Proceedings of the 2nd ICEENG Conference, 23-25 Nov. 1999

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Military Technical College
Kobry Elkobbah,
Cairo, Egypt

2nd International Conference
on Electrical Engineering
ICEENG 99

PERFORMANCE ANALYSIS OF SYNCHRONOUS DIGITAL HIERARCHY (SDH) POINTERS

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I- Abstract

In this paper, a mathematical derivation for the probability to interpret an erroneous pointer in the SDH network is carried out. The derivation is based on International Telecommunication Union Telecommunication standardization section (ITU-T) recommendations for both the steady state and at the beginning of new data. The poses and cones of the ITU-T recommended way are discussed. To validate the analytically derived equations, a simulation program for a communication channel utilizing the SDH technique is developed. The results of both the analytical derived equations and the simulation program are presented. The obtained results are very close to each other.

II- Key Words

SDH, SONET, Performance, Analysis, Probability, Pointers

III- Introduction

It is known that both SDH and Synchronous Optical NETwork (SONET) technologies will be widely deployed in access metropolitan and toll networks to transport both today's and tomorrow's services (e.g., high quality video, high resolution imaging, high speed data) [1]. Contrary to the existing asynchronous hierarchy (Plesiochronous Digital Hierarchy (PDH)), the advantage of the SDH/SONET format is the ease with which the signals can be manipulated (e.g. multiplexed, added, dropped and/or cross-connected). The SDH/SONET standard also eliminates the back-to-back intermediate interfaces that are required in today's PDH transmission networks. The higher level of integration possible with SDH/SONET leads to fewer components at transmission nodes and less susceptibility to failures and errors due

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to both intrinsic causes and maintenance activities [2]. In SDH the administration unit (AU) pointer provides a method of allowing flexible and dynamic alignment of the virtual container (VC) within the AU frame. It means that the VC is allowed to "float" within the AU frame. Thus the pointer is able to accommodate differences not only in the phases of the VC and section overhead (SOH) but in the frame rates as well. Interpreting an erroneous pointer is very critical because all the received data in that frame will be erroneous data and will be lost. That is why ITU-T has proposed an excellent algorithm to improve the pointer immunity against communications channel errors [3]. The pointer is composed of two bytes (16 bits) as shown in figure 1. NDF is enabled if a new pointer value is sent, otherwise it is disabled. In NDF a single error can be easily detected and corrected.

N	N	N	N	S	S	I	D	I	D	I	D	I	D	I	D

10 bits pointer value

N New data flag (NDF)
SS Administration unit type

I Increment
D Decrement

NDF is enabled when at least 3 out of 4 bits match "1001".
NDF is disabled when at least 3 out of 4 bits match "0110".

Negative justification (Invert 5 D-bits and accept majority vote).
Positive justification (Invert 5 I-bits and accept majority vote).

Figure 1. SDH Administration Pointer Coding.

IV- ITU-T rules for interpreting the AU pointers:

The following summarizes the rules recommended by ITU-T for interpreting the AU pointers [3].

1) During normal operation, the pointer locates the start of the VC within the AU frame.
2) Any variation from the current pointer value is ignored unless a consistent new value is received three times consecutively or it is preceded by one of rules 3. 4 or 5.
3) If the majority of the I-bits of the pointer word are inverted, a positive justification operation is indicated. Subsequent pointer values shall be incremented by one.
4) If the majority of the D-bits of the pointer word are inverted, a negative justification operation is indicated. Subsequent pointer values shall be decremented by one.
5) If the NDF is set to 1001, then the coincident pointer value shall replace the current one at the offset indicated by the new pointer value regardless of the state of the receiver.

