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The Convergence In Analysis Of Cylindrical Structures

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Abstract

This paper is devoted to discuses the convergence in analysis of free standing cylindrical structure due to a plane wave illumination. The numerical solution is based on the Conjugate Gradient - Fast Fourier Transform (CG-FFT) iterative scheme in the spectral domain.

Key Words

Wave scattering, Antennas

I. Introduction

In the last decade, the scattering and radiation from non-planar structures has received the attention in practical applications; e.g. cylindrical microstrip transmission line [1], Frequency Selective surfaces and radomes [2], corrugated struts for antenna systems [3], etc... . For simple structures, where the media inside and outside the structure are similar, the Green's function in an isotropic media can be used in the analysis [4]. The structures of microstrip antenna and arrays require an appropriate Green's function which is normally derived in the spectral domain for simplicity. However, the analysis of non-planer structure illuminated by a plane wave is characterized as a doubly convergence problem; the number of harmonics that represent the incident wave, and the number of Floquet's modes that describe the scattered field. Unfortunately, the convergence in the analysis of non-planer structure has not been a point of interest in most published works.

In this paper, the discrete spectral domain technique is used in analysis of one- and two-dimensional scattering problems from cylindrical structures. The technique is based on sampling the surface current and field with a convenient rate that outline the structure accurately. Although this technique may requires large computer resources (storage and processing time), it has been found accurate in the analysis of arrays with complex element configurations. One additional advantage is that the same formulation and computer code can be used for both patch and aperture arrays. However, another convergence problem arises with the selected technique of analysis

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which is the solution convergence in terms of the sampling rate. In section II, the problem formulation in the spectral domain is introduced in a suitable form for the discrete analysis. In section III, some numerical results, based on the Conjugate Gradient - Fast Fourier Transform (CG-FFT) iterative scheme in the spectral domain, are presented to illustrate the convergence in the analysis. Finally, the conclusion is presented.

II. Formulation

Consider a problem of free standing uniform array of patches conformed to a surface of circular cylinder of radius R and arranged in the orthogonal directions φ and z, as shown in Fig.1, with longitudinal displacement D_z and angular displacement $D_\varphi = \frac{2\pi}{I}$, I being the number of elements around the cylinder. For non-uniform angular displacement or patches with different geometries, one has to choose $D_\varphi = 2\pi$.

Spectral Domain Representation Of Cylindrical surface signals:

Let $f(R,z,\varphi)$ being a cylindrical surface signal with linear phase shift across the doubly periodic surface as

$$f(R,z,\varphi) = g(R)e^{-ik_{\varphi_o}\varphi}e^{-ik_{z_o}z} \qquad (1)$$

where k_{φ_o} is an integer and k_{z_o} is a real constant. Assume that the spatial domain of periodicity is divided into N_1 and N_2 segments in z and φ directions respectively such that the value of $f(R,z,\varphi)$ can be assumed constant over each cell, and denoted by $f(R,z_{n_1},\varphi_{n_2})$ for the cell (n_1,n_2). Hence

$$f(R,z_{n_1},\varphi_{n_2}) = f_{n_1,n_2} = g_{n_1,n_2}(R)e^{-ik_{\varphi_o}\varphi_{n_2}}e^{-ik_{z_o}z_{n_1}};\; n_1 = 1,N_1, and\; n_2 = 1,N_2 \qquad (2)$$

where

$$z_{n_1} = \frac{D_z}{N_1}(n_1-.5), and \;\; \varphi_{n_2} = \frac{D_\varphi}{N_2}(n_2-.5).$$

