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TRELLIS FREQUENCY AND PHASE MODULATION IN FADING MOBILE SATELLITE CHANNELS

by

*Prof. Dr. R. El-awady * , M. A. Bahie-Eldin**, and Eng. L. F. Gergis **

ABSTRACT

Most satellite communication systems are power and bandwidth-limited. In band limited nonlinear satellite applications, various researchers have shown that respectable coding gains can be achieved by Ungerboeck TCM (trellis coded modulation) codes [1], which improve error performance without reducing data rates or requiring more bandwidth than conventional uncoded schemes. This is achieved by channel coding with expanded sets of multilevel phase signals, such that the free Euclidean distance is maximized. A coded modulation format is defined over an expanded set of signals varying both in frequency and phase modulation (FPM). This special technique combines Frequency Shift Keying (FSK) and Phase Shift Keying (PSK) modulations. The FPM signal has been transmitted through nonlinear satellite transponder which exhibits amplitude and phase distortion (AM/AM and AM/PM), respectively, [2]. The TCM / FPM schemes demonstrate improved performance over TCM / MPSK systems on AWGN and the fading channels.

I. INTRODUCTION

The use of error correcting codes can improve the performance of a digital satellite communication at the expense of bandwidth expansion. For power - limited channels, one may trade the bandwidth expansion for transmitted power in order to achieve a desired system performance. In the past, coding and modulation have been considered as two separate parts of a digital communication system. High performance is achieved by lowering the code "rate" and increasing the number of redundancy in the code at a cost of decoding complexity and bandwidth expansion.

Ungerboeck [1] presented a method that integrates a convolutional code with a bandwidth - efficient modulation scheme. Significant coding gain can be demonstrated compared to the uncoded schemes without requiring additional bandwidth. This technique is called Trellis Coded Modulation (TCM). In TCM systems, a redundant 2^{m+1} - ary signal set is used in order to transmit m bits in each signaling interval. The m input bits are encoded by a rate $m\ /(m+1)$ trellis encoder and $m+1$ encoded bits are mapped to the signal points of the 2^{m+1} - ary signal set in such a way that the minimum Euclidean distance between channel - signal sequences is maximized. These

* FACULTY OF ENGINEERING MANSOURA UNIVERSITY
**EGYPTIAN ARMED FORCES

available signal space dimensions. It uses two data shaping pulses and two carriers, which are pairwise quadrature in phase, to create two more dimensions than what are available in QPSK and minimum shift keying (MSK). The proposed scheme thus increases the bandwidth efficiency by a factor of two over QPSK and MSK without altering the bit energy requirement substantially. The FPM signal, in the absence of any additional constraint, does not maintain a constant envelope. But for nonlinear channels, the constant envelope in the modulated signal may be an important desirable feature [6]. The main objective for this study is to present a generalization of modulation scheme that is less sensitive to channel nonlinearities [7]. Section 3 considers the study of error probability for the proposed TCM/FPM scheme.

II. SYSTEM MODEL

The system under consideration is illustrated by the block diagram in Fig.1. It consists of trellis encoder, modulator and mapper , the satellite transponder and fading channel. At the receiver, the faded and distorted signals are decoded by Viterbi decoder.

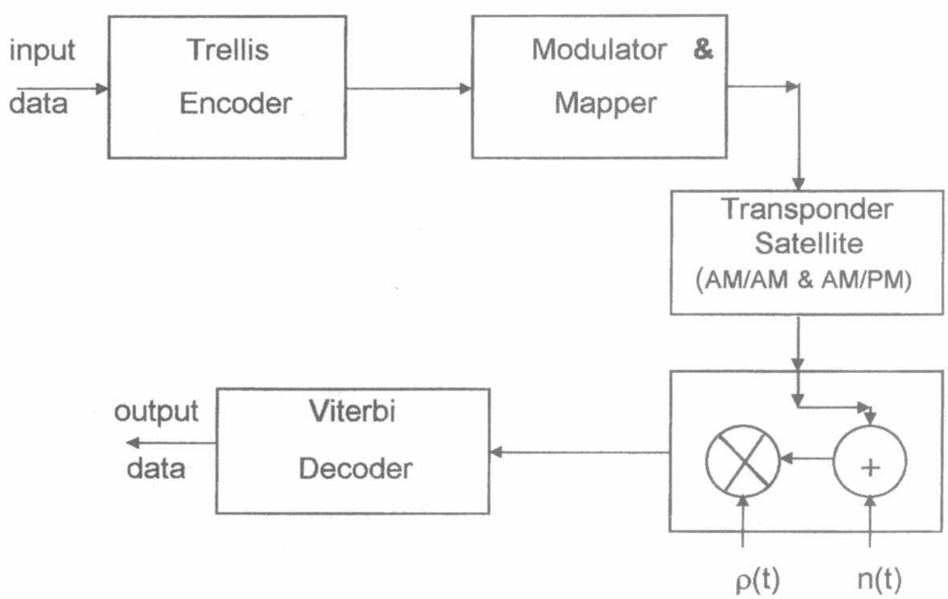

Fig. 1. System Model "Block Diagram"

