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IMPLICIT TIME-DEPENDENT METHOD FOR THE CALCULATION OF TRANSONIC BLADE-TO-BLADE FLOWS WITH STRONG SHOCK WAVES

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ABSTRACT

Based on Euler equations, an implicit time-dependent method is applied for the calculation of the transonic blade-to-blade flows with strong shock waves. The numerical method is based on a fully implicit time difference scheme. The stabilizing correction method is utilised as an ADI sequence to break down the two dimensional operator into two operators. These operators are selected so that at each time step the variables are determined independently of each other. The work presented here is an adaptation, to turbomachinery flows, of the basic method previously publicized and well applied in the internal nozzle flow.

The purpose of this paper is to examine the ability of this method to predict the blade-to-blade transonic flow with strong shock waves. Detailed studies of the grid system and corresponding problems related to the boundary (inlet,exit and periodicity) conditions. Results are presented for the ECDEV transonic compressor cascade with prescribed pressure ratio and with a strong passage shock. The results obtained are compared with those obtained by an explicit method utilised at ONERA - FRANCE.

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INTRODUCTION

In the practical design of transonic cascades, the designer usually assumes the shape of the shock according to the experimental investigation or the computational prediction. Accordingly, the shock loss and flow downstream of the shock are estimated [1] .

In the experimental investigation of transonic cascades, the shock configuration obtained by optical measurement appears as a band rather than a line. Thus, the shock position is not clearly defined.

In the computational prediction, the solution is generally difficult because of the mixed hyperbolic-elliptic character of the problem. Even though, two families of approaches are employed to calculate such transonic flow problem; the mixed difference approach using the steady state formulation and the time dependent approach using the unsteady formulation.

In the mixed difference approach, the potential function and/or the stream function are used to express the flow field. The main advantage of this approach is to reduce the unknown variables to one or two variables. The difficulty of the stream function method, which is the non uniqueness of density, has been overcome recently either by integrating the velocity gradient equation to determine the velocity distribution [2-3] or by splitting the velocity vector into a potential (compressible) and a rotational (incompressible) part [4].

In the time dependent approach, the unsteady form of Euler equations are integrated until steady state is reached. This system of equations is of hyperbolic character with respect to time.

The solution of the Euler equations is the most common way of computing the transonic flow with strong shock waves, since it is not generally possible to assume irrotational or isentropic flow.

Using finite difference techniques, the unsteady Euler equations may be solved by either explicit or implicit schemes.

The explicit schemes, which are easy to implement, suffer from long computational times due to the time step limiting stability criterion (CFL).

The implicit schemes, which are free from this restriction on the time step, is difficult to implement because of the non linear form of the equations.

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In the present paper a two dimensional inviscid transonic flow in cascade is presented. The time dependent approach with an implicit finite difference technique is used. The method is an extension of the approach recently developed [5] and applied to the internal flows in convergent divergent nozzles [6-7].

GOVERNING EQUATIONS

Considering the unsteady inviscid flow, the conservation equations of mass, momentum and energy can be written as follows

Continuity

$$\frac{\partial}{\partial t}(\rho) + \mathrm{div}(\rho\vec{V}) = 0 \qquad (1.a)$$

Momentum

$$\frac{\partial}{\partial t}(\rho\vec{V}) + \mathrm{div}(\rho\vec{V}\otimes\vec{V}) + \mathrm{grad}\ p = 0 \qquad (1.b)$$

Energy

$$\frac{\partial}{\partial t}(\rho H-p) + \mathrm{div}(\rho H\vec{V}) = 0 \qquad (1.c)$$

where the character $\otimes$ designate the tensorial product of two vectors. The flow properties are velocity V, density ρ and pressure p. The total enthalpy H for a perfect gas and constant specific heats, is defined by

$$H = \frac{\gamma}{\gamma-1}\ \frac{p}{\rho} + \frac{1}{2}V^2 \qquad (1.d)$$

where γ is the specific heat ratio.