The Discrete Fourier Transformation (DFT) of $f(R,z,\varphi)$ from the domain (R,z,φ) into the spectral domain (R,k_z,k_φ) can be expressed as

$$\tilde{f}_{p,q} = \tilde{f}(R,k_{z_p},k_{\varphi_q}) = \frac{1}{N_1N_2}\sum_{n_1=0}^{N_1-1}\sum_{n_2=0}^{N_2-1} g_{n_1,n_2}e^{-i2\pi(\frac{n_1p}{N_1}+\frac{n_2q}{N_2})} \qquad (3)$$

for $p = -\frac{N_1}{2},\frac{N_1}{2}-1$ and $q = -\frac{N_2}{2},\frac{N_2}{2}-1$. The discrete components $\tilde{f}_{p,q}$ are called the coefficients of the Floquet's modes expansion of $f(R,z,\varphi)$, and defined at

$$k_{z_p} = \frac{2\pi}{D_z} p + k_{z_o}; \; -\frac{\pi}{D_z} N_1 \le k_{z_p} < \frac{\pi}{D_z} N_1,$$
$$k_{\varphi_q} = \frac{2\pi}{D_\varphi} q + k_{\varphi_o}; \; -\frac{\pi}{D_\varphi} N_2 \le k_{\varphi_q} < \frac{\pi}{D_\varphi} N_2.$$

Since $g(R)$ has a constant value over the surface, $\widetilde{f}_{p,q}$ has a dominant mode equals $g(R)$ at (k_{z_o}, k_{φ_o}), and zero elsewhere. If the signal on the surface is a sum of two or more signals each has a linear phase shift around the cylinder, the location of Floquet's modes of each are similar to the others regardless the order of arrangement. This point allows the simultaneous representation of multi-components signal in the spectral domain.

Incident Plane Wave Expansion

For a plane wave incidence from angles φ' and ϑ' with the x and z axes respectively, the electric field at the cylinder surface is derived from the unit amplitude TE_z and TM_z vector potentials given by

$$A_{TE/TM}(R,z,\varphi) = e^{ikR\cos((\varphi-\varphi')\sin\vartheta'} e^{ikz\cos\vartheta'} \tag{4}$$

where k is the wave number. It can be expanded in terms of an infinite number of Floquet's modes as [5]

$$A_{TE/TM}(R,z,\varphi) = \sum_{k_{\varphi Q}=-\infty}^{\infty} i^{k_{\varphi Q}} J_{k_{\varphi Q}}(kR\sin\vartheta') e^{ik_{\varphi Q}\varphi} e^{ikz\cos\vartheta'} \tag{5}$$

where k_{φ_Q} is an integer. The location of the Floquet's modes is defined in the domain (R, k_z, k_φ) at intervals of length 1 on an axis parallel to k_φ axis and shifted down in k_z direction by $k\cos\vartheta'$. Note that Eqn.(5) represents an infinite number of cylinder signals each has a constant amplitude $i^{k_{\varphi Q}} J_{k_{\varphi Q}}(kR\sin\vartheta')$ with linear phase shift across the surface. For a truncated number of modes equals the number of angular samples N_2, the coefficients of the Floquet's modes can be obtained using DFT of N_2 samples around the surface.

Dyadic Green's Function Of Cylindrical Structure:

The tangential scattered field at discrete locations on a cylinder surface can be written in terms of the Floquet's modes expansion coefficients as

$$\begin{bmatrix} E^s_\varphi(R,z_{n_1},\varphi_{n_2}) \\ E^s_z(R,z_{n_1},\varphi_{n_2}) \end{bmatrix} = \sum_{k_{z_p}=-\frac{N_1}{2}}^{\frac{N_1}{2}-1} \sum_{k_{\varphi_q}=-\frac{N_2}{2}}^{\frac{N_2}{2}-1} \begin{bmatrix} \widetilde{E}^s_\varphi(R,k_{z_p},k_{\varphi_q}) \\ \widetilde{E}^s_z(R,k_{z_p},k_{\varphi_q}) \end{bmatrix} e^{i2\pi(\frac{n_1 p}{N_1}+\frac{n_2 q}{N_2})} . \tag{6}$$

for truncated numbers of modes N_1 and N_2 in k_z and k_φ directions respectively, where p and q denote the longitudinal and angular Floquet's modes respectively. The tilled over the quantity denotes the quantity in the spectral domain.

