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DESIGN OF MULTIBAND FRACTAL ARRAY FACTORS WITH REDUCED ARRAY SIZE

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Abstract *– The multiband behavior of the fractal Kaiser-Koch array factor is described in this paper. The fractal array factors presented keep the same shape at several bands because they are constructed from self-similar curves. A Kaiser window as a generating pulse function is proposed for the design of Koch-array factors. Kaiser windows are characterized by having the lowest side lobes of all windows in the transformed domain. The result of applying such a technique would result in an array current distribution having lower side lobes, with a reduced array size after setting a threshold beyond which the elements are eliminated.*

Key Words — Multiband Antennas, Fractals

1. Introduction

In practice, it is often necessary to design an antenna system that will yield desired radiation characteristics. A very common request is to design an antenna whose far field pattern exhibits a desired distribution, having narrow beamwidth and low side lobes, decaying minor lobes, and so forth. The design objective is to find the system configuration and excitation distribution that yields an acceptable radiation pattern. This method of design is often referred to as synthesis [1]. In our case the radiating system is expected to possess a multiband behavior. A multiband behavior is one in which the side lobe ratio and radiation pattern characteristics are held similar at several frequency bands. Most fractal objects have self-similar shapes, which means that some of their parts have the same shape as the whole object but at different scales [2], [3]. Fractal array factors are dealt with in [5], where the visible range is always centered at a secondary lobe that has the same shape as the total pattern.

In this paper, the generation and behavior of the fractal Kaiser-Koch array factor is studied. The pattern is based on a well-known set of fractal curves, the Koch curves, generated from a Kaiser window generator. The array will radiate through a scaled version of the whole array factor when the visible region is altered by means of a change in the operating frequency. This paper unfolds as follows, **section (2)** reviews Fractal Koch Curves, **section (3)** describes Fractal Radiation Patterns, **section (4)** illustrates the Koch-Pattern Construction Algorithm, **section (5)** gives Array Pattern Synthesis, **section (6)** presents the Kaiser-Koch Array, **section (7)** presents the Kaiser-Koch Computer Simulation Results and **section (8)** ends with a Conclusion.

2. Fractal Koch Curves

The Koch-curve was first introduced in 1904. As given in [4], we begin with a straight line. This initial object is also called the initiator. Partition it into three equal parts, then replace the middle third by an equilateral triangle and take away its base.

This completes the basic construction step. A reduction of this figure, made of four parts, will be reused in the following stages. It is called the generator. Thus, we repeat, taking each of the resulting line segments, partitioning them into three equal parts, and so on.

3. Fractal Radiation Patterns

In this section the design of fractal array factors is presented. The pattern-construction algorithm is quite similar to that of the Koch curves, but is modified to provide a functional form. The main feature of this pattern is that each lobe of the curve is equal to the whole pattern. When the array radiates at longer wavelength, the visible range is reduced and only a fraction of the whole array factor appears in the radiation pattern. Every time, the visible region is reduced around one of the secondary lobes and the visible pattern is kept the same as the original one. The Koch pattern is illustrated in fig.1. The curve has been constructed from six iterations (M=6), therefore the curve will keep its similarity properties at six different scales. By adding a progressive phase β, the visible range is always centered at a secondary lobe that has the same shape as the total pattern. The following progressive phase is taken as

$$\beta = kd \tag{1}$$

Where k is the wave number, d is the spacing between the elements. The visible range covers the interval $\{0, 2kd\}$ for each frequency. Therefore, a frequency reduction by a factor of $(\frac{1}{\delta})^n$ would reduce the visible region by δ^n around a secondary lobe, where δ and n are positive integers. We will have a set of bands scaled by a factor of $1/\delta$ with in which the array factor will remain the same. Also it can be seen that the proposed arrays keep a similar radiation pattern at several bands, the pattern magnitude is not held constant. As frequency is reduced, the magnitude is also reduced.