For the calculation of a steady flow, it is now classical to simplify the solution of the Euler equations by replacing the unsteady energy equation by the condition of constant total enthalpy,

$$H = \mathrm{Const.} \qquad (1.e)$$

This assumption is a central feature of the pseudo-unsteady method [8] which becomes exact even in the presence of shock waves as long as the flow becomes steady. This simplification reduces the system to three equations in the two-dimensional case, which saves computing time and memory.

In the numerical solution of fluid flow problems using finite difference techniques, the treatment of field boundaries not coincident with coordinate surfaces involves complex and inaccurate procedures. To correctly treat these boundary conditions, coordinate surfaces are chosen so that all body surfaces are bring onto coordinate surfaces.

With respect to an arbitrary curvilinear coordinates ζ_j, The continuity and the momentum equations (1.a and 1.b) can be expressed as

$$\frac{\partial}{\partial t}(F) + G = 0 \qquad (2)$$

...

where

$$F = \begin{bmatrix} F_0 \\ F_1 \\ F_2 \end{bmatrix} \quad \text{and} \quad G = \begin{bmatrix} G_0 \\ G_1 \\ G_2 \end{bmatrix}$$

with

$$F_0 = \rho\sqrt{g} \quad , \quad F_r = \rho \underline{v}^r \sqrt{g} \quad ,$$

$$G_0 = \frac{\partial}{\partial \zeta_s}(F_s) \quad \text{and} \quad G_r = \frac{\partial}{\partial \zeta_s}(F_r F_s / F_0) + g^{rs}\sqrt{g}\, \frac{\partial}{\partial \zeta_s}(p) + \Gamma^r_{sq}\, (F_s F_q / F_0)$$

The Christoffel symbol Γ^r_{sq} is defined by

$$\Gamma^r_{sq} = \frac{1}{2}\, g^{rm} \left\{ \frac{\partial}{\partial \zeta_q}(g_{ms}) + \frac{\partial}{\partial \zeta_s}(g_{mq}) - \frac{\partial}{\partial \zeta_m}(g_{sq}) \right\}$$

In these equations superscripts or subscripts indices take on the values 1,2, and the Einstein's summation convention is employed.

The covariant and contravariant components of the metric tensor (g_{rs} and g^{rs}) are expressed in terms of the first derivatives of the mapping functions

$$\zeta_r = \zeta_r \; (x_s)$$

defining the system of curvilinear coordinates ζ_r (r=1,2) as function of the cartesian coordinate system x_s (s=1,2).

g is defined by the determinant of the matrix $|g_{rs}|$, and v^r denoting the contravariant velocity components.

NUMERICAL METHOD

For the numerical solution, the system of equations (2) are discretized between two time levels using a backward time difference scheme

$$F^{n+1} = F^n - \tau\, G^{n+1} + O(\tau^2) \qquad (3)$$

where the superscript n represents, here, the time level and τ is the time step. It is assumed that the solution is known at the n level, t^n, and is desired at the n+1 level, t^{n+1}.

G^{n+1} represents a non linear differential function in which the non linear terms ($F_r F_s / F_0$) are linearized as follows

$$(F_r F_s / F_0)^{n+1} = (F_r F_s / F_0)^n + \left\{ \frac{\partial}{\partial F_q}(F_r F_s / F_0) \right\}^n f_q$$

, for all $q = 0, r$ and s .

$$f_q = F^{n+1}_q - F^n_q$$

The system of equations (3) can be written in the form

$$\{ I + \tau \mathcal{B} \} f = - \tau G^n \qquad (4)$$

where $\mathcal{B}$ represent a matrix operator in which the spatial derivatives $\partial/\partial\zeta_1$ and $\partial/\partial\zeta_2$ are replaced by an approximate difference operators δ_1 and δ_2 using a suitable second order three points in space along each direction.

The solution of these three difference equations using one step solution requires an important storage location and tends to be time consuming. In order to obtain an economical scheme, the ADI method of stabilizing correction type is used as follows

$$\{ I + \tau \mathcal{B}_1 \} \tilde{f} = - \tau G^n$$

$$\{ I + \tau \mathcal{B}_2 \} f = \tilde{f}$$

with

$$\mathcal{B}_1 + \mathcal{B}_2 = \mathcal{B}$$

where I is the unit matrix.

