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MILITARY TECHNICAL COLLEGE

CAIRO - EGYPT

FINITE ELEMENT PREDICTION OF UNSTEADY HEAT TRANSFER IN HOLLOW BLOCKS

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ABSTRACT

This paper presents a numerical method for describing the transient heat transfer through hollow cement blocks via the employment of finite element technique. The block under consideration is assumed to be of rectangular cross-section and composed of two parts, a solid inside which an enclosure (space) is located. The outer surface of the block is exposed to the solar flux. The temperature distribution inside the block as function of space coordinates and its variation with time are the unknown variables of the problem. This leads to a parial differenial equation in the temperature. Finite element technique is used to obtain the solution of the problem for different boundary conditions. The effect of the presence of the enclosure is investigated by comparing homogeneous and hollow blocks results.

NOMENCLATURE

A	area of the triangular element, m^2
a	length of the enclosure, cm
C	specific heat, kJ/kg °C
[C]	capacity matrix
{F}	nodal force vector
f,g	temperature and its gradient functions respectively.
[K]	stiffness matrix
K	thermal conductivity, W/m °C
L_1	length of the cement block under investigation, cm
L_2	height of the cement block, cm
N	interpolation function
n	unit vector
q	solar heat flux, W/m^2
T	temperature, °C
t	time, s

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Greek :

ρ	density, kg/m^3	α	fraction of energy absorbed
Ω	bounded domain	Γ	domain boundary, Fig. 3

Superscripts :

e	element
$\cdot$	derivative with respect to time

Subscripts :

1,2,3,4	refers to sides, Fig. 3
a	air
i	initial
sol	solid

1. INTRODUCTION

The problem of heat transfer through hollow cement blocks becomes of fundamental importance at the present time. The subject has been motivated by recent orientations in the field of building material technology, which incline towards the use of hollow cement blocks that have many advantages. Among these advantages are being light, noise absorbent, low cost, etc. The work reported here was undertaken towards the study of heat transfer through hollow cement blocks in both solid and enclosure. It is assumed that the top surface of the block is subjected to solar heat flux while its bottom is maintained at the interior environment building temperature. The intent of the present study is to obtain a numerical solution for the temperature distribution through the block in multi-dimensional space.

Basically, the problem is treated as a two-dimensional transient analysis. Heat absorbed by the block from the top surface or librated to the interior of the building depends mainly on the thermal properties of the block and enclosure. A differential equation formulation is convenient for inhomogeneous media. The solution region is thus for two different media, mainly the solid portion of the block and the air entraped inside the enclosure. The complexity of this equation is such that the analytical solutions exist for only extremely simple models and homogeneous media [1,2]. Moreover, many of the practical applications include various kinds of boundary and initial conditions which provide additional factors that further complicate the mathematical treatment. Therefore, a numerical method has to be employed based on an accurate model which is essential for this type of analysis. Accordingly, difficulties associated with these features can be easily overcomed by finite element formulation. The formulation allows the material properties to be different in both solid and the enclosure of the cement block. The importance of heat transfer problems through solids has attracted a large number of investigators who have tried a variety of different techniques to obtain the solution. The literature covers both experimental and theoretical work in this area, but it deals

with homogeneous material. In particular, natural convection inside the enclosure was investigated for different configurations and heated surfaces of the enclosure, [3,4,5,6]. In the present work, the block is assumed to be heated from the top surface, and cooled from the bottom; this leads to a negligible convective heat transfer. Radiation inside the enclosure is also neglected. Thus, the only prevailing mode of heat transfer is conduction through the hollow cement block.
The objective of the present analysis is to develope a computational model to solve for the temperature history inside the cement block due to the incident solar flux on the top surface. The effect of the enclosure inside the cement block on the temperature distribution is indicated. Following this introduction, the physical problem is developed and certain fundamental relationships are presented in the next sections.