Coded Frequency and Phase Modulation

In the case of binary orthogonal FSK, two pairs of signals, $\cos(\omega_c \pm \omega_d)t$ and $\sin(\omega_c \pm \omega_d)t$ are generated, where $2\omega_d$ is the separation between the frequencies. If $\omega_c T = \lambda\pi$, with λ an integer, and the minimun separation between the frequencies is $2 f_d = 1 / T$ where $T = 2 T_b$ is the duration of 4-ary, the signal constellation is four dimensional [7].

Considering those signals which are combination of binary FSK and M-ary PSK. The two frequencies are taken as $2 f_d = h / T$ which lead to the two pairs :

i) $\cos (\omega_c t \pm h\pi t / T)$ rad/s

ii) $\sin (\omega_c t \pm h\pi t / T)$ rad/s

where h is the modulation index.

The scheme could be explained by means of an example. In [6], A convolutional encoder with rate R = 2/3 has been used to expand 8-PSK signal constellation sets in which different combinations of the three bits at the output of the encoder select one of the eight phase shifts. We use *one* of the three bits at the output of the convolutional encoder, at each T interval, to select one of the two carrier frequencies (the FSK part of the modulation) and the remaining *two* bits to select one of the four phase shifts to be applied to the corresponding carrier. This proposed scheme is referred to as *2FSK / QPSK* coding. The block diagram of this example is shown in Figure. 2.

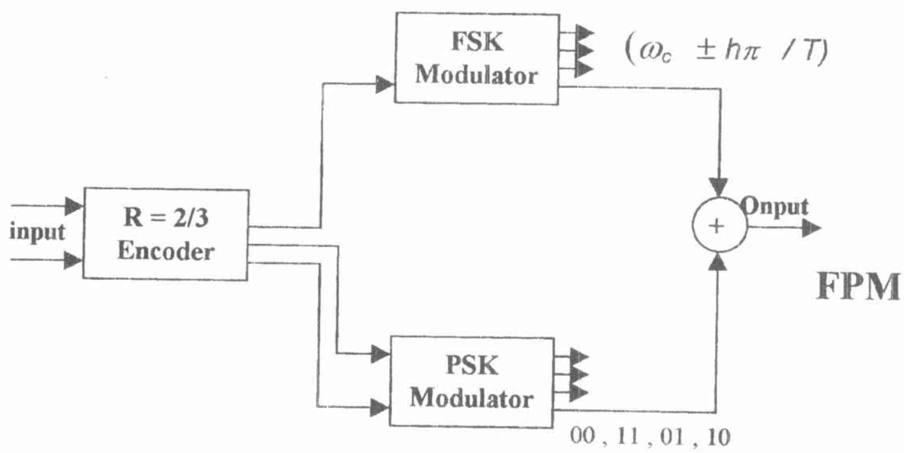

Fig. 2 . Block Diagram of the proposed 2FSK / QPSK

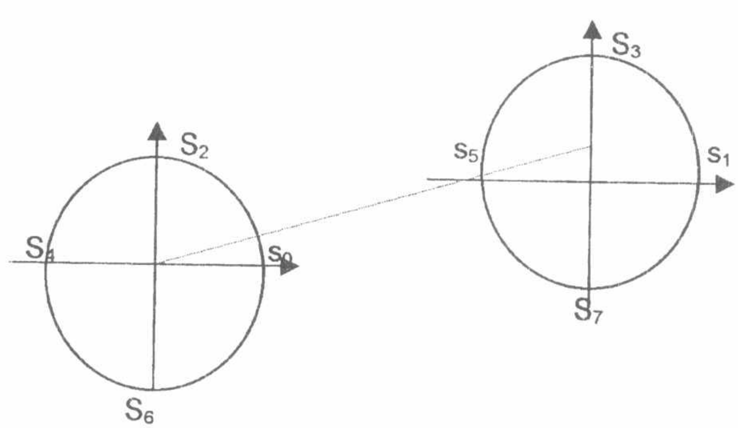

Fig. 3 . The 2FSK / QPSK signal set sketched as a pair of two - dimensional 4PSK [6]

Expurgated Phase Codes

The performance of the transmitted signal can be improved without increasing the spectral efficiency [7], by expurgating the signal phase codes. The signal constellation for this case is shown in Figure .3.