The coefficients of the matrices $\mathcal{B}_1$ and $\mathcal{B}_2$ are chosen so that the unknown variables f_0 , f_1 and f_2 are determined independently of each other at each fractional steps. For more detail see [7].

BOUNDARY CONDITIONS

1- Inlet and outlet flow conditions

In order to avoid the problem of overspecification or underspecification of the inlet/outlet conditions, the number of specified dependent variables at these boundaries is determined by the theory of characteristic. From this theory, it is clear that the number of boundary conditions required should be determined by the origin of the characteristic lines pointed from the outside to the inside of the calculated region.

Let the inlet and outlet boundaries coincide with the lines along which the coordinate ζ_1 is constant. The system of equations (2), after decoupling $\partial/\partial\zeta_1$, can be rewritten as

$$\frac{\partial}{\partial t}(F) + A \frac{\partial}{\partial \zeta_1}(F) = C$$

where,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -v^1 v^1 + R_2 g^{11}(2H+V^2) & 2v^1 - 2R_2 g^{11} v_1 & -2R_2 g^{11} v_2 \\ -v^1 v^2 + R_2 g^{12}(2H+V^2) & v^2 - 2R_2 g^{12} v_1 & v^1 - 2R_2 g^{12} v_2 \end{bmatrix}$$

and C is the sum of the ζ_2-derivative terms and the source term. v_1 and v_2 are the covariant velocity components. R_2 is defined by

$$R_2 = (\gamma - 1)/2\gamma$$

...

The slope of the characteristic lines in the $(t-\zeta_1)$ plan, $\lambda=d\zeta_1/dt$, is obtained by solving the equation

$$\det.(A-\lambda I) = 0$$

where I is the unit matrix. It is clear that λ_1, λ_2 and λ_3, the three roots of this equation, are the slopes of the three characteristics which carry the information necessary to define F_0, F_1 and F_2 at every point.

For the form of the unsteady Euler equations considered in this work, the eigenvalues of the coefficient matrix A are

$$\lambda_1 = v^1 \quad , \quad \lambda_{\substack{2\\3}} = v^1(R_1 \pm \tilde{R}_1)$$

where

$$\tilde{R}_1 = R_1\sqrt{1 + (\frac{a^2}{\sqrt{V}}g^{11} - 1)/\gamma R_1^2}$$

with $R_1=(\gamma+1)/2\gamma$, a is the local speed of sound.

From the above equations, it is clear that

$$\lambda_1 > 0 \quad , \quad \lambda_2 > 0$$

and $\lambda_3 < 0$ if $(v^1/\sqrt{g^{11}})<a$ (Subsonic B.)

or $\lambda_3 > 0$ if $(v^1/\sqrt{g^{11}})>a$ (Supersonic B.)

This result can be interpreted physically by considering the plane made up of the ζ_1 coordinate and time . From figure (1), it can be seen that the point a_1 is fed information from the time level $t=t^n$ along the lines *ba*, *ca* and *da*.

The following table summarizes the number of the boundary conditions which must be imposed at the inlet and outlet boundaries.

Boundary	subsonic	supersonic
inlet	2	3
outlet	1	0

For subsonic inlet and outlet boundaries, three variables must be specified , two at inlet and one at outlet.

For cascade flow computation, it is convenient to specify the total pressure and the flow direction at the inlet, and the static pressure at the outlet boundary.

2- Periodicity condition

On the bounding lines upstream and downstream of the blade row, this condition is easily satisfied by treating points on these lines as interior points and then equating values at corresponding points on the two boundaries.

This periodicity applied immediately downstream of the trailing edge is found to be sufficient to satisfy the Kutta-condition at the trailing edge and then no explicit Kutta-condition need to be applied.