2. BASIC EQUATIONS AND FINITE ELEMENT FORMULATION

2.1. Basic Equations

When heat is being transferred through a medium, the basic governing equation for the transient heat conduction in two dimensions can be expressed in the typical form :

$$\partial(k(\partial T/\partial X))/\partial X + \partial(k(\partial T/\partial Y))/\partial Y = \partial(\rho C T)/\partial t \quad (1)$$

Equation (1) refers to the balance of the thermal energy, where T is a continuous temperature function of the global space (X,Y) and time (t), Eq. (1) is definde over simply connected domain Ω bounded by the boundary Γ .
The boundary-value problem for Eq. (1) involves the solution of a function T such that

$$T = f \qquad \text{on} \quad \Gamma_1 \qquad (2)$$

$$n^T k \nabla T = g \qquad \text{on} \quad \Gamma_2 \qquad (3)$$

where f and g are given functions of (X,Y), n is the outward unit vector normal to Γ . The boundary surfaces Γ_1 and Γ_2 are subregions of Γ . More detailed description of the boundary and initial conditions will be discussed in the section where the numerical examples are presented.

2.2. Finite Element Formulation and Discretization

The finite element equations are drived using the space discretization by the general weighted residual method where the weighting functions are taken as the interpolation functions. The temperature field is approximated by appropereiate piecewise linear polynomials. Thus by following the conventional Galerkin procedure [7,8,9] , the variation of the temperature throughout the domain of interest, Ω, is approximated in terms of the nodal values T_s as :

$$\bar{T}\,(X,Y,t) = \sum_s N_s(X,Y)\; T_s(t)\;, \qquad s=1,2,3 \tag{4}$$

where N_s are the usual interpolation functions, [9], defined piecewise element by element and given by :

$$N_s = (1/2A)\,(a_s + b_s X + C_s Y)\;, \qquad s= 1,2,3 \tag{5}$$

where for s=1 ,

$$a_1 = X_2 Y_3 - X_3 Y_2 \tag{6}$$

$$b_1 = Y_2 - Y_3 \tag{7}$$

$$C_1 = X_3 - X_2 \tag{8}$$

and the other components being given by cyclic permutation of the subscripts in the order 1,2,3 as depicted in Fig.1. The element area A is given by :

$$2\,A = \mathrm{Det}\begin{bmatrix} 1 & X_1 & Y_1 \\ 1 & X_2 & Y_2 \\ 1 & X_3 & Y_3 \end{bmatrix} \tag{9}$$

If the approximations given by Eq. (4) are substituted in Eq. (1), a residual is obtained which is then minimized using Galerkin procedure. This requires that the integral of weighted error over the domain Ω must be zero, with the interpolation functions N_r being utilized as the weighted functions, i.e.

$$\int_\Omega N_r\,[\,\partial(k\,(\partial\bar{T}/\partial X)/\partial X + \partial(k\,(\partial\bar{T}/\partial Y)/\partial Y$$

$$-\partial(\rho\, C\,\bar{T}\,)/\partial t]\;\; d\Omega = 0 \tag{10}$$

The application of Green's theorem to Eq. (10), after satisfying the boundary conditions given by Eqs. (2) and (3),

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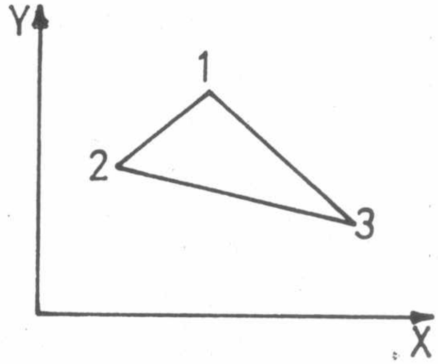

Fig. 1 Triangular element with linear temperature variation.