III. ERROR PROBABILITY FOR TCM/FPM SCHEME

In this section, the performance upper bounds for TCM / FPM over AWGN and Rayleigh fading will be considered. The symbol constituting the FPM set for the 2FSK / MPSK is defined by [6] :

$$S(t) = \sqrt{2E_s/T}\ \cos[\omega_c t + \Psi(t,\alpha) - \theta_i] \quad (1)$$

$$\theta_i \in \{0, \pi/2, \pi, 3\pi/2\}.$$

where the information carrying phase is [8]

$$\Psi(t,\alpha) = 2\,\omega\, h \int_{-\infty}^{t} \sum_{i=-\infty}^{\infty} \alpha_i\, g(\tau - iT)\, d\tau \quad (2)$$

with E_s is the symbol energy, T is the symbol time and ω_c is the carrier frequency .The transmitted signals mapped by set patitioning, are shown in Figure.4. The FPM signal is assumed to be transmitted over an additive white Gaussian channel having a one sided power spectral density N_0 . Thus the received signal at the receiver is :

$$r(t) = S(t,\alpha) + n(t) \qquad -\infty < t < \infty \quad (3)$$

where n(t) is a Gaussian random process with zero mean and one-sided power spectral density N_0 . A maximum Likelihood sequence estimation (MLSE) detector is used to choose the infinitely long sequence α' which minimizes the error probability.

The average bit error probability for trellis coded signals for any type of modulation scheme is upper bounded by the union bound [4]

$$P \le \sum_{x}\sum_{x'\in C} N(x, x')\, p(x)\, P(x \rightarrow x') \quad (4)$$

where N(x,x')is the number of bit errors that occur when the sequence x is transmitted and the sequence $x' \neq x$ is chosen by the decoder, $p(x)$ denotes the priori probability of transmitting x, C is the set of all coded sequence , and $P(x \rightarrow x')$ represents the pairwise error probability.

The upper bound is evaluated using the transfer function bound approach applied to TCM in [3-5]. It is conditioned on the amplitude vector $\rho = (\rho_1, \rho_2, \ldots\ldots, \rho_n)$, the pairwise error probability is given by

$$P(x \rightarrow x'/\rho) \le \exp\left\{-(E_s/4N_0)\, d^2(x,x')\right\} \quad (5)$$

where

$$d^2\,(x\,,\,x') \cong \sum_{L=1}^{L} \rho^2_L \,|\, x - x' \,|^2 \qquad (6)$$

which represents the square of *Euclidean distance* between the two symbol sequences x and x'. ρ_L is the normalized fading amplitude for the *Lth* transmission interval, and L is the length of the error sequence $x' \neq x$.