The procedure of deriving thé cylindrical Green's function in the spectral domain is similar to that for planar structure where the scattered field is decomposed into those TE_z and TM_z. The solution of the wave equation in cylindrical coordinates, and forcing the electric field to be continuos through the surface and relating the magnetic field to the induced electric current, result [2]

$$\begin{bmatrix} \tilde{E}_\varphi(R,k_{z_p},k_{\varphi_q}) \\ \tilde{E}_z(R,k_{z_p},k_{\varphi_q}) \end{bmatrix} = \begin{bmatrix} G_{\varphi\varphi} & G_{\varphi z} \\ G_{z\varphi} & G_{zz} \end{bmatrix} \begin{bmatrix} \tilde{J}_\varphi(R,k_{z_p},k_{\varphi_q}) \\ \tilde{J}_z(R,k_{z_p},k_{\varphi_q}) \end{bmatrix} \qquad (7)$$

where

$$G_{\varphi\varphi} = -\frac{60\pi^2}{kR}\left[(kR)^2 J'_{k_{\varphi_q}}(k_{\rho_p}R) H'^{(2)}_{k_{\varphi_q}}(k_{\rho_p}R) + \left(\frac{k_{\varphi_q}k_{z_p}}{k_{\rho_p}}\right)^2 J_{k_{\varphi_q}}(k_{\rho_p}R) H^{(2)}_{k_{\varphi_q}}(k_{\rho_p}R)\right]$$

$$G_{\varphi z} = G_{z\varphi} = \frac{60\pi^2}{k} k_{\varphi_q} k_{z_p} J_{k_{\varphi_q}}(k_{\rho_p}R) H^{(2)}_{k_{\varphi_q}}(k_{\rho_p}R)$$

$$G_{zz} = -\frac{60\pi^2}{k} k_{\rho_p}^2 R J_{k_{\varphi_q}}(k_{\rho_p}R) H^{(2)}_{k_{\varphi_q}}(k_{\rho_p}R)$$

and $J_{k_{\varphi_q}}(k_{\rho_p}R)$ and $H^{(2)}_{k_{\varphi_q}}(k_{\rho_p}R)$ are the Bessel and second type Hankel functions of order k_{φ_q}. The radial wave number k_{ρ_p} of the Floquet's mode (p,q) is given by

$$k_{\rho_p} = \begin{cases} \sqrt{k^2 - k_{z_p}^2} \; ; & k > k_{z_p} \\ -i\sqrt{k_{z_p}^2 - k^2} \; ; & k \le k_{z_p} \end{cases}.$$

III. Numerical Results

The results in this section discuses the convergence in analysis of cylindrical structures. The first three cases are one-dimensional problems where the convergence in terms of the angular plane wave modes, angular scattered Floquet's modes, and angular sampling rate are discussed. The fourth case is a two dimensional problem where the convergence in terms of the longitudinal Floquet's modes is discussed.

Angular Expansion Modes of an Incident Plane Wave

Consider an infinitely extended solid cylinder in z of size $kR = 4$ illuminated by TE_z plane wave with $\varphi' = 0^o$. Since there is no discontinuity in the structure, the sufficient number q of the angular Floquet's modes equals the angular expansion modes $Q \le N_2$ of the incident plane wave. Let $Q = LB$ where B is an integer defined by the Carson's rule in Bessel function expansion $B = Int.(2(kR+1))$, and L is a constant to be selected such that Q becomes integer. In Fig.2, the RCS (σ / λ) is computed for L equals 1, 1.5, and 2 with N_2=128 that fulfills a high sampling rate equals $32/\lambda$. It is obvious that the two curves of L=2 (solid line) and 1.5 (plus scatterer) are coincident. Reducing the number Q with L=1 (circle scatterer) causes some deviations from the two other cases, but produces a similar results to that given in [2] where Q is selected such that no evident contribution to the surface current from the higher modes. Therefore, it is more convenient to discuss the convergence with the scattered field rather than the surface current. From the above, the proper number Q has to fulfill the criteria