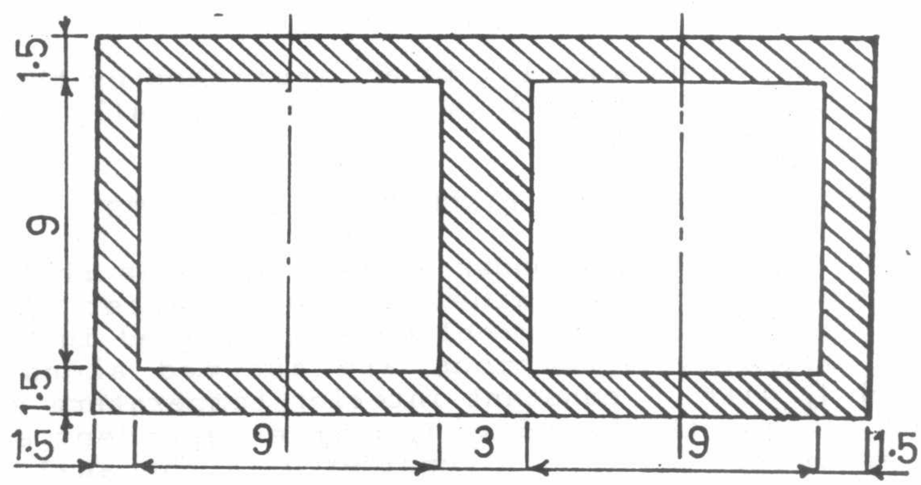

Fig. 2 Actual cross-section of the hollow cement block. (Dim. in cm).

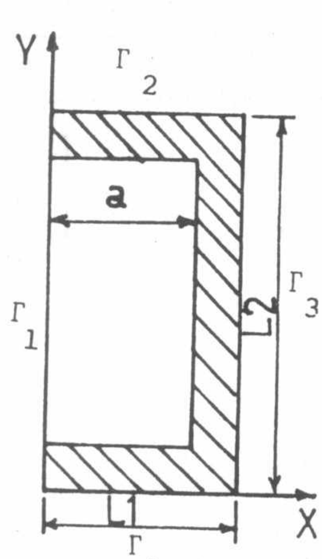

Fig. 3 Domain (Ω) for two-dimensional analysis.

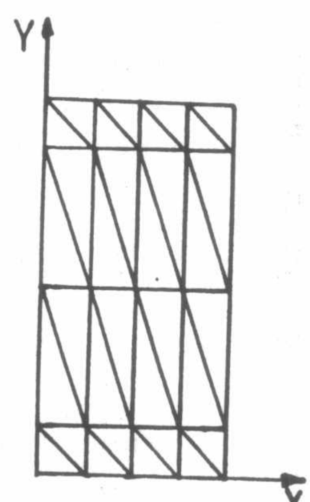

Fig. 4 :Finite element discretization.

leads to the solution in matrix form as :

$$[k]\{T\} + [C]\{\dot{T}\} = \{F\} \tag{11}$$

where

$$k_{rs} = \sum^{e} \int_{\Omega^e} k\,[(\partial N_r/\partial X)(\partial N_s/\partial X) + (\partial N_r/\partial Y)(\partial N_s/\partial Y)]\,d\Omega \tag{12}$$

$$C_{rs} = \sum^{e} \int_{\Omega^e} \rho\, C\, N_r\, N_s\, d\Omega \tag{13}$$

The numerical solution of Eq. (11) is accomplished by the use of the backward difference method as an unconditionally stable time integration scheme [10].

3. SAMPLE PROBLEMS AND DISCUSSION OF RESULTS

The cross-section of the block under investigation is shown in Fig. 2. It is possible to examine only one quarter of the section, due to symmetry of the problem, as presented in Fig.3. Non-conducting boundary conditions are assumed along the boundaries Γ_1 and Γ_3 with constant interior temperature along the boundary Γ_4. Along the boundary Γ_2 a time-dependent solar heat flux is assumed. The domain considered in the analysis, Fig. 3, is then subdivided into the finite element mesh as indicated in Fig. 4. The values of the required physical properties (e.g. the thermal conductivity, density and specific heat) for both the cement block and the enclosure are given in table (I).

Table (I) : Numerical values of the parameters used.

Parameter	Value	Parameter	Value
k_{sol}	0.232 W/m°C	a/L_1	0.0, 0.25, 0.5, 0.75
k_a	0.027 W/m°C		
ρ_{sol}	1200 kg/m^3		
ρ_a	1.15 kg/m^3	L_1	6.0 cm
C_{sol}	1.045 kJ/kg°k	L_2	12.0 cm
C_a	1.002 kJ/kg°k	α	0.2

Initial temperature has been assumed constant throughout the block. The following initial and boundary conditions are adopted.

