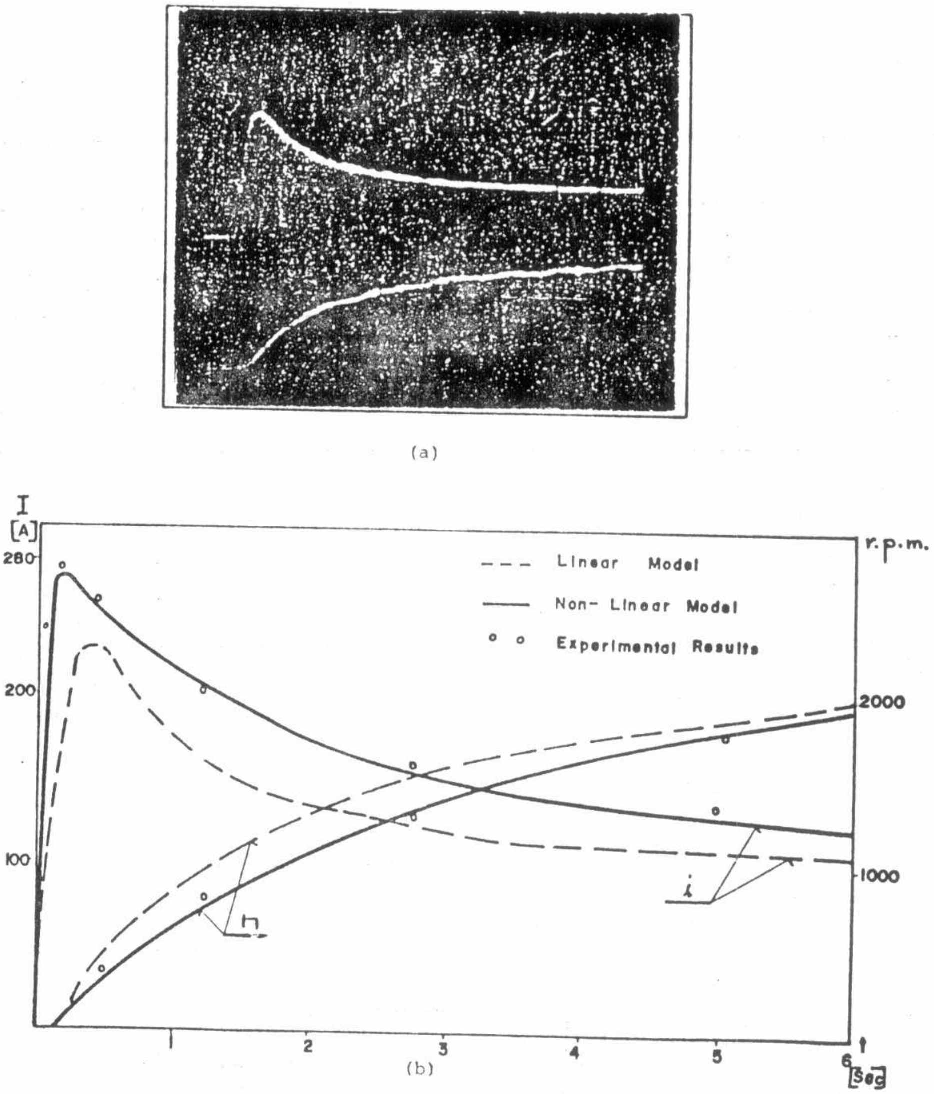
COMPUTATIONAL PROGRAM AND RESULTS

The starting system differential equations are deduced by substituting the values of K and L from the table into the equations (5)&(6). The flow chart of the computer program used for solving the starting system differential equations is represented in (Fig.8)



The following remarks are observed from the experimental and computational results shown in (Fig.9a,b):

- The peak current in the case of the non linear model is approximately 20% higher than in the case of the linear one.
- The steady state current for the non linear model is about 7% higher than that for the linear one.
- The rate of change of the motor speed for the linear model is higher than that for the non linear one.
- The computer results for motor current and speed obtained by adopting the non linear model are reasonable compared with the experimental one. The small difference between both sets of results may be due to the armature reaction which was neglected during the analysis.



(a)

(b)

Fig.9 The experimemtal and computational results.

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CONCLUSION

The analytical representation of the starting system used with the motor vehical depends essentially on the properties and characteristics of the individual elements of this system (battery, D.C. series motor, and engine).

In this work, a linear and non linear mathematical models for the starting system under investigation have been analysed. In the non linear model, the saturation in the magnetisation curve is approximated by a simple and easy computation method. It was found that the results of the starting characteristics obtained using the non linear model are very close to the experimental results.

The mathematical model and analytical representation of the starting system enable to study and analyse the effect of the system parameters on its transient behaviour.

REFRENCES

[1] JU.M. Galkin "Automotive Electrical Equipment"
[2] William H. Crouse "Automotive Electrical Equipment"
[3] A.H. Abdelhamid "M.Sc. Thesis: Transient Analysis of D.C. Drive"