$$Q \geq 3Int.(kR+1). \tag{8}$$

Angular Expansion Modes of The Scattered wave

Consider a problem of uniform array consists of four strips conformed to a cylinder of size $kR = 4$ with a strip width equals half the angular displacement D_φ. This problem requires an angular Floquet's modes number q greater than Q. For q equals 1, 1.5, and 2 times Q, the σ / λ due to a TM_z plane wave illumination from angle $\varphi' = 0^o$ directly on the center of the first strip are shown in Figure 3.a using N_2=128 with Q fulfills the criteria given by En.(8). It is obvious that there is a good convergence with $q \geq 1.5Q$; compare the solid line and the plus scatterer curves. The third curve denoted by circle scatterer for $q = Q$ deviates slightly in the angular sector of the main lope. Further reduction in q causes a significant change in computed σ / λ. For the three selected values of q, no significant differences have been noted in the induced surface current. In Fig.3.b, the longitudinal surface current J_z where the edge effect and the symmetry behavior can be noted. In the following, q is chosen to be equal $2Q$.

Angular Sampling rate:

Consider a problem of a half cylinder of size $kR = 4$ as shown in Fig.4. Assume that the structure is illuminated by TE_z plane wave with $\varphi' = 180^o$ directly on the center of the conductor. Choose Q=20 and q=40, and choose N_2 equals 128, 64, or 32 to achieve a sampling rate equals 32, 16, or 8/λ, respectively. The absolute value of the total E_φ (incident and scattered) at a circle of radius R is shown in Fig.4 for the three selected sampling rate. It can be noted that the conductor boundary condition is fulfilled in the three cases, with accepted convergence in the computed field at the cylinder aperture. It has been noted that further reduction in the sampling rate may causes some deviations in the scattered field. Therefore, 8 samples/λ is the minimum angular sampling rate if it could outline the structure properly, and has to be greater than Q.

Longitudinal Expansion Modes Scattered wave

Consider a two-dimensional problem of longitudinal periodic metallic rings with ring width equals half the length of periodicity, as shown in Fig.5, with $kR = 4$ and D_z =.6λ. Choose $q = Q = 20$, and N_2=64 with N_1=32, 16, or 8. Note that in the discrete spectral analysis, the number of longitudinal expansion modes equals the number of longitudinal samples N_1. In Figs.5.a and b, σ / λ is shown in the two cases of TM_z and TE_z plane wave illumination with $\varphi' = 0^o$ for the three selected values of N_1. In both Figures, there is a convergence with increasing N_1. One has to note that increasing the longitudinal sampling rate extends the spectral domain in k_z direction and increases the number of modes with complex radial wave number. Some difficulties may aries in the numerical computation of the Bessel function with complex argument due to its positive exponential charactristics. Therefore, the proper longitudinal sampling rate has to outline the structure in the longitudinal direction accuratley, and aviods the overflow and the round-off errors in the computation. From the results in Fig.5, 32 samples/ λ is a proper rate for the considered problem.

IV. Conclusion

The angular and longitudinal sampling rates in the discrete spectral domain analysis of cylindrical structures are governed by the necessary numbers of the longitudinal and angular Floquet's modes that describe the scattered field accurately without numerical computation problems. The number of angular samples N_2 is related to the number of angular expansion modes by

$$N_2 \geq q \geq 1.5Q \text{, with minimum } N_2 \geq 8 / \lambda, and Q \geq 3 Int.(kR+1)$$

In addition, the number of longitudinal samples N_1 must be greater than the necessary number of longitudinal Floquet's modes p that guarantee the convergence in the analysis without overflow or round-off error in the computation. A longitudinal sampling rate around 32 samples/ λ is suggested for testing the soultion convergence.

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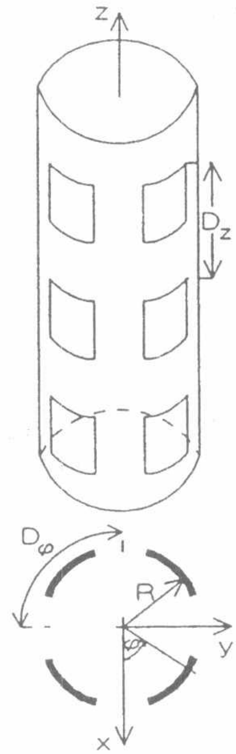