a) Initial conditon : $T_i = 20\ ^\circ C$

b) Boundary conditions :

$$\partial T / \partial X = 0 \qquad \text{on } \Gamma_1 \qquad (14)$$

$$q = 800\, \alpha \sin \pi t / (10 \times 3600) \qquad \text{on } \Gamma_2 \qquad (15)$$

$$\partial T / \partial X = 0 \qquad \text{on } \Gamma_3 \qquad (16)$$

$$T = 20\ ^\circ C \qquad \text{on } \Gamma_4 \qquad (17)$$

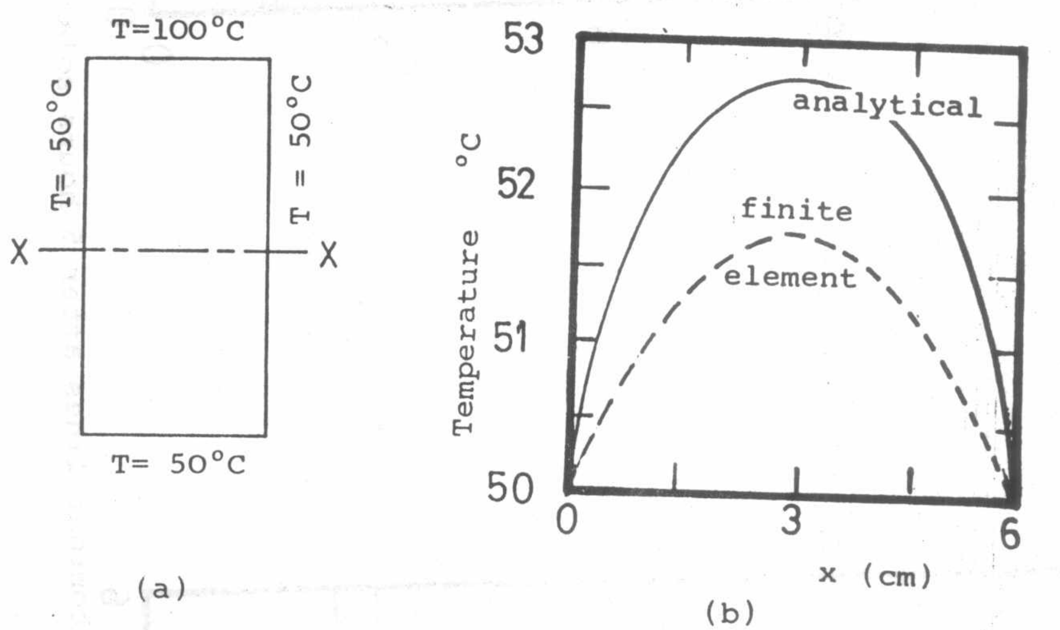

Fig. 5 Boundary conditions and the centerline (X-X) temperature distribution

The validity of the numerical results is usually checked with the corresponding analytical solutions whenever possible. In the present analysis, the numerical results, obtained using the finite element formulation, for the steady state conditions are compared with the analytical solution, [1], in the case of homogeneous solid. Fig. 5b shows a comparison of the results obtained via the conventional Galerkin formulation of the finite element method with those estimated analytically. The results are based on the boundary conditions indicated in Fig. 5a . A good agreement is observed between the two solutions with maximum deviation of 2 %. The effect of the enclosure on the temperature distribution inside the block is considered.