According to the rules listed above, we may divide pointer interpretation into two time domains. These are, when the pointer value is new (NDF is enabled) and when the pointer value is not new (NDF is disabled).

i- When the pointer value is new (NDF is enabled)

If the bit error rate of a communication channel is Q and the pointer contains 10 bits (only 10 bits of the 16 bits of the pointer are user to determine the pointer value). The probability of receiving a correct pointer value is:

$$P_{CNEW} = (1 - Q)^{10} \tag{1-a}$$

The probability of receiving an erroneous pointer value is:

$$P_{ENEW} = 1 - P_{CNEW}$$

$$P_{ENEW} = 1 - (1 - Q)^{10} \tag{1-b}$$

At practical values of Q (Q <<1) we have

$$P_{ENEW} \approx 10Q \tag{1-c}$$

ii- When the pointer value is not new (NDF is disabled)

An erroneous pointer value may be interpreted in any of the following four scenarios:

a- According to rule 2 above, an erroneous pointer value will be interpreted if three successive erroneous pointers in each of which the same bit pattern is received.

b- According to rule 3 above, pointer value will be incremented (after a positive justification operation) if at least three of the I-bits of the pointer word are received erroneously inverted.

c- According to rule 4 above, pointer value will be decremented (after a negative justification operation) if at least three of the D-bits of the pointer word are received erroneously inverted.

d- According to rule 5 above, an erroneous pointer value will be interpreted if the NDF is erroneously enabled (at least three of the N bits were erroneously inverted) and at least one of the bits carrying the pointer value is erroneously received.

Scenarios a:

If we have *k* bits, the probability to receive *j* erroneous bits from them is:

$$P_{ejk} = \frac{k!}{j!\,(k-j)!} Q^{j} (1-Q)^{k-j} \tag{2}$$

where P_{ejk} is probability to have *j* erroneous bits from *k* bits.

To receive the same bit (one of the 10 bits that determine the pointer value) erroneously three successive times we have k=3 and j=3.

$$P_{e33} = \frac{3!}{3!\,0!} Q^3 (1-Q)^0$$

$$P_{e33} = Q^3 \tag{3}$$

According to equation (2), the probability to receive a correct pointer value is

$$P_{EA} = (P_{CNDF})^3 (10 P_{e33}\ (1-P_{e33})^9)$$

Where P_{CNDF} is probability to receive a correct NDF indicator

We receive correct NDF if one or none erroneous bits were received from the four NDF bits. And according to equation (2) we have:

$$\begin{aligned} P_{CNDF} &= P_{e14} + P_{e04} \\ &= 4Q(1-Q)^3 + (1-Q)^4 \\ &= (1-Q)^3 (1+3Q) \end{aligned}$$

$$P_{EA} = 10Q^3 (1-Q)^9 (1+3Q)^3 (1-Q^3)^9 \tag{4-a}$$

At practical values of Q (Q <<1) we have:

$$P_{EA} \approx 10Q^3 \tag{4-b}$$

Scenarios b and c:

An erroneous increment (or decrement) will be received if at least three of the I (or D) bits are received inverted and NDF is disabled (i.e. NDF is received correctly).

$$P_{EC} = P_{EB} \tag{5}$$

$$P_{EB} = P_{CNDF} (P_{e35} + P_{e45} + P_{e55})$$

According to equation (2) we have:

$$P_{e35} = 10Q^3 (1-Q)^2$$

$$P_{e45} = 5Q^4 (1-Q)$$

$$P_{e55} = Q^5$$

Then

$$P_{EB} = Q^3 (1+3Q)(1-Q)^3 (10 - 15Q - 6Q^2) \tag{6-a}$$

At practical values of Q (Q <<1) we have

$$P_{EB} = 10Q^3 \tag{6-b}$$

Scenario d:

According to rule 5 above, an erroneous pointer value will be interpreted if the NDF is erroneously enabled (at least three of the N bits are erroneously inverted) and at least one of the bits carrying the pointer value is erroneously received.