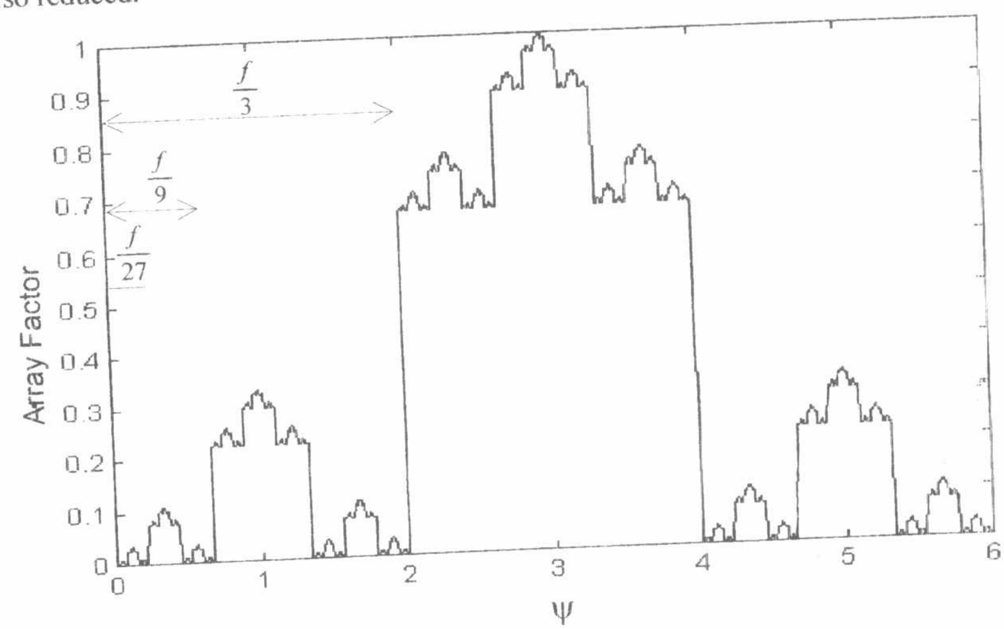


Fig.1 The Koch array Factor, constructed from M=6, δ=3, α=1

4. Koch-Pattern Construction Algorithm

In this section, a compact functional form is presented for describing the pattern construction procedure and hence, corresponding array current distribution. The Koch pattern construction algorithm is important for obtaining the array current distribution. Constructing such patterns is very simple. A periodic pulse-train in the spatial frequency domain ψ is assumed, its width is scaled by a factor of δ and its amplitude by a factor of $\alpha\delta$. After iterating this scheme M times, the M resulting patterns are superimposed to obtain a Koch pattern as the one shown in fig.1. For the pattern in fig.1, a rectangular pulse window and a log period δ=3, and an amplitude factor α=1 were chosen for generating the pattern with M=6 iterations. The analytical expression for each generating pulse train $T(\psi)$ can be written as

$$T(\psi) = F(\psi) * \sum_{n=-\infty}^{\infty} \delta(\psi - n\psi_T) \quad (2)$$

where $F(\psi)$ is the single pulse function which in general can be taken to have any arbitrary shape such as a rectangular window, blackman window, raised cosine or a Kaiser window. The period of the pulse train is denoted ψ_T. From eqn.2, the analytical expression for the Koch pattern $K(\psi)$ after adding M-scaled pulse trains is

$$K(\psi) = \sum_{p=0}^{M-1} F(\delta^p \psi) * \{\frac{1}{\alpha^p \delta^p} \sum_{n=-\infty}^{\infty} \delta(\psi - n\frac{\psi_T}{\delta^p})\} \quad (3)$$

The number M of iterations used to construct the curve determines the number of times the curve will look similar under a δ factor scaling transformation. M is the number of bands in which the array will have a similar pattern [5].

5. Array Pattern Synthesis and Corresponding Current Distribution

Once the fractal array factor is defined, the relative current distribution between the elements that would generate such a self-similar array factor is derived numerically. This distribution can be numerically computed by taking the inverse Fourier transform (IFT) of the Koch-array factor. By taking the inverse Fourier transform of eqn.3, the element distribution k(z) will be

$$k(z) = \frac{1}{\psi_T} \sum_{p=0}^{M-1} \frac{1}{\delta^p} f(\frac{z}{\delta^p}).\left[\frac{1}{\alpha^p} \sum_{n=-\infty}^{\infty} \delta(\psi - n.d.\delta^p)\right] \quad (4)$$

The corresponding computed array structure would require a large number of elements (N). The number of elements may be calculated as

$$N = \delta^M \quad (5)$$

6. Kaiser-Koch Array

The array current distribution is basically a superposition of the inverse transforms of the pulse generator [5]. It is then better to choose a pulse generator having a low side-lobe level transform to allow a better truncation of the Koch arrays. The blackman window is characterized for having low side lobes in the transformed domain. It is then preferable to choose a train of Blackman pulses to generate the Koch patterns instead of rectangular ones. The main advantage of the Blackman-Koch pattern is that the array relative current distribution has lower side-lobes and a better confinement around the central elements fig.4. Therefore, instead of truncating the tips of the array, a threshold level can be set to observe which elements are important in the pattern synthesis and which are not [5].