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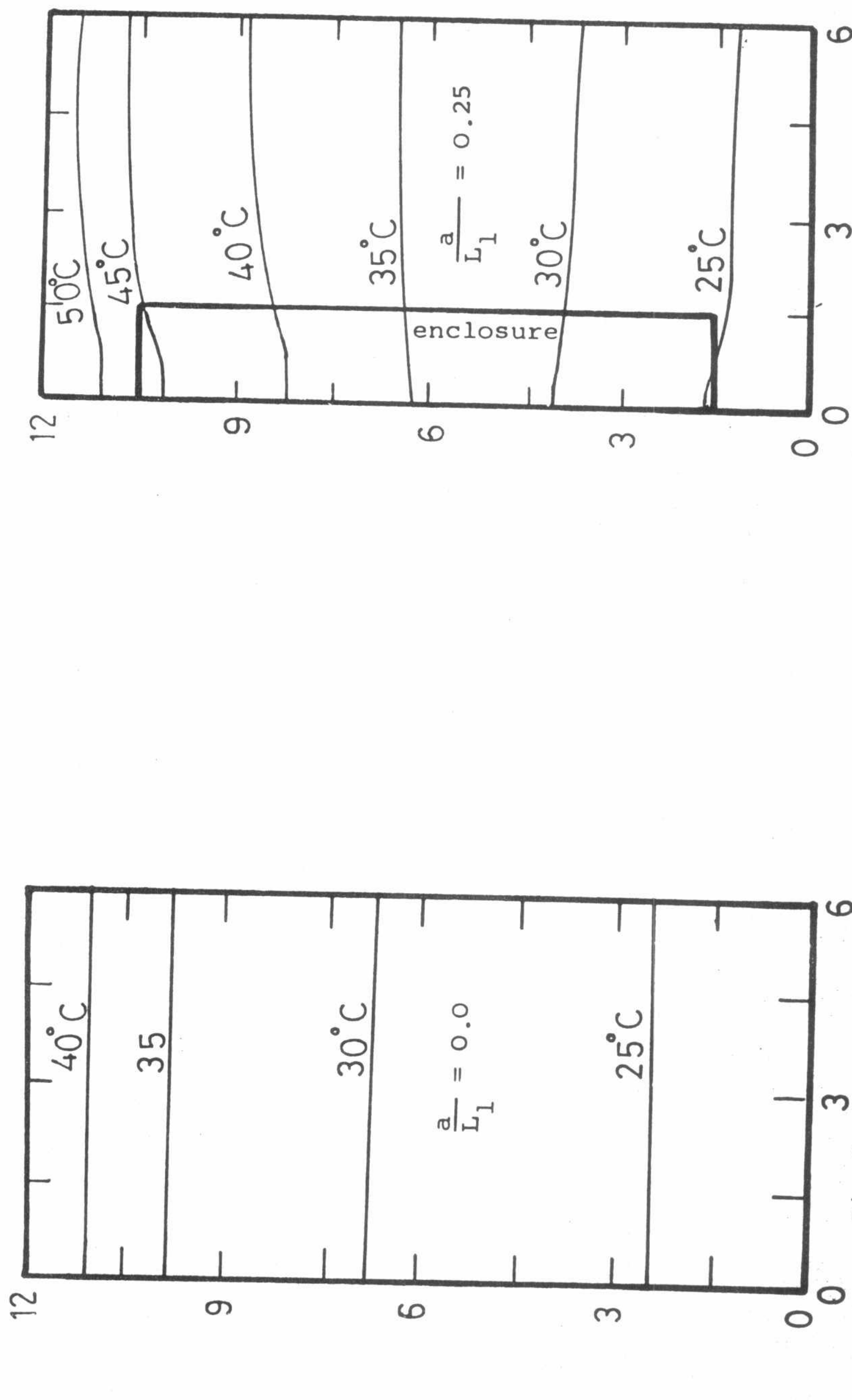

Fig.6 Isothermal contour lines after 5 hours solar time.