$$P_{ED} = P_{ENDF} \; P_{EP}$$

$$P_{EP} = 1 - (1-Q)^{10}$$

$$\begin{aligned} P_{ENDF} &= P_{e34} + P_{e44} \\ &= 4Q^3(1-Q) + Q^4 \\ &= Q^3(4-3Q) \end{aligned}$$

$$P_{ED} = Q^3(4-3Q)(1-(1-Q)^{10}) \qquad (7\text{-a})$$

At practical values of Q (Q <<1) we have:

$$P_{ED} = 40Q^4 \qquad (7\text{-b})$$

According to [4] the final total error probability P_{ET} is:

$$P_{ET} = P_{EA} \cup P_{EB} \cup P_{EC} \cup P_{ED}$$

$$\begin{aligned} P_{ET} = {} & (P_{EA} + P_{EB} + P_{EC} + P_{ED}) - \\ & (P_{EA} \cap P_{EB} + P_{EA} \cap P_{EC} + P_{EA} \cap P_{ED} + P_{EB} \cap P_{EC} + P_{EB} \cap P_{ED} + P_{EC} \cap P_{ED}) + \\ & (P_{EA} \cap P_{EB} \cap P_{EC} + P_{EA} \cap P_{EC} \cap P_{ED} + P_{EB} \cap P_{EC} \cap P_{ED}) - \\ & (P_{EA} \cap P_{EB} \cap P_{EC} \cap P_{ED}) \end{aligned} \qquad (8)$$

Since the bit error rate (Q) in the normal communication channels is very small compared to the unit (Q << 1), then the second, third and fourth terms in equation (8) may be neglected. This will significantly simplify the equation with a very minor effect on the accuracy.

$$P_{ET} \approx P_{EA} + P_{EB} + P_{EC} + P_{ED} \qquad (9)$$

iii- Error propagation

If an erroneous pointer was interpreted, all the following pointers will subsequently be interpreted erroneously until one of the following two events happen:

- The NDF is disabled and a correct value is received three times consecutively

  or
- The NDF is erroneously enabled and a correct pointer value is received.

Then the probability for error propagation is:

$$P_{ep} = 1 - (P_{CNDF} * P_{CNEW})^3 - P_{ENDF} * P_{CNEW}$$

According to the ITU-T recommendations, at least three frames must be sent before changing the value of a pointer in SDH. Thus the final probability of interpreting an erroneous pointer in an SDH system is

At new data:

$$P_{E-NEW-POINTER} = P_{ENEW}(3+P_{ep}+p^2{}_{ep}+P_{ep}{}^3+\ldots\ldots\ldots\ldots)$$
$$= P_{ENEW}(3+\frac{P_{ep}}{1-P_{ep}}) \qquad (10\text{-a})$$

At practical values of Q (Q <<1) we have:

$$P_{E-NEW-POINTER} \approx 30Q \qquad (10\text{-b})$$

In the steady state:

$$P_{E-POINTER} = P_{ET}(3+P_{ep}+p^2{}_{ep}+P_{ep}{}^3+\ldots\ldots\ldots\ldots)$$
$$= P_{ET}(3+\frac{P_{ep}}{1-P_{ep}}) \qquad (11\text{-a})$$

At practical values of Q (Q <<1) we have:

$$P_{E-POINTER} \approx 90Q^3 \qquad (11\text{-b})$$

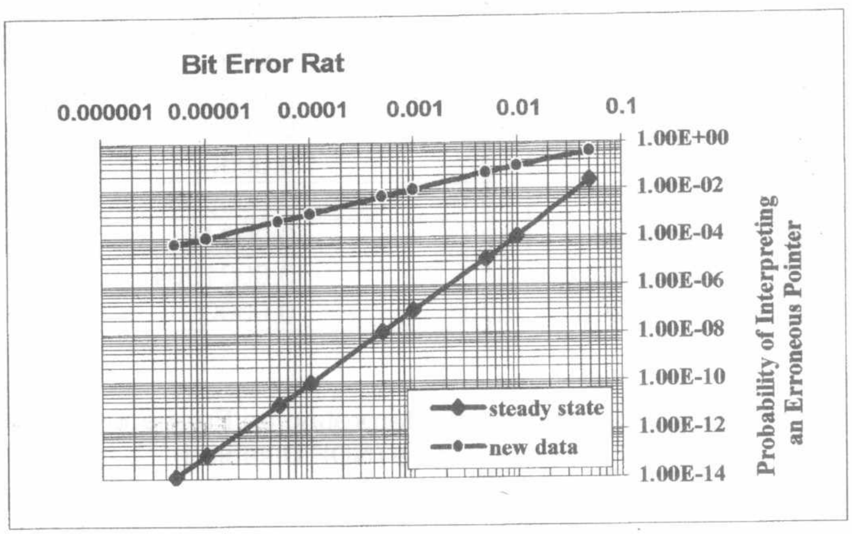

Figure 2. Probability of Interpreting an Erroneous Pointer

From equation (1), it is clear that the probability of interpreting an erroneous pointer at the beginning of new data is proportional to Q while in the steady state and according to equations (4) through (11), that probability is proportional to Q^3. It is clear that in the steady state the pointer is strongly immunized against communication errors. This is one advantage of the ITU-T recommended manner of interpreting the pointer. The disadvantages of this recommendations are the low immunity against errors at the beginning of new data (as shown in Figure 2) and error propagation in the event an erroneous pointer was interpreted.

V- Simulator Program

A simulator program was written in FORTRAN to simulate a communication channel utilizing the SDH technique. The program is composed of several cascaded modules. Each module simulates a block of the simulated channel. The communication channel errors were simulated by a random error source using the random function generator built into the FORTRAN package. At the receiver, a

Table 1. Probability of Interpreting an Erroneous Pointer
New Frames (NDF Enabled).

Bit Error Rate Q	Simulator Program (NDF Enabled).		Probability of Interpreting an Erroneous Pointer (NDF Enabled).	
	No. of Transmitted Frames	No. of Erroneously Interpreted Pointers	Simulator Results	Analytically Calculated
0.1	1000	630	0.63	0.651322
0.05	1000	420	0.42	0.401263
0.01	5000	480	0.096	0.0956179
0.005	10000	498	0.0498	0.0488899
0.001	50000	490	0.0098	0.00995512
0.0005	100000	507	0.00507	0.00498877
0.0001	500000	493	0.000986	0.00099955
0.00005	1000000	480	0.00048	0.000499888
0.00001	5000000	502	0.0001004	0.0000999955
0.000005	10000000	500	0.00005	0.0000499989

Table 2. Probability of Interpreting an Erroneous Pointer
Steady State (NDF Disabled).

Bit Error Rate Q	Simulator Program (NDF Disabled).		Probability of Interpreting an Erroneous Pointer (NDF Disabled).	
	No. of Transmitted Frames	No. of Erroneously Interpreted Pointers	Simulator Results	Analytically Calculated
0.10000	100	32	0.32	0.502345
0.05000	1000	23	0.023	0.240143E-01
0.01000	100000	9	0.9E-4	0.100717E-03
0.00500	1000000	12	0.12E-4	0.118731E-04
0.00100	100000000	9	0.9E-7	0.909428E-07

statistical analysis on the received signal was done. The analysis results insured that the bit error rate at the receiver side is very close to that given to the simulator. The simulator was used to validate the analytically derived equations at different bit error rates. To reduce the time loss in the event of any unpredictable problem, the intermediate results and variables are recorded once for every one- percent of the processing cycle. Running the simulator on a Pentium 233 MMX computer, about 200 frames per second were processed. To reduce the duration of the processing cycle, high bit error rates were used during the simulation process. Five cycles were carried out and the average results are provided in Tables 1 and 2.

VI- Conclusion

1-By comparing the output of the simulator program and the results of the analytically derived equations provided in Table 1 and Table 2, we notice that:

i- When the bit error rate is high ($Q = 0.1$ which is not practical), there are some deviations between the output of the simulator program and the results of the analytically derived equations. These deviations are due to the approximation we made during the derivation of the equations which is valid only for $Q << 1$.

ii- As the bit error rate decreases (becoming more practical), both the analytical and the simulated results become very close to each other. This means that the analytically derived equations and the simulator program are correct at practical values of bit error rates.

2-From equations 1 and 10, we notice that the probability of interpreting an erroneous pointer in the case of new frames (NDF enabled) is too high (about ten times the bit error rate). If we consider the error propagation, the probability will be greater than 30 times the bit error rate. An error-correcting algorithm is strongly recommended, especially for the pointer at the first frame transmitted over a satellite (or microwave) communications channels where the bit error rates may at times exceed 0.00001.

VII- References

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