The main advantage of using Kaiser windows is that they are characterized of having the lowest side lobes of all windows in the transformed domain. If Kaiser windows are used in the design of fractal patterns, one would expect of them an array current distribution having lower side lobes, with a more reduced array size. In this case, the single pulse function is taken to be a Kaiser window. Therefore, one can choose a train of Kaiser pulses to generate the Koch patterns. Patterns resulting from Kaiser pulses maintain a smooth shape with the same similarity properties of the Koch array factor. A Kaiser window [6] is given by

$$w_K(nT) = \frac{I_o(\beta)}{I_o(\gamma)} \qquad \text{for } |n| \le \frac{N-1}{2}$$

$$w_K(nT) = 0 \qquad \textit{otherwise} \tag{6}$$

where $I_o(x)$ is the zeroth-order modified Bessel function of the first kind, γ is the Kaiser independent parameter and

$$\beta = \gamma\sqrt{1-\left(\frac{2n}{N-1}\right)^2} \tag{7}$$

7. Kaiser-Koch Computer Simulation Results

Kaiser windows allow an independent control of the mainlobe width by altering the γ factor. Smaller values for γ result in a wider beamwidth, while larger values result in a narrower beamwidth. Figs. 2 and 3 show several configurations of Kaiser-Koch patterns with the corresponding current distribution. Not only the main lobe width is controllable, but also the current distribution and its side lobe level. Thus, controlling the threshold level that discerns which important elements are chosen for the pattern synthesis and which are not. This is illustrated in fig.3. It is clear that the current side lobe is inversely proportional with γ. Larger values of γ reduce the current side lobe way below the Blackman window. The result of applying such a technique is shown in the bottom case in fig.4. Comparing both windows for $M = 6$, $\delta = 3$ and $\alpha=4$, the Kaiser's current distribution reached a minimum of 10^{-13} while that of the Blackman reached a minimum of 10^{-10}.