3- Blade surface condition

The blade surface is taken as a surface of ζ_2=const. Along this surface flow tangency is imposed by putting the normal velocity component ($v^2/\sqrt{g^{22}}$) equal to zero, then

$$F_2 = \rho\sqrt{g}\ v^2 = 0.$$

GRID GENERATION

It is well known that a carefully constructed mesh is essential in transonic computations in order to obtain a sufficient resolution for capturing shocks.

In a blade to blade surface, it is possible to construct one of the three types of the grid meshes shown in figure 2 . A comparison between these types is illustrated in the following table

type	Inlet/exit surfaces	grid shape	N°of grid points	application of periodicity cond.	all inlet variables are imposed
1	⊥ to the axial dir.	orthogonal	too large	approximate (by interp.)	if axial Mach N°> 1
2	⊥ to the tangent of L.E & T.E.	pseudo orthogonal.	suitable	approx. or exactly	if relative Mach N°> 1
3	⊥ to the axial dir.	distorted	suitable	exactly	if axial Mach N°> 1

APPLICATIONS AND RESULTS

In order to evaluate the effectiveness of this method, the present code is applied to a test case and the results are compared with the results obtained by another operational code |9|. The ECDEV profil cascade (figure 3), utilised at ONERA as a test cascade, is taken as a test case.

Two flow conditions are computed:

1) P.R = 0.7 & 2) P.R = 0.66

where P.R is the ratio of the static exit pressure to the total inlet pressure.

The computed results, shown in figures 4,5 and 6, represent respectively the static pressure distribution on the blade surfaces, the iso-Mach and the iso-bar contours on the blade to blade passage for the two cases of the pressure ratio.

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For these two conditions, the shock close completely the blade to blade passage (choked flow). In the case of P.R=0.7, the shock intersect the upper and lower surfaces at a distance ($\frac{x}{c}$) approximately equal to 0.75 and 0.2 from the leading edge. In the case of P.R=0.66, the shock is moved downstream and it intersect the upper surface at the trailing edge, and the lower surface at $(x/c) \doteq 0.4$.

The solution obtained by the present implicit method is compared with the Euler explicit code |9| utilised in ONERA. Figures 4.c and 6.b show the static pressure distribution on the blade surface and the isobar contours on the blade to blade passage obtained by the explicit code for the case of P.R=0.66.

It can be seen that the shock position obtained by the two methods agrees very well. Compared with the explicit method, a better shock thickness is obtained by the present implicit code (see figure 6).
The results presented here are obtained on a CYBER 170/750 at ONERA research center using 86X11 mesh points. The computation time and the number of iterations needed by each method is depicted in the following table

Code	N°of iter. to s.s.	total time (sec)	time/iter/ point (sec)	time/point to s.s(sec)
explicit	1500	560	0.0004	0.6
implicit	100	220	0.0023	0.23

Thus the present implicit method is approximately 2.5 times faster than the explicit code |9|.

CONCLUSION

The present implicit time dependent method can give good results for the solution of transonic choked flow through cascades.

Compared with other numerical results, it seems to us that our results have a better shock capturing properties and it converges rapidly.

Writing the set of equations with an arbitrary curvilinear coordinates, the programing code can be adapted to various types of flow problems and have high flexibility.

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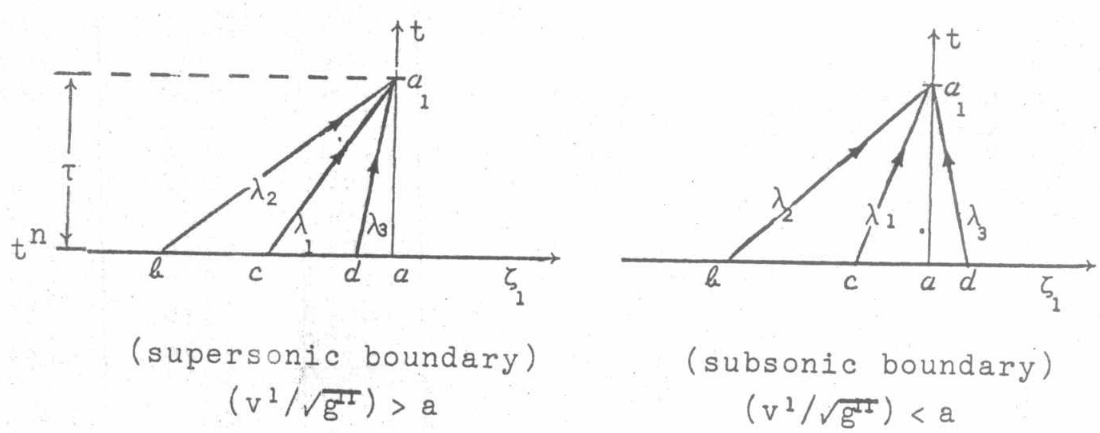