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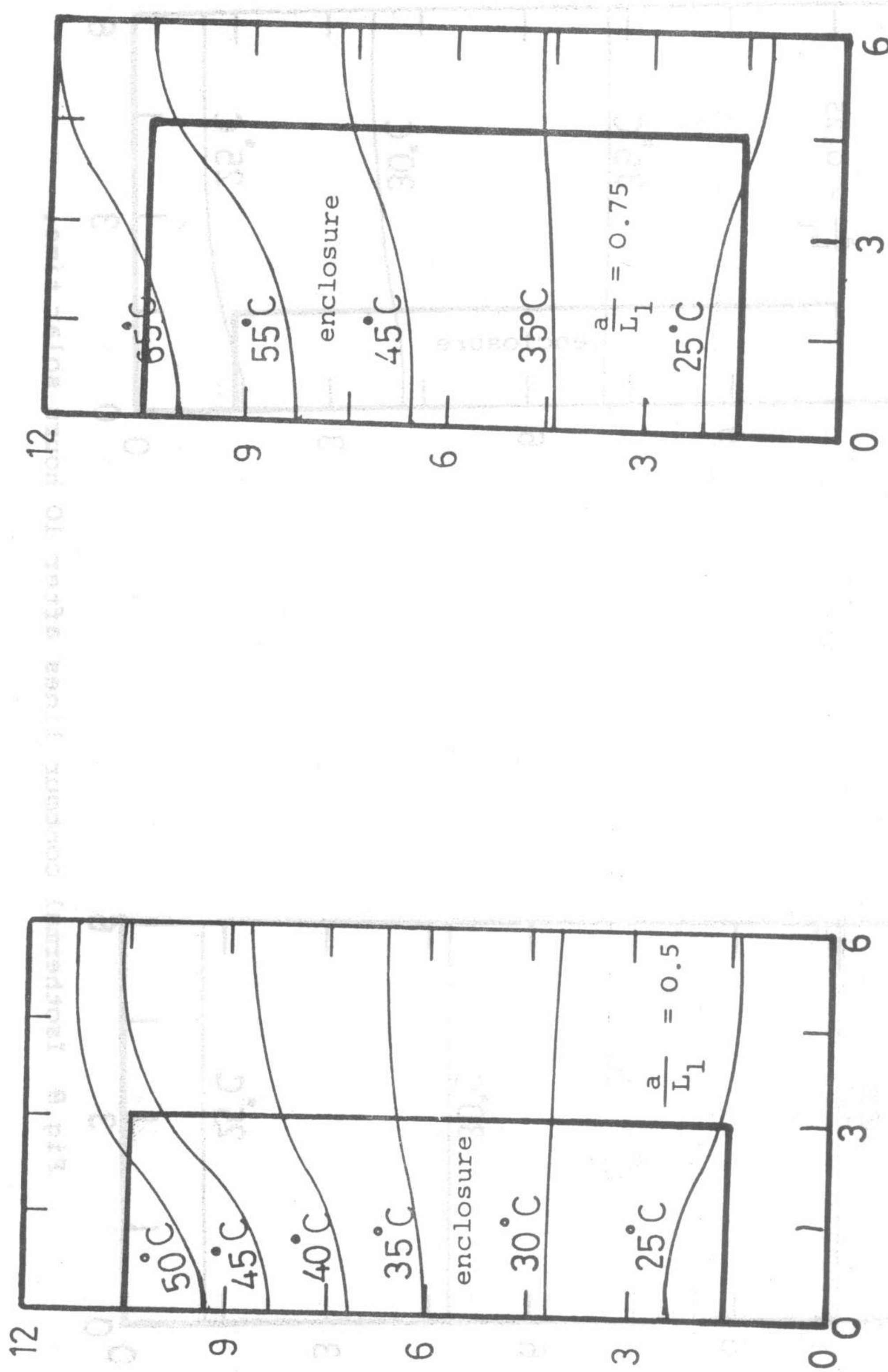