Fig.1 Geometry of surface periodicity

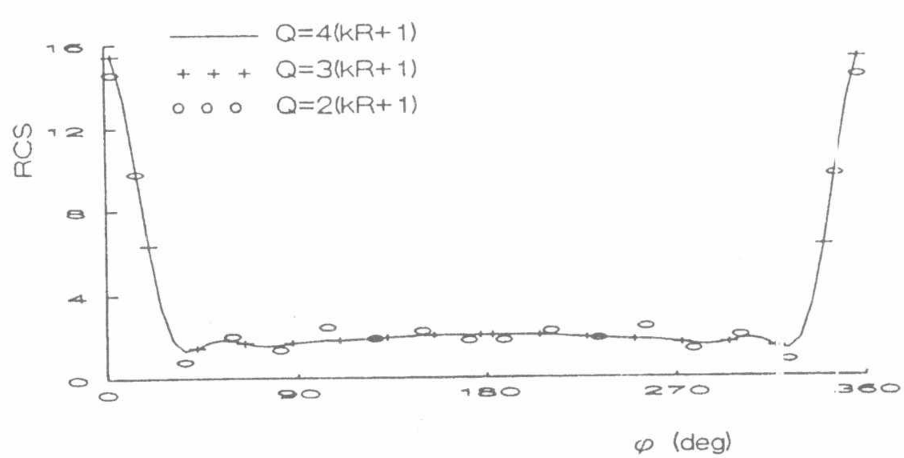

Fig.2 RCS of a solid cylider of size kR=4 due to TM_z plane wave illumination for various number Q

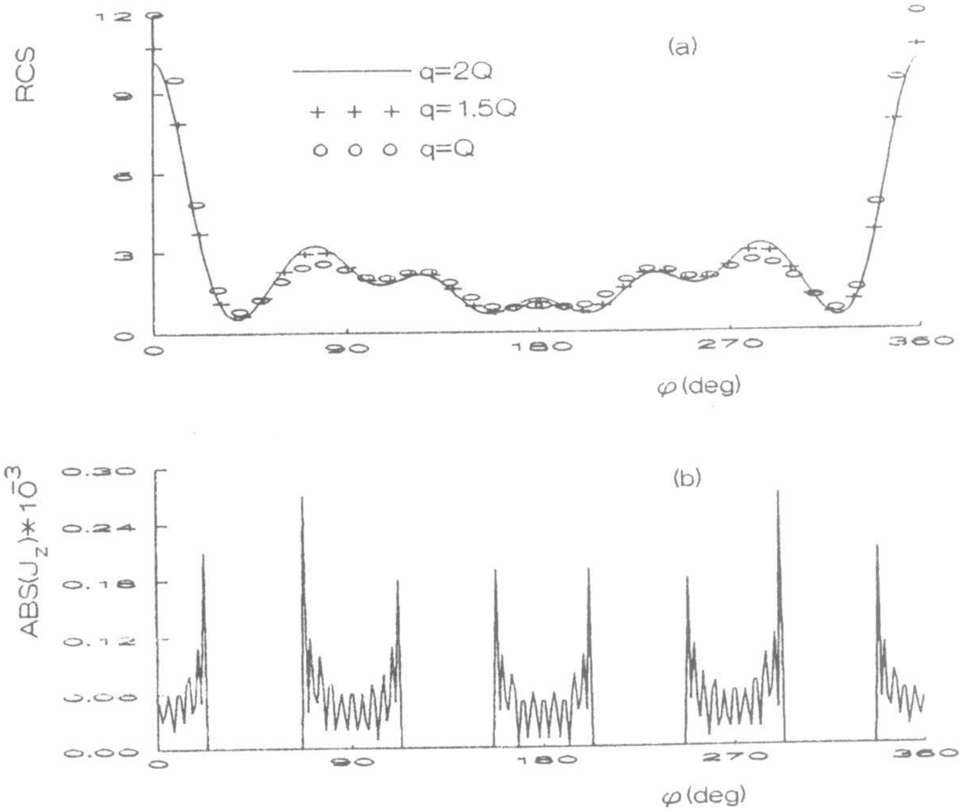

Fig.3 (a) the RCS for various q, and (b) J_z vs. φ due to TM_z plane wave illumination from $\varphi=0^{\circ}$ of uniform array of 4 strips