Fig.1-Characteristic lines in the $(\zeta_1\text{-}t)$ plan.

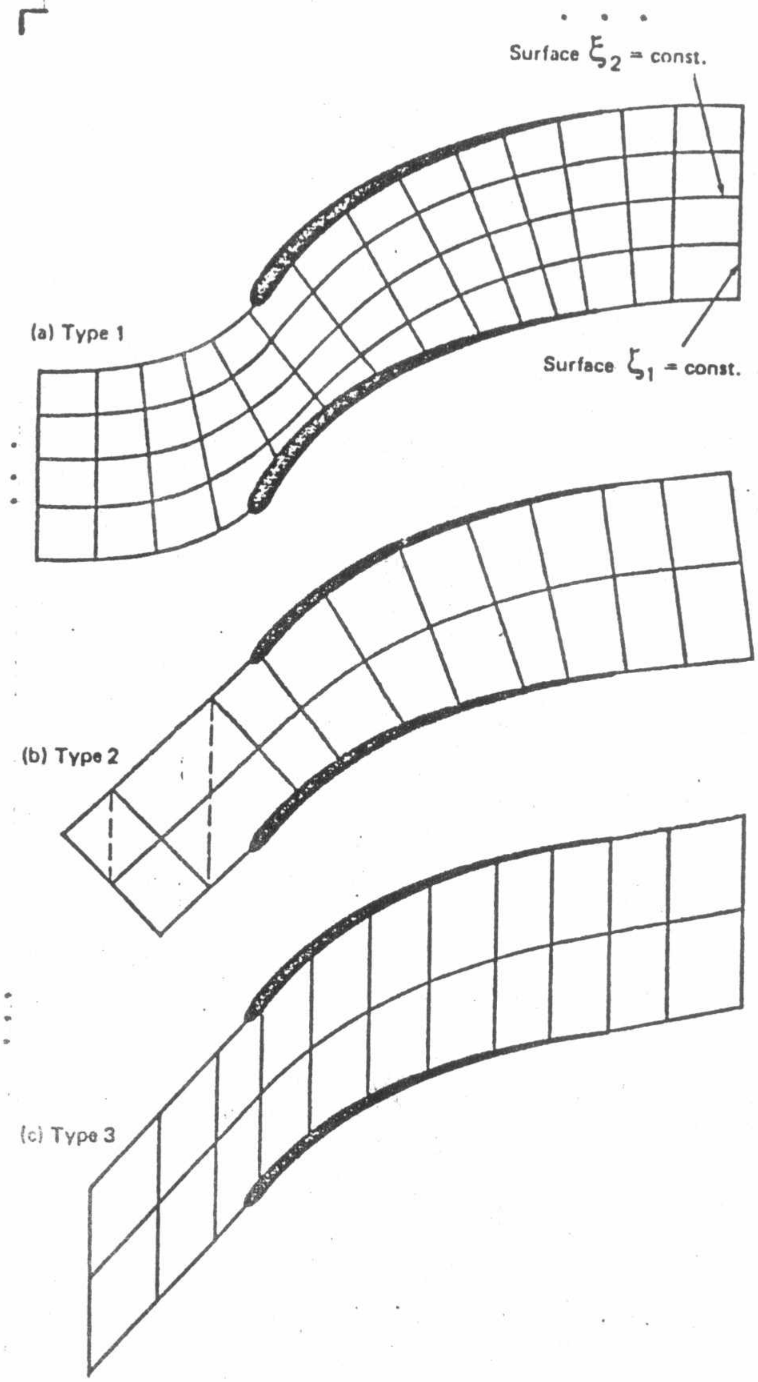


Fig.2-Different types of mesh arrangements for the blade to blade computation.