Fig.7 Isothermal contour lines after 5 hours solar time

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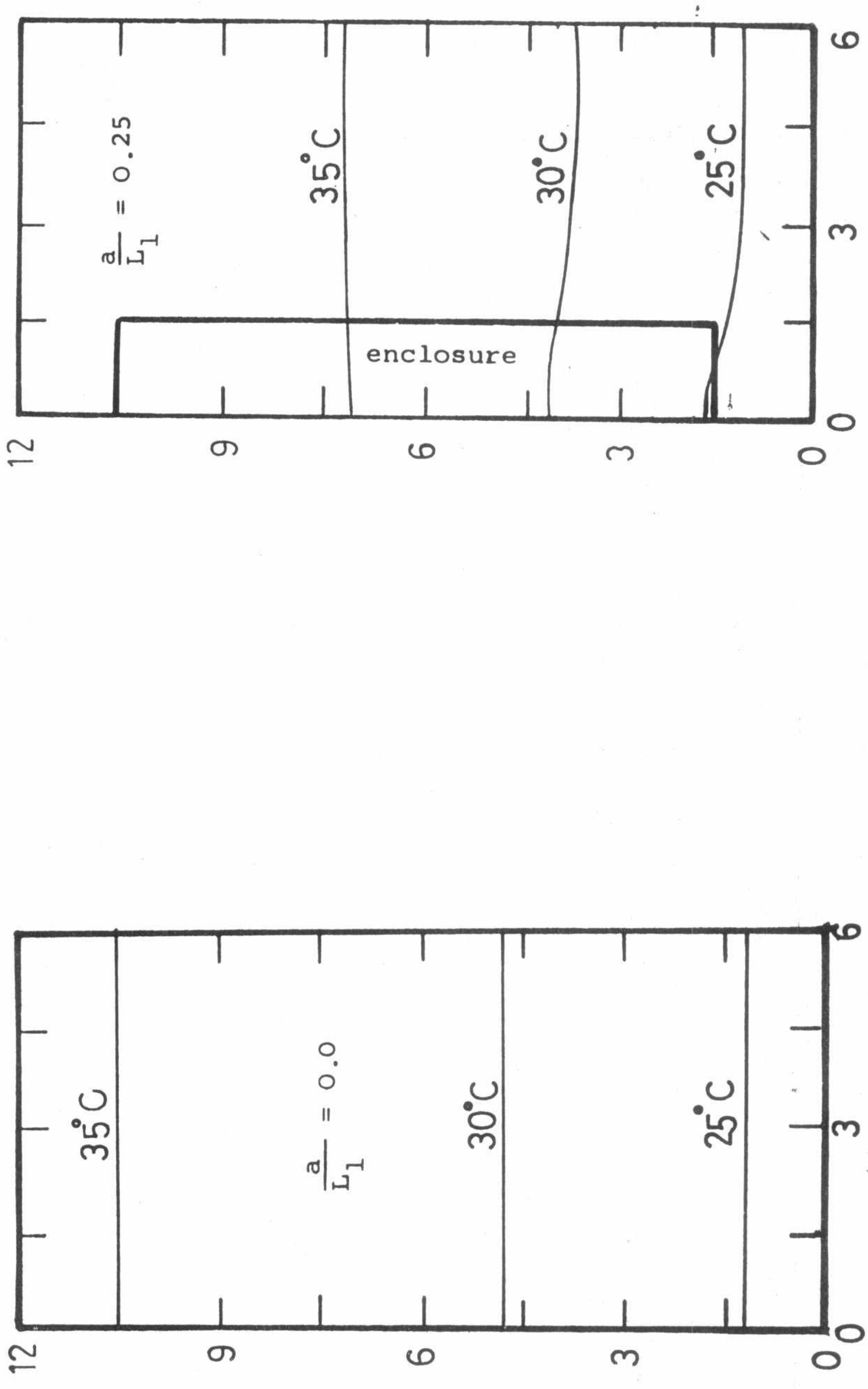

Fig.8 Isothermal contour lines after 10 hours solar time.