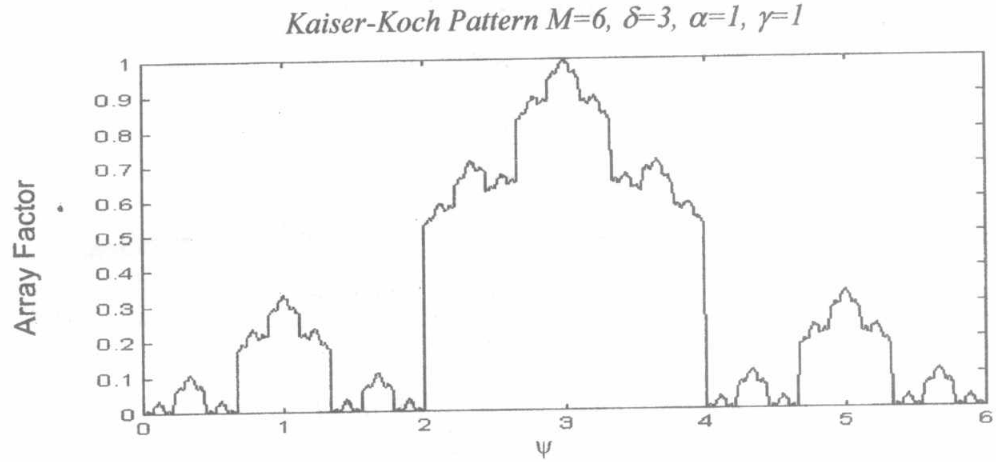


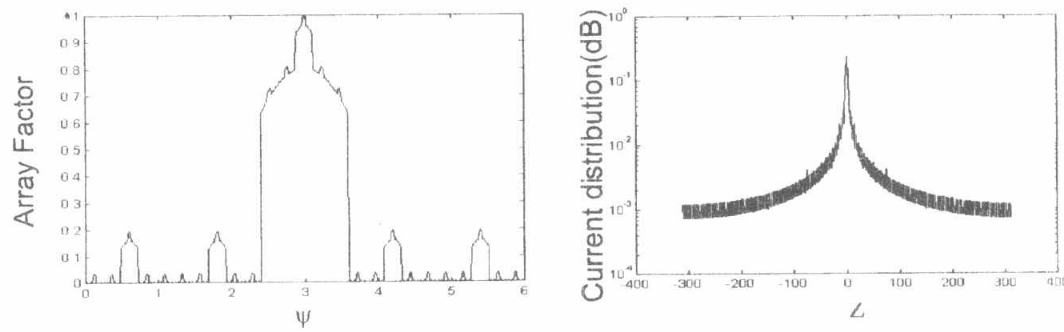
Fig.2 The Kaiser-Koch array Factor, constructed from M=6, δ=3, α=1, γ=1

The two bottom cases in fig.3 indicate that the array relative current distribution is independent of the reduction factor α. The reduction factor is chosen in the last case in fig.3 to improve the pattern's SLR and for smoothing its shape. Also both patterns resulting from either a Blackman or a Kaiser window generating pulse are very much similar to each other, shown in Fig.4 (a). That is, it is possible to obtain the Blackman window characteristics while maintaining a lower value for the current distribution, Fig.4 (c). In table.1, a threshold value is set at different levels and the corresponding elements that are important in the pattern synthesis, that keep the self-similar behavior at several bands are observed. For larger thresholds, -30dB, the number of elements are greatly reduced for lower values of γ, reaching only 8 elements with γ=5. For smaller thresholds, -130dB, the number of elements are reduced for larger values of γ, reaching only 92 elements with γ=20. The main reason for this result is that most of the elements are confined in a very low level (below –130dB) for larger values of γ. Whereas, for smaller values of γ, the elements are distributed over all the levels, experiencing different amplitudes.

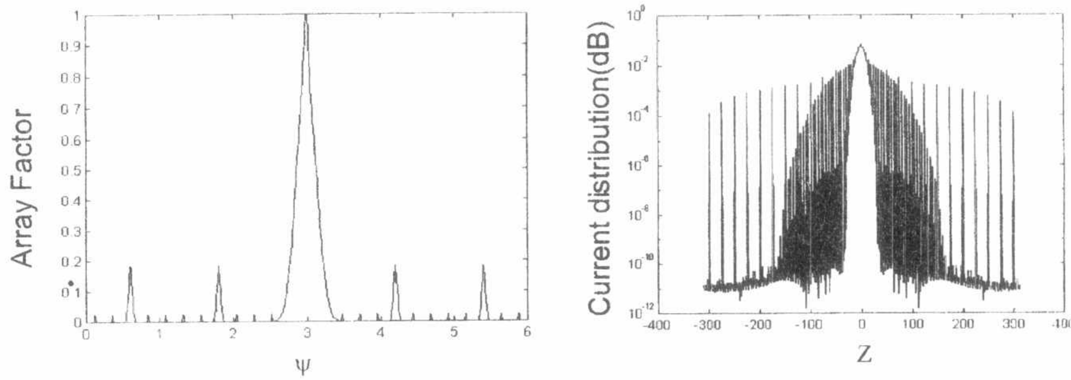
Threshold (dB) / Window type	Number of array elements at different threshold levels for different Windows as a generating pulse with a reduction factor α=1 and α=4										
	-30	-40	-50	-60	-70	-80	-90	-100	-110	-120	-130
Blackman, α=4	8	12	18	22	30	42	60	86	126	178	256
Kaiser, γ=20, α=4	12	18	26	34	42	54	60	72	82	90	92
Kaiser, γ=15, α=4	12	18	22	30	36	44	54	62	72	74	82
Kaiser, γ=10, α=4	10	12	18	24	30	36	40	52	72	142	563
Kaiser, γ=5, α=4	8	8	14	20	44	150	533	675	705	727	729

Table.1 A comparison between different generating pulse functions. The main construction parameters are M = 6 and δ = 3