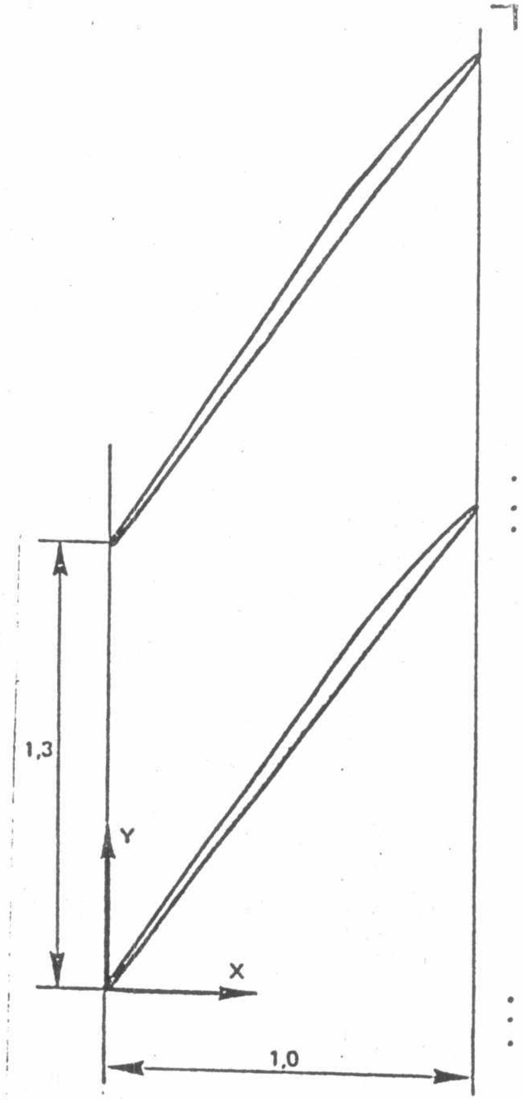
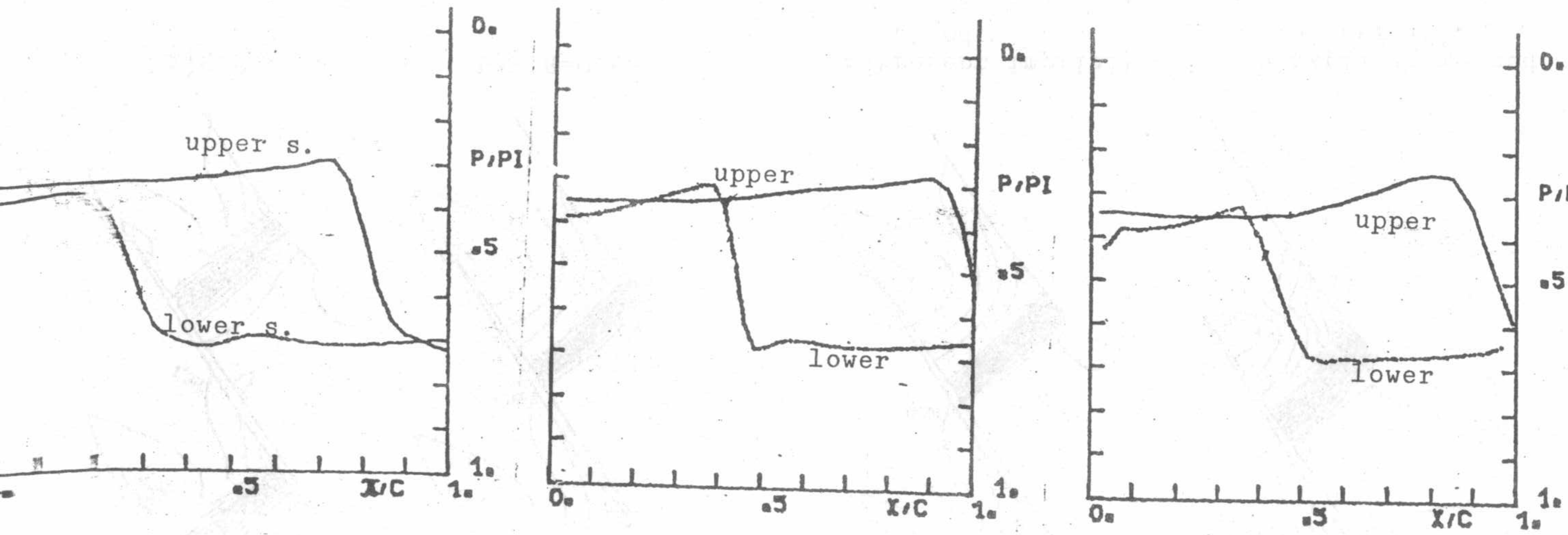