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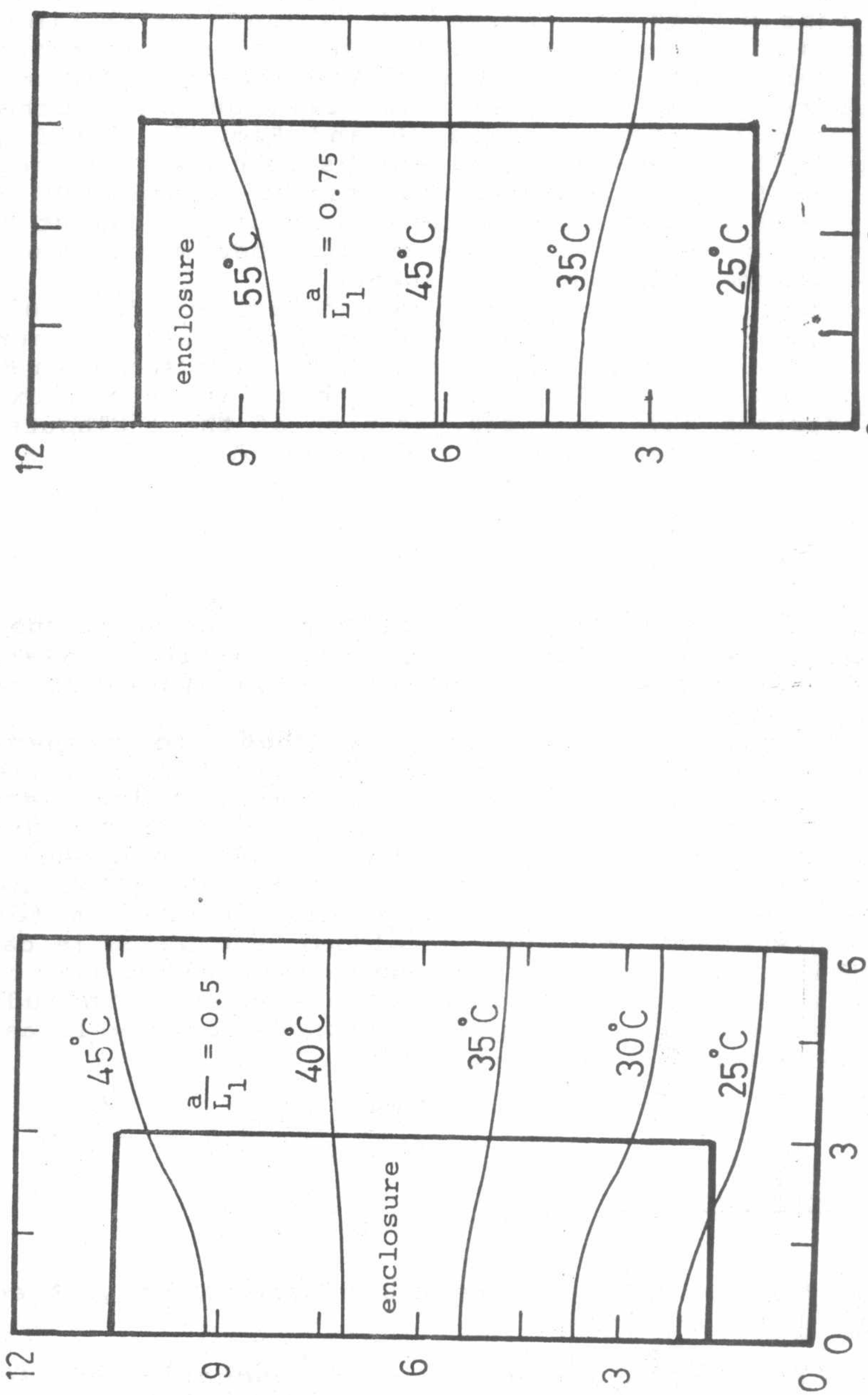

Fig.9 Isothermal contour lines after 10 hours solar time.

The solution of Eq. (1) subject to the initial and boundary conditions given by Eqs. (14)-(17) is shown in Fig. 6- Fig. 9. The isothermal contour lines are presented to indicate the response of the cement block to the time dependent solar heat flux considered over an interval of 10 hours solar time. For the purposes of illustration, the contours are superposed on the cement block at selected times. The effect of the size of the enclosure, after 5 hours solar time, on the isothermal contour lines is shown in Fig. 6 and Fig. 7. The figures suggest that the presence of the enclosure causes an increase in the top surface temperature. This may be interpreted by the increase in the average thermal resistance of the cement block. Also, Fig. 7, as the size of the enclosure becomes larger the top surface temperature becomes higher.
Fig. 8 and Fig. 9, illustrate the same behaviour after 10 hours of solar time; it could be noticed that the top surface temperatures have smaller values , than those shown in Fig. 6 and Fig. 7, due to the decrease in the solar heat flux. The above discussions show that the presence of the enclosure does modify the thermal behaviour of the cement block.

CONCLUSIONS

The finite element method was implemented to solve the two-dimensional transient heat conduction equation. Galerkin's procedure was used with the interpolation functions as the weighting functions.
Numerical results are presented to show the temperature distribution. The analysis shows that as the size of the enclosure increases the top surface temperature increases for the boundary conditions given by Eqs. (14)-(17). The present analysis shows that the thermal behaviour of the cement block is affected by the presence of the enclosure, this may be useful in obtaining more accurate load estimation for air conditioning systems. The model developed here which is capable of describing the behaviour of the cement block under time dependent heating conditions can be extended to include the effect of variation of the physical properties as well as more complicated boundary conditions.

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