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SQUARING UP AND REGULATION IN A CLASS OF LINEAR MULTIVARIABLE SYSTEMS

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ABSTRACT

Non-square linear multivariable systems means that the number of inputs and the number of outputs of these systems are not equal. This type of systems has certain difficulties in many control aspects. Squaring a non-square multivariable system, is done by making the number of inputs equal to the number of output; this is accomplished either by squaring up or by squaring down operations. In this paper the problem of squaring up certain non-square linear multivariable system is solved by finding additional inputs or outputs such that the resulting square system has arbitrarily zero locations. The problem of regulation of this type of multivariable systems is tackled, based on the fact that non-square linear multivariable systems almost always possess no zeros. The zeros introduced by squaring up operations, are located at the position of an equal number of system poles and consisting fixed modes. By using certain output feedback, developed by the parameters of the system, the remaining poles are asymptotically assigned to arbitrary locations and the output response is nearly regulated. The proposed method is simple, efficient and easily programmed using MATLAB to deal with practical large scale systems.

KEY WORDS

Fixed modes, Squaring-down, Zeros, Multivariable systems,

1. INTRODUCTION

In the design of multivarible feedback systems, often feedback loops are introduced between a selected set of measured output variables and an equal number of independent control inputs. Thus, given a non-square system, the first stage of design essentially consists of squaring that system. The squaring process may be either a squaring down or a squaring up, and both of them introduces additional invariant zeros. The zeros of the corresponding T.F have to be located in the left hand side of the s-plane.

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The squaring down problem was thoroughly studied in the last decade [1], [2] and [3] . It is found that squaring down with the constraint of additional zeros being in the left hand side of the s-plane cannot in general be accomplished with static compensator alone. We are thus lead to the use of dynamic compensators, thereby increasing the order of the system.

On the other hand, squaring up of certain class of systems is accomplished by finding additional inputs or outputs such that the resulting square system has arbitrarily zero locations [4]. In this paper the problem of regulation for non-square systems is tackled, based on the fact that these systems almost always possess no zeros [1], [2]. The zeros, introduced by squaring up operations, are located at the position of an equal number of system poles and consisting of fixed modes [5]. By using a static output feedback, developed by the parameters of the system, the remaining poles are asymptotically assigned to arbitrary locations and the output response is nearly regulated.

The paper is organized as follows: Section 2 is devoted for the regulation of non-square systems presented by their triplet (A,B,C) with CB having full rank, while conclusions of the work are found in Section 3.

2. REGULATION OF NON-SQUARE SYSTEMS WITH CB HAVING FULL RANK

Consider a linear multivariable system represented by the following state space equations:

$$\left.\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}\right\} \qquad (1)$$

where x, u, and y are, respectively , n, m, and p-dimensional vectors. Without loss of generality, assume that B and C are of full rank, $m > p$ and rank $CB = p$. Regulation of that system is carried out in two steps.

Step 1. Squaring Up the System

Squaring up is done by finding additional (m-p) x n output coupling matrix such that the system matrix takes the form

$$
P(s) = \begin{array}{c} \\ \left[\begin{array}{cccc} sI - A_{11} & -A_{12} & \vdots & B_1 \\ -A_{21} & sI - A_{22} & \vdots & B_2 \\ \cdots & \cdots & \cdots & \cdots \\ C_{11} & C_{12} & \vdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ C_{21} & C_{22} & \vdots & 0 \end{array}\right] \end{array} \begin{array}{c} * \\ m \\ n-m \\ \\ P \\ \\ m-p \end{array} \qquad (2)
$$

where $[C_{21}\ C_{22}]$ are to be determined.