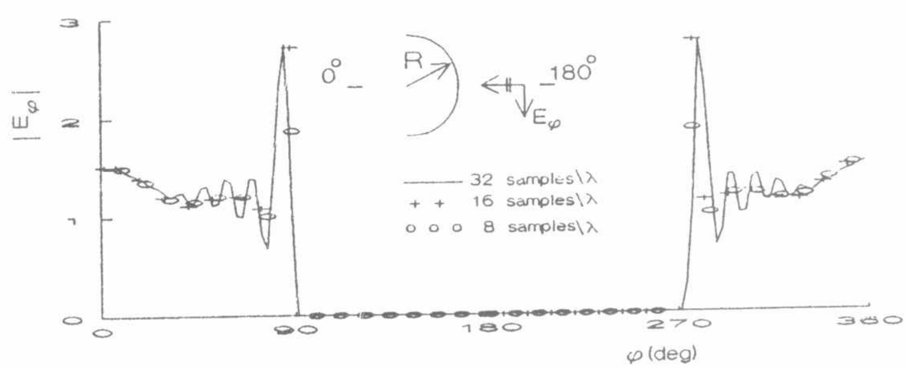

Fig.4 E_{φ} at circle of radius R due to TE_z plane wave illuminat. of a half cylinder of size kR=4 from $\varphi=0$ for various N_2

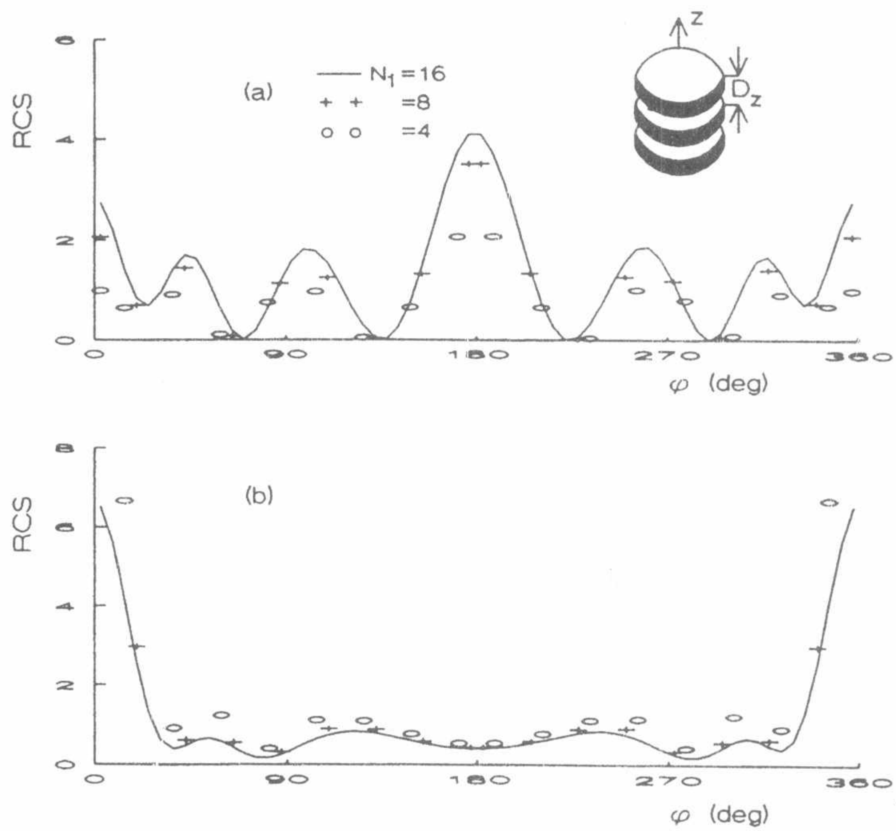

Fig.5 The RCS vs. φ from array of rings of size kR=4 due to (a) TM_z (b) TE_z plane wave illumination for various N_1