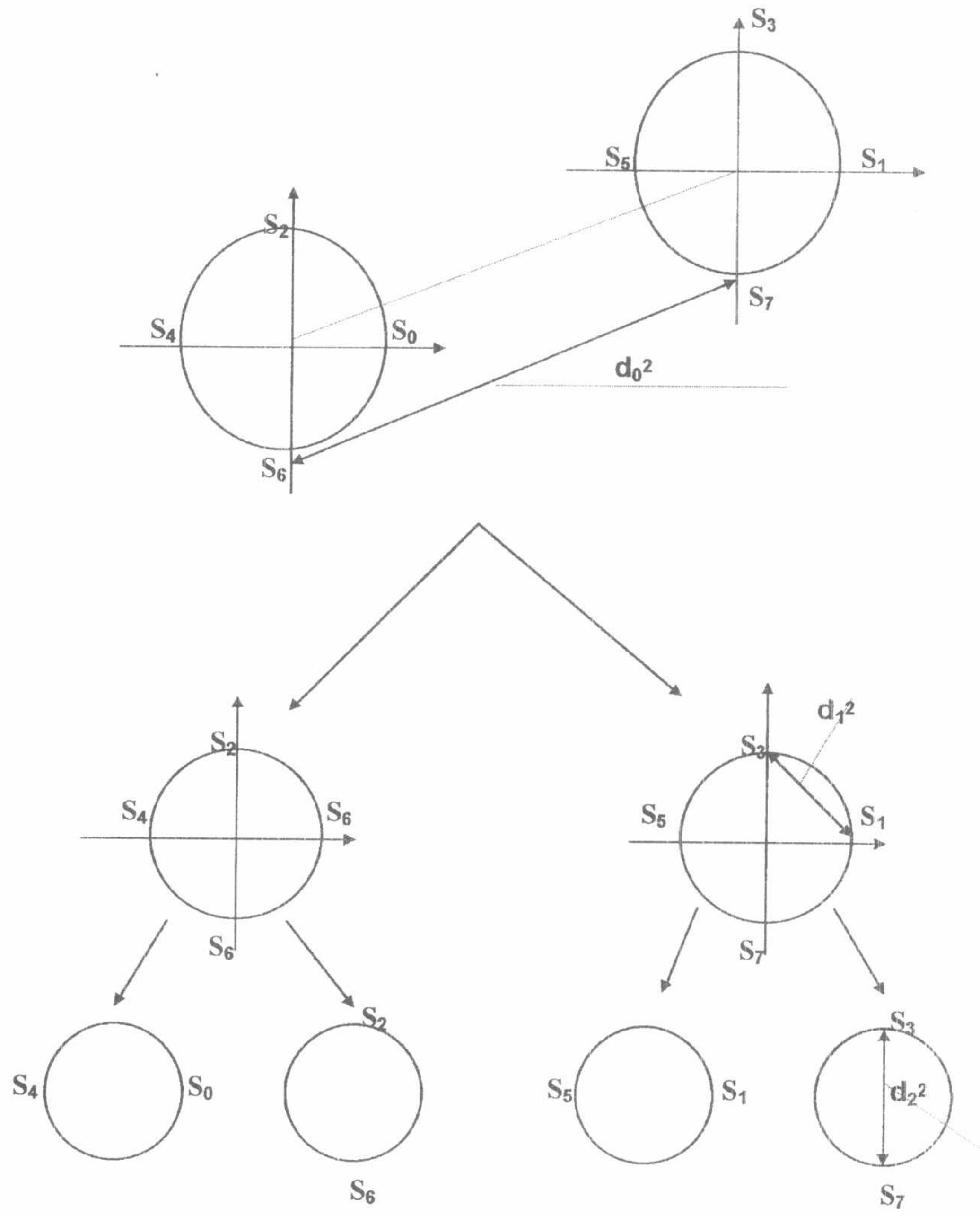

Fig..6 . Set Partitioning of 2FSK / QPSK

For mobile satellite communication, multipath fading produces a received signal with an amplitude which can be modeled by Rician statistics with parameter K representing the ratio of the power in the direct (line-of-sight) and specular component to that in the diffuse component. The probability density function for the fading amplitude ρ is given by [2].

$$P(\rho) = 2\rho(1+K)\exp[-K-\rho^2(1+K)] \cdot I_0\left(2\rho\sqrt{K(1+K)}\right) \qquad \rho \geq 0$$
$$= 0 \qquad \text{otherwise} \qquad (7)$$

where $I_0(x)$ is the zero-order modified Bessel function of the first kind. Averaging (5) over the probability density function of (7) gives [3]

$$P(x \rightarrow x') \leq \prod_{n=1}^{L} \frac{1+K}{1+K+(E_s/4N_0)\,|x_n - x'_n|^2} \cdot \exp\frac{-K(E_s/4N_0)\,|x_n - x'_n|^2}{1+K+(E_s/4N_0)\,|x_n - x'_n|^2} \qquad (8)$$

which can be written in the form

$$P(x \rightarrow x') \leq \exp(-E_s \cdot d^2/4N_0) \qquad (9)$$

with

$$d^2 = \prod_{n=1}^{L} \left\{ \frac{|x_n - x'_n|^2\,K}{1+K+(E_s/4N_0)\,|x_n - x'_n|^2} + \left(\frac{E_s}{4N_0}\right)^{-1} \ln\left[\frac{1+K+(E_s/4N_0)\,|x_n - x'_n|^2}{1+K}\right] \right\}$$

where $K = 0$ (the case of Rayleigh fading)

It is shown that d^2 is the sum of the logarithms of the squared Euclidean distances (each weighted by $E_s/4N_0$). The upper bound on pairwise error probability for this special case becomes [6].

$$P(x \rightarrow x') \leq \left(\prod_{n=1}^{L} (E_s/4N_0)\,|x_n - x'_n|^2\right)^{-1} \qquad (10)$$

IV. FUNDAMENTAL PROPERTIES OF NONLINEAR CHANNELS

Minimum Euclidean Distance

The minimum Euclidean distance between all possible pairs of the transmitted signal sequences is the most useful criterion, and it is one of the most important characteristics of an error event. It is used to evaluate the error performance for trellis coded modulation channels. The normalized squared Euclidean distance between the signals S(t) and S'(t) , is given by [8] :

$$d^2 = (1/ 2\, E_S) \int^{\infty} [\ S(t) - S'(t)\ ]^2\ dt \qquad (11)$$

equation (11) can be rewritten as

$$= \sum_{0}^{\infty} (1/ 2\, E_S) \int_{iT_S}^{(i+1)T_S} [\ S(t) - S'(t)\ ]^2\ dt \qquad (12)$$

The *i*th summand is the *i*th *incremental normalized squared Euclidean distance* d_i^2 , and is defined as :

$$d_i^2 = (1/ 2\, E_s) \int_{iT_S}^{(i+1)T_S} [\ S(t) - S'(t)\ ]^2\ dt \qquad (13)$$

To compute (11), we evaluate the integral (13) with assuming $\omega T_S >>1$and defining the phase difference at time iT_s by [8]

$$\Delta\Psi_i = \Psi (iT_s , \alpha) - \Psi (iT_s , \alpha') \qquad (14)$$

α and α' are the information sequence corresponding to s(t) and s'(t) ,respectively. Recalling (14) and let S(t) and S'(t) be signal of the form :

$$\sqrt{2E_s / T}\ \cos (\omega t + \Omega_i\, t + \theta_i) \qquad (15)$$

where $\Omega_i = 2\pi h_i / T$ is a frquency deviation during the interval. The square distance between S(t) and S'(t) attributable to the *i*th interval is

$$\int_{iT_S}^{(i+1)T_S} [\ S(t) - S'(t)\ ]^2\ dt$$

$$= \int S^2 (t)\, dt + \int S'^2 (t)\, dt - 2\int S(t)\, S'(t)\ dt$$

$$\approx 2\, E_s - 2 \int S(t)\, S'(t)\ dt \quad (\text{for } \omega \text{ large}) \qquad (16)$$

The cross - product integral is :

$$(2\, E_s / T) \int \cos (\omega t + \Omega_1\, t + \theta_1) \cos (\omega t + \Omega_2\, t + \theta_2)\, dt$$

$$= E_s / T \left. \frac{\sin[(\Omega_1 - \Omega_2)t + (\theta_1 - \theta_2)]}{(\Omega_1 - \Omega_2)} \right|_{t = iT_s}^{t=(i+1)T_s} + \varepsilon$$

where $\varepsilon \to 0$ as $\omega \to \infty$. Let $\Delta\Psi_i = (\Omega_1 - \Omega_2)\, iT_s + (\theta_1 - \theta_2)$ be the total phase differences at the beginning and at the end of the *i*th interval. then

$$d_i^2 = \begin{cases} \dfrac{1 - \sin\Delta\Psi_{i+1} - \sin\Delta\Psi_i}{\Delta\Psi_{i+1} - \Delta\Psi_i} & ; \quad \Delta\Psi_{i+1} \neq \Delta\Psi_i \\ 1 - \cos\Delta\Psi_i & ; \quad \Delta\Psi_{i+1} = \Delta\Psi_i \end{cases} \qquad (17)$$

From (17), it is clear that d_i^2 depends only on the *phase difference* at the beginning and at the end of the *i*th interval and is symmetrical with respect to $\Delta\Psi_i$ and $\Delta\Psi_{i+1}$.

Downlink Signal-to-Noise ratio (S/N)d

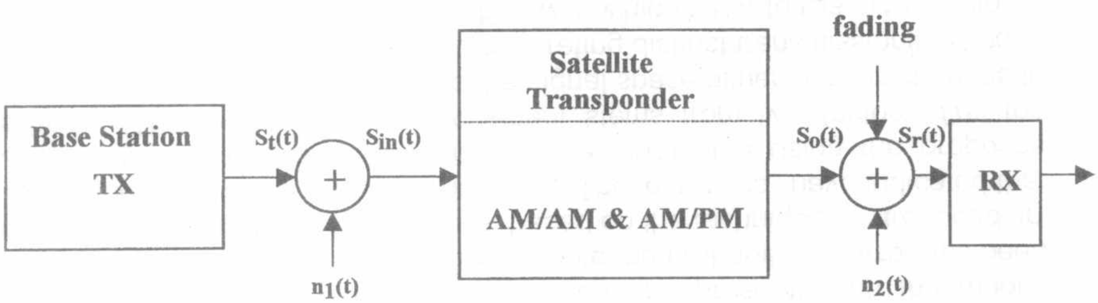

Fig. 5 . Block Schematic Diagram of Mobile Satellite Communication System

The block schematic diagram of the two - link mobile satellite communication system under consideration is shown in Figure 5. The transmitted up-link signal $S_u(t)$ is amplified by the TWT amplifier onboard satellite transponder, and during this process the signal is subjected to AM /AM and AM / PM distortions. The output down-link signal S_d (t) ,from the transponder is represented as [2]

$$S_o(t) = F[S_u(t)] \cos\{\omega_c t + \theta(t) + \Phi[S_u(t)]\} \qquad (18)$$

where F[.] and Φ[.] represent the amplitude nonlinearity conversion (AM/AM) and phase nonlinearity conversion (AM/PM), respectively, [2]. Figure.6. illustrates the probability of error of the proposed TCM/FPM system as a function of E_b / N_o under the Nonlinearity distortions.

V. NUMERICAL RESULTS

TCM/FPM signal sets have the advantage of providing higher values of Euclidean distances when compared to their equivalent TCM/MPSK signal sets. In the analysis of TCM/FPM, it is observed that the set partitioning designed to maximize these Euclidean distanes. Applying the technique developed in section II to generate numerical results for the performance of trellis coded FPM schemes. Figure 7 displays plots of error probability versus E_b/N_0 for uncoded BPSK, different coded modulation scheme and TCM/FPM in the presence of AWGN only. The same modulation techniques were illustrated in Figure 8 over Rayleigh fading and AWGN. The results in Figures 7 and 8, demonstrate that the TCM/FPM has superior advantageous perfo.rmance analysis over the other coded and uncoded different modulation approaches compared.

VI. CONCLUSION

In this paper, the performance analysis of trellis coded Frquency / Phase modulation scheme was estimated and presented on the mobile satellite channels. The formula to compute the probability of error has been derived. An extensive error analysis based on the techniques introduced in [3-5] and [9], have been carried out. Those results have indicated that significant performance improvements are obtainable using the proposed scheme. This performance improvement stems from two factors: *first* the expurgated trellis codes increase the signal space dimension which leads to maximize the Euclidean distance separating distinct transmissions, *second* the higher values of output back-off power, which allow to reach to the linear characteristics of the high power amplifier in the satellite transponder.

REFRENCES

1) **Gottfried Ungerboeck** ,"Channel Coding with Multilevel / Phase Signals" IEEE Transactions on Information Theory,Vol. IT-28, No. 1,pp. 55-67, January 1982.

2) **Tri T. Ha,** " Digital Satellite Communications " McGraw-Hill ,2nd Edition 1990.

3) **Ezio Biglieri, Dariush Divsalar, Peter J. Mclane** and **Marvin K. Simon,** Introduction to Trellis - Coded Modulation with applications, Macmillan Publishing Company 1991.

4) **Slimane Ben Slimane** and **Tho Le-Ngoc,**" Tight Bounds on the Error Probability of Coded Modulation Schemes in Rayleigh Fading Channels" IEEE Transactions on Vehicular Technolgy, Vol. Vt- 44, No. 1,pp.121-129, February 1995 .

5) **Ross G. Mckay, Peter J. McLane** and **Ezio Bigliere,**" Error Bounds for Trellis - Coded MPSK on a fading Mobile Satellite Channel" IEEE Transactions on Communications, Vol. 39, No. 12,pp. 1750-1761, December 1991

6) **Shalini S. Periyalwar,** and **Solomon M. Fleisher,** " Multiple Trellis Coded Frequency and Phase Modulation "IEEE Transactions on Communications, Vol. 40, No. 6, pp. 1038-1046, May 1992.

7) **Char - Dir Chung,** " Differential Detection of Quadrature Frequency/Phase Modulated Signals " IEEE Transactions on Communications, Vol. Com-47, No. 4, pp. 546-556, April 1999.

8) **Iwao Sasase** and **Shinsaku Mori,** " Multi-h Phase - Coded Modulation " IEEE Communications Magazine, pp. 46-56, December 1991.

9) **John G. Proakis,** Digital Communications, Mc-Graw - Hill 1995.

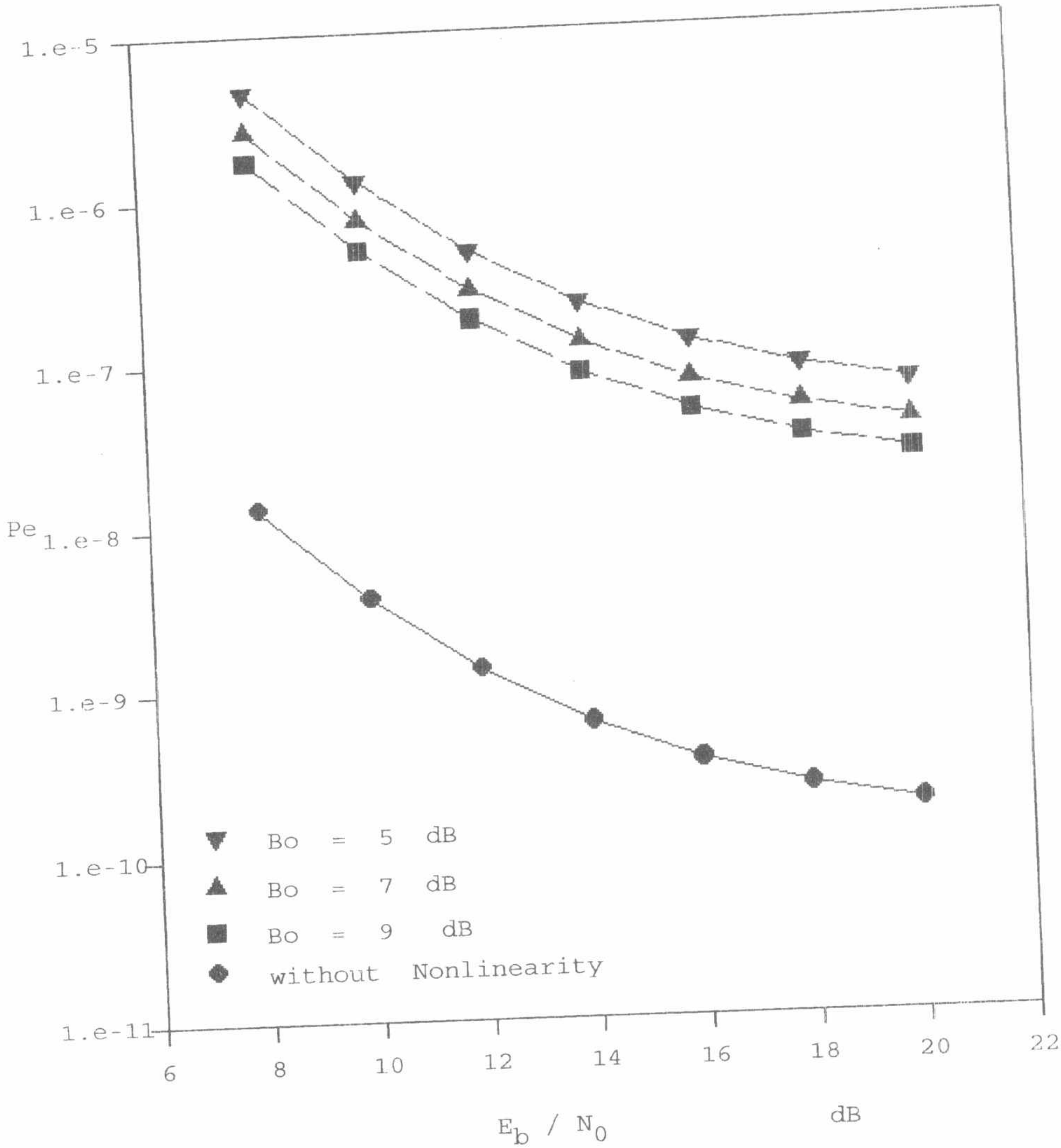

Fig.6 . Performance of TC - 2FSK/QPSK Due to Nonlinearity under Rayleigh fading and AWGN