Starting by determining a similarity transformation matrix T such that

$$
\bar{A} = TAT^{-1}, \ \bar{B} = TB = \begin{bmatrix} I \\ \cdots \\ 0 \end{bmatrix} \qquad \text{and} \qquad \bar{C} = CT^{-1}
$$

The system matrix of the squared system will be

$$
P(s) = \left[\begin{array}{cccc} sI - \bar{A}_{11} & -\bar{A}_{12} & \vdots & I \\ -\bar{A}_{21} & sI - \bar{A}_{22} & \vdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ \bar{C}_{11} & \bar{C}_{12} & \vdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ \bar{C}_{21} & \bar{C}_{22} & \vdots & 0 \end{array}\right] \begin{array}{c} * \\ m \\ n-m \\ \\ p \\ \\ m-p \end{array} \qquad (3)
$$

The invariant zeros of the squared system are the eigenvalues of

$$
\left(\bar{A}_{22} - \bar{A}_{21} \begin{pmatrix} \bar{C}_{11} \\ \cdots \\ \bar{C}_{21} \end{pmatrix}^{-1} \begin{pmatrix} \bar{C}_{12} \\ \cdots \\ \bar{C}_{22} \end{pmatrix} \right)
$$

Now, we turn our attention to the computation of $[\ \bar{C}_{21}\ \bar{C}_{22}]$ such that the invariant zeros of the squared system could be arbitrarily located.

$\bar{C}_{21}$ is chosen such that the matrix $\begin{pmatrix} \bar{C}_{11} \\ \hline \bar{C}_{21} \end{pmatrix}$ has full rank. This could always be achieved if $\bar{C}'_{21}$ is in the null space of $\bar{C}_{11}$ and any numerical algorithm in the literature, such as a singular value decomposition could be used to determine it [4].

To determine $\bar{C}_{22}$ such that the zeros are arbitrarily assigned, we argue as follows:

Consider the matrix

$$\left(\bar{A}_{22} - \bar{A}_{21} \begin{pmatrix} \bar{C}_{11} \\ \cdots \\ \bar{C}_{21} \end{pmatrix}^{-1} \begin{pmatrix} \bar{C}_{12} \\ \cdots \\ \bar{C}_{22} \end{pmatrix} \right)$$

$\begin{pmatrix} \bar{C}_{12} \\ \hline \bar{C}_{22} \end{pmatrix}$ is similar to a state feedback matrix acted upon a system represented by

$$\left(\bar{A}_{22} \ , - \bar{A}_{21} \begin{pmatrix} \bar{C}_{11} \\ \cdots \\ \bar{C}_{21} \end{pmatrix}^{-1} \right);$$

The matrix $\begin{pmatrix} \bar{C}_{12} \\ \hline \bar{C}_{22} \end{pmatrix}$ could be written as follows

$$\begin{pmatrix} \bar{C}_{12} \\ \hline \bar{C}_{22} \end{pmatrix} = \begin{pmatrix} \bar{C}_{12} \\ \cdots \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \cdots \\ \bar{C}_{22} \end{pmatrix} = \hat{C}_{12} + \hat{C}_{22}$$

The problem of determining $\bar{C}_{22}$ reduces to finding a state feedback $\hat{C}_{22}$ such that the matrix

$$\left\{ \left[\bar{A}_{22} - \bar{A}_{21} \begin{pmatrix} \bar{C}_{11} \\ \hline \bar{C}_{21} \end{pmatrix}^{-1} \hat{C}_{12} \right] - \bar{A}_{21} \begin{pmatrix} \bar{C}_{11} \\ \hline \bar{C}_{21} \end{pmatrix}^{-1} \hat{C}_{22} \right\}$$

has the desired eigenavlues.

Step 2, Regulation of the Squared System

Based on the fact that pole-zero cancellation prevents excessive output response in speeding up the closed loop dynamic [6],[7]. Regulation is achieved by assigning the zeros of the squared system to the positions of a subset of poles and then applying certain output feedback controller.

The squared system will be a system with (C_{sq} B) having full rank; where C_{sq} is the output coupling matrix of the squared system. The output feedback controller takes the form $(C_{sq}\ B)^{-1}$ J; where J is an arbitrarily diagonal matrix with specified eigenvalues .

By speeding up the response of the system; the poles of the closed loop system will be assigned, asymptotically, to the locations of the eigenvalues of the matrix J [7], [8]. Consequently, the output response of the squared system will be nearly regulated.