Fig.3-Geometry of the ECDEV profil cascade.

X	Y(upper)	Y(lower)
0	0	0
.0125	.0228	-.0096
.0625	.1028	.0594
.1125	.1826	.1330
.1625	.2623	.2067
.2125	.3418	.2805
.2625	.4212	.3544
.3125	.5005	.4283
.3625	.5796	.5024
.4125	.6587	.5765
.4625	.7376	.6507
.5125	.8154	.7248
.5625	.8915	.7986
.6125	.9651	.8721
.6625	1.0356	.9448
.7125	1.1032	1.0169
.8125	1.2303	1.1585

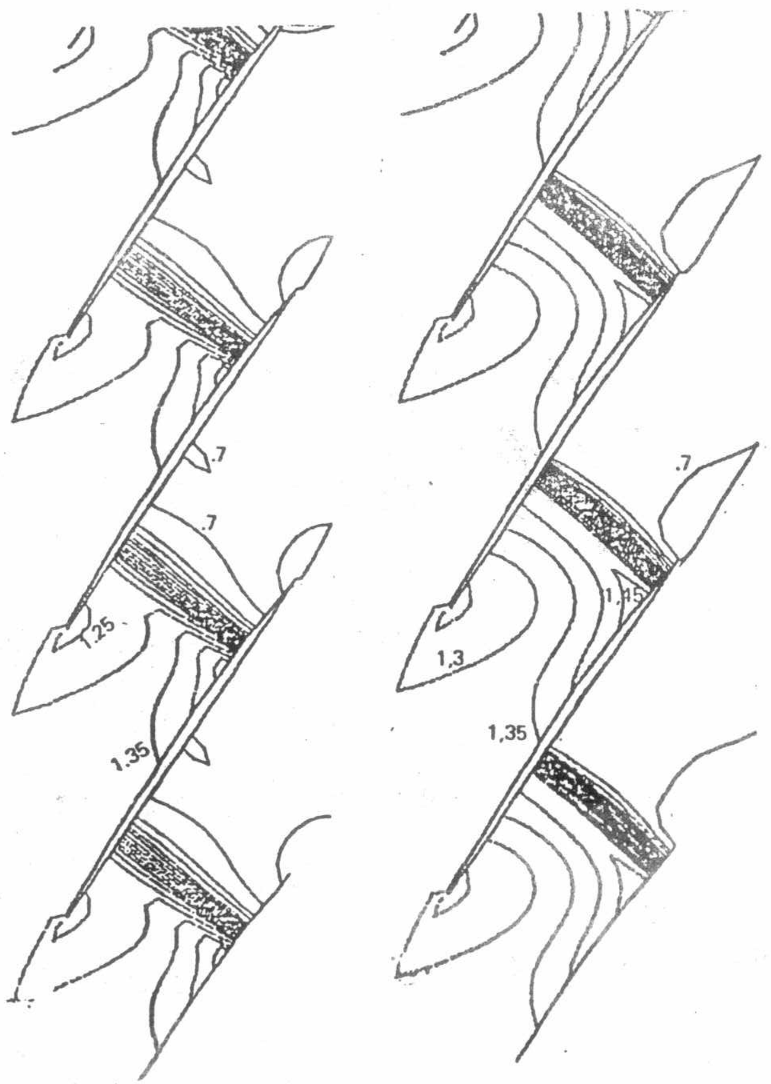