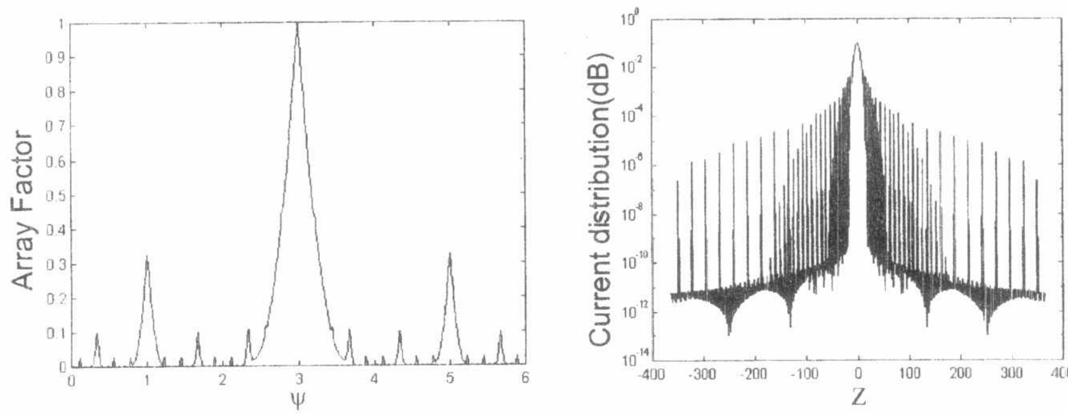
Kaiser-Koch Pattern M = 4, $\delta = 5$, $\alpha = 1$, $\gamma = 1$



Kaiser-Koch Pattern $M = 4,\ \delta = 5,\ \alpha = 1,\ \gamma = 20$



Kaiser-Koch Pattern $M = 6,\ \delta = 3,\ \alpha = 1,\ \gamma = 20$



Kaiser-Koch Pattern $M = 6,\ \delta = 3,\ \alpha = 4,\ \gamma = 20$

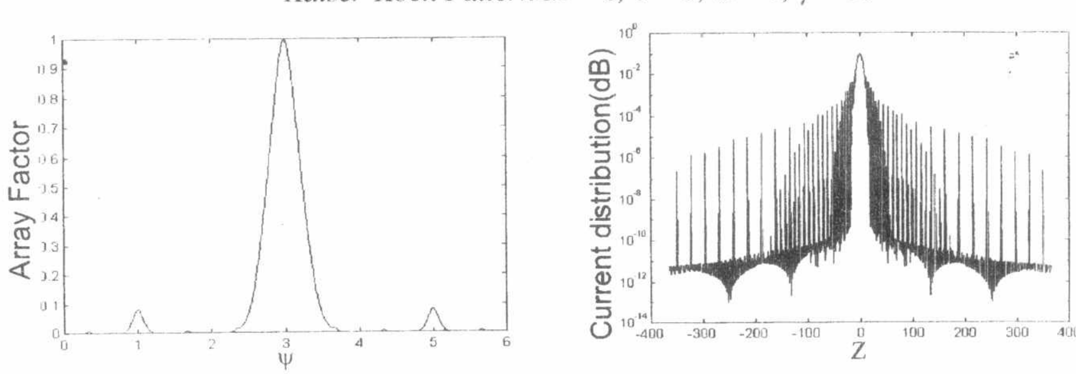
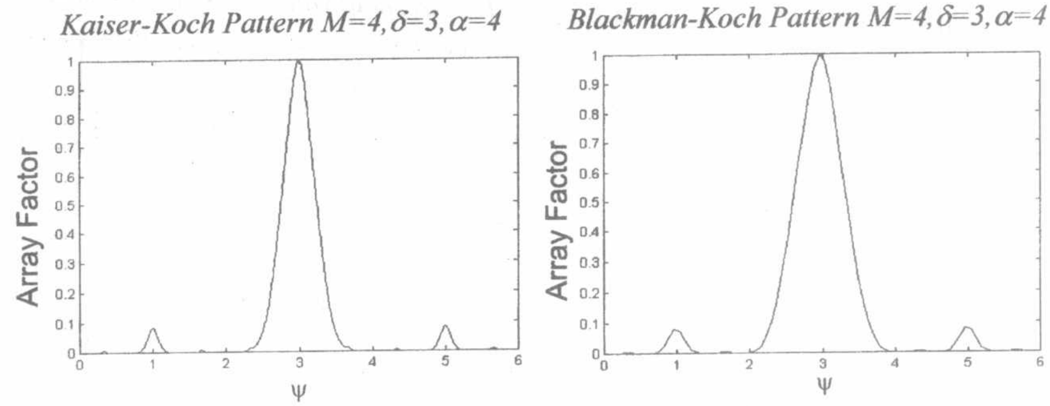