If it is possible to augment the input output coupling matrix D such that rank D = m - p , then we argue as follows:

The system matrix of the squared system will take the form

$$\begin{array}{cc} & \begin{array}{ccccc} \quad\quad\quad\quad\quad\quad\quad\quad & * & p & * & m-p\ * \end{array} \\ \bar{P}(s) = & \left[\begin{array}{ccccccc} sI - A_{11} & -A_{12} & \vdots & B_{11} & \vdots & B_{12} \\ -A_{12} & sI - A_{22} & \vdots & B_{21} & \vdots & B_{22} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ C_{11} & C_{12} & \vdots & 0 & \vdots & 0 \\ \cdots & \cdots & \cdots & \cdots & & \cdots \\ C_{21} & C_{22} & \vdots & 0 & \vdots & D \end{array}\right] \begin{array}{c} p \\ n-p \\ \\ p \\ \\ m-p \end{array} \end{array} \quad (4)$$

By applying certain similarity transformations $\bar{P}(s)$ could be transformed to

$$\bar{\bar{P}}(s)=\begin{bmatrix} sI-\bar{A}_{11} & -\bar{A}_{12} & \vdots & \bar{B}_{11} & \bar{B}_{12} \\ -\bar{A}_{21} & sI-\bar{A}_{22} & \vdots & \bar{B}_{21} & \bar{B}_{22} \\ \cdots & \cdots & \cdots & & \cdots \\ I & O & \vdots & 0 & 0 \\ \bar{C}_{21} & \bar{C}_{22} & \vdots & 0 & D \end{bmatrix} \tag{5}$$

The invariant zeros of the system are the zeros of the invariant polynomials of the matrix

$$Z(s)=\begin{bmatrix} -\bar{A}_{12} & \vdots & \bar{B}_{11} & \bar{B}_{12} \\ sI-\bar{A}_{22} & \vdots & \bar{B}_{21} & \bar{B}_{22} \\ \cdots & \cdots & \cdots & \cdots \\ \bar{C}_{22} & \vdots & 0 & D \end{bmatrix} \tag{6}$$

By carrying certain elementary operations, we obtain

$$\bar{Z}(s)=\begin{bmatrix} -\bar{A}_{12}-\bar{B}_{12}D^{-1}\bar{C}_{22} & \bar{B}_{11} & 0 \\ sI-\bar{A}_{22}-\bar{B}_{22}D^{-1}\bar{C}_{22} & \bar{B}_{21} & 0 \\ 0 & 0 & D \end{bmatrix} \tag{7}$$

The invariant zeros of the system are the eigenvalues of

$$\left[\bar{A}_{22}+\bar{B}_{22}D^{-1}\bar{C}_{22}-\bar{B}_{21}\bar{B}_{11}^{-1}(\bar{A}_{12}+\bar{B}_{12}D^{-1}\bar{C}_{22})\right]$$
$$=\left[\bar{A}_{22}-\bar{B}_{21}\bar{B}_{11}^{-1}\bar{A}_{12}-(\bar{B}_{21}\bar{B}_{11}^{-1}\bar{B}_{12}-\bar{B}_{22})D^{-1}\bar{C}_{22}\right]$$

with loss of generality D could be taken identity; consequently, $\bar{C}_{22}$ is found such that the (n - p) zeros are arbitrarily assigned. C_{21} is arbitrarily chosen.

Remark:

Squaring up the system by adding auxiliary outputs necessitates measuring the system states; hence it is necessary that all the states of the system has to be accessible.

3. CONCLUSIONS

Based on the fact that zeros of linear systems are the source of unbounded peaking for output response. Squaring-up of non-square systems with CB having full rank is made such that the zeros of the squared system are located at the position of a subset of poles and consist of fixed modes. The pole-zero cancellation prevents excessive output response in speeding up the closed loop dynamics. The remaining poles are asymptotically assigned to certain location by using an output feedback and the output response is, nearly, regulated.

Squaring up and regulation of non-square systems with CB having rank deficient is left for future work.

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