a)present implicit method for P.R=0.7

b)present implicit method for P.R=0.66

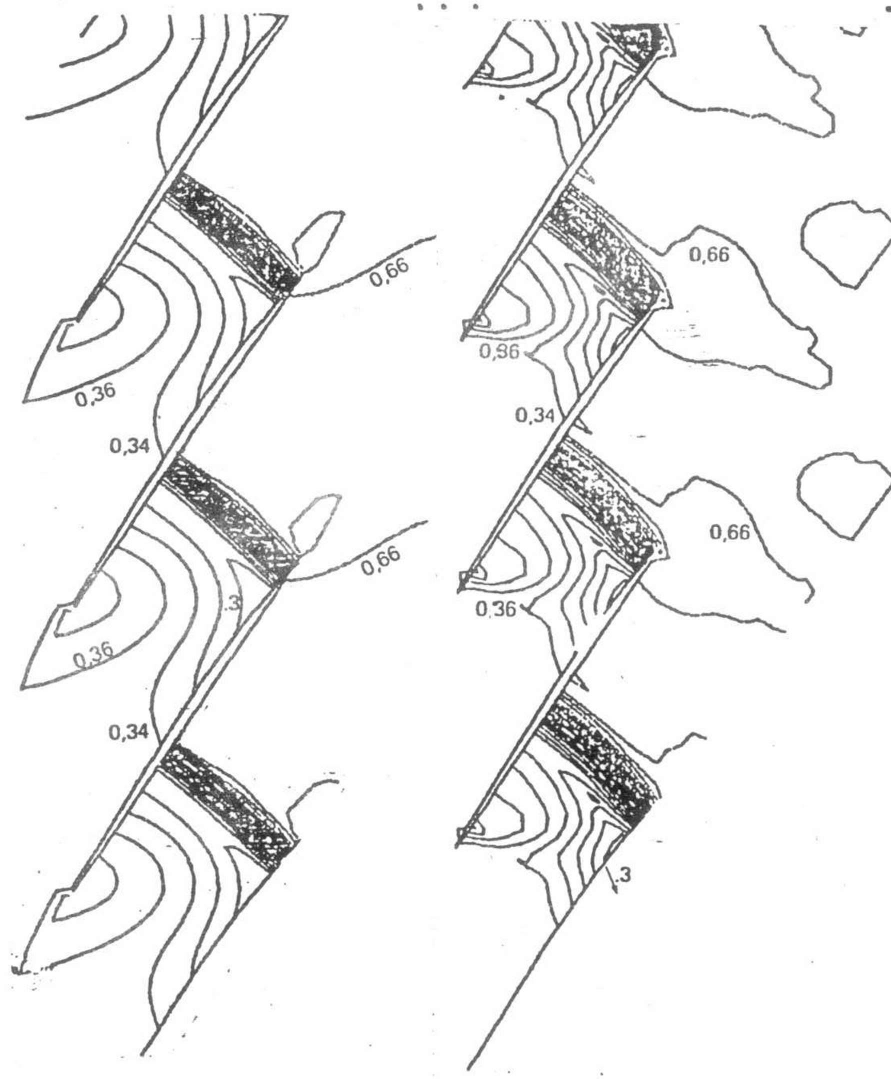
c)explicit method of Ref.[9] for P.R=0.66

Fig.4-Pressure ratio distribution along the airfoil cascade surfaces.



a)P.R=0.7 b)P.R=0.66

Fig.5- Iso-Mach contours on the blade to blade passage.



a)Present implicit method. b)Explicit method of Ref.[9].

Fig.6- Isobar contours on the blade to blade passage for P.R=0.66.