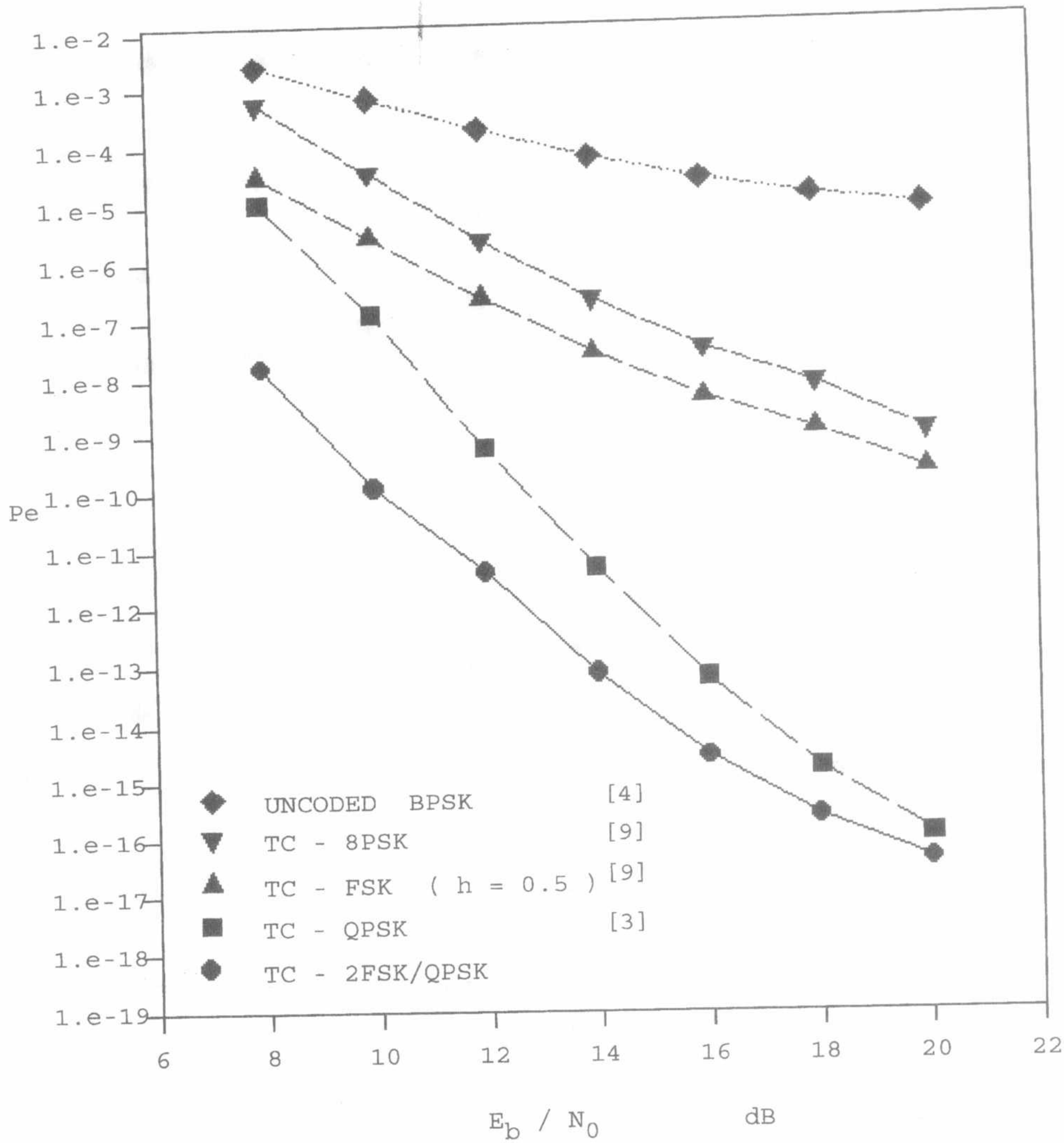

Fig.7 . Performance of Different Modulation Technique under AWGN

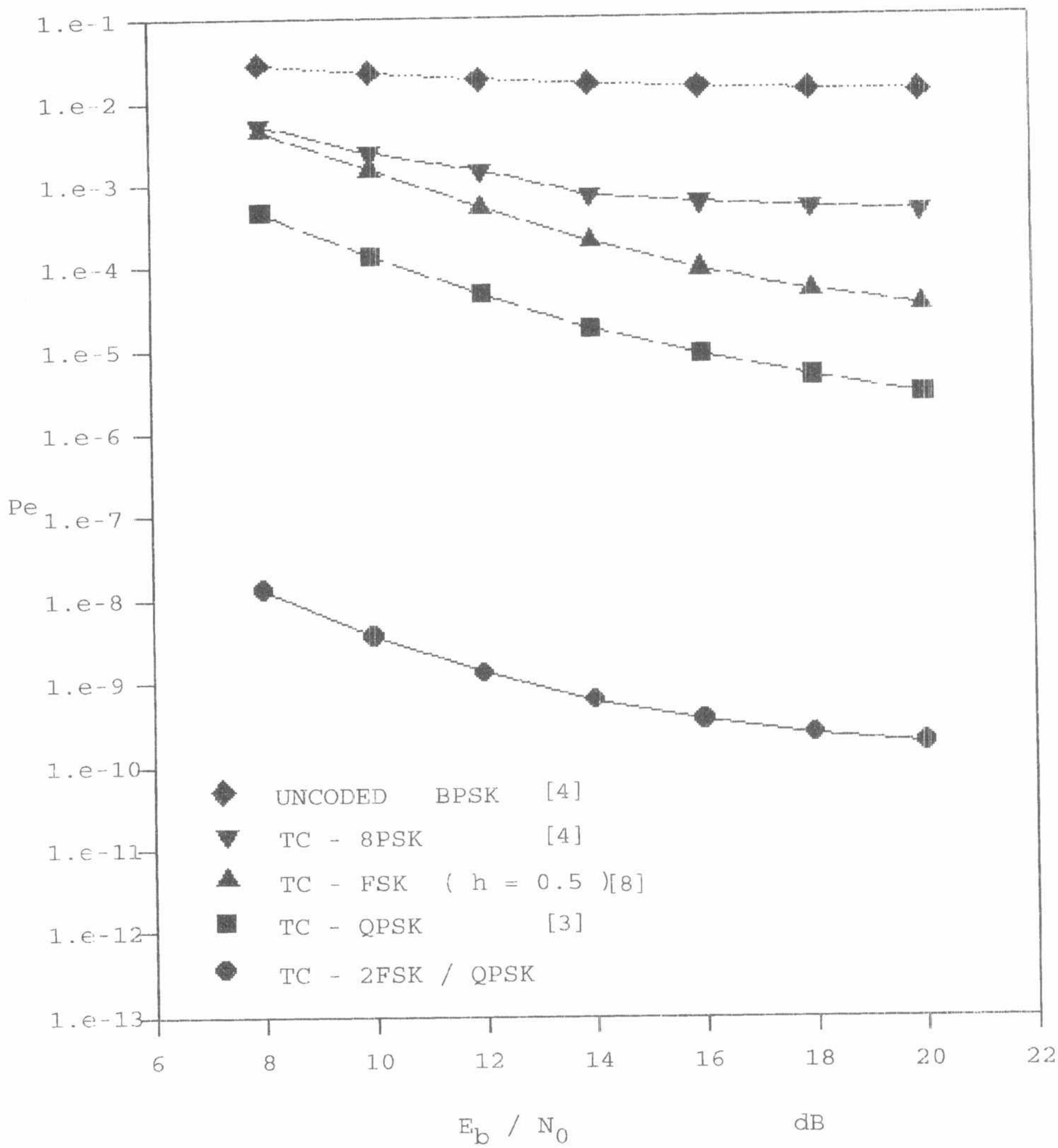

Fig. 8 . Comparison of Different Modulation Techniques under Rayleigh Fading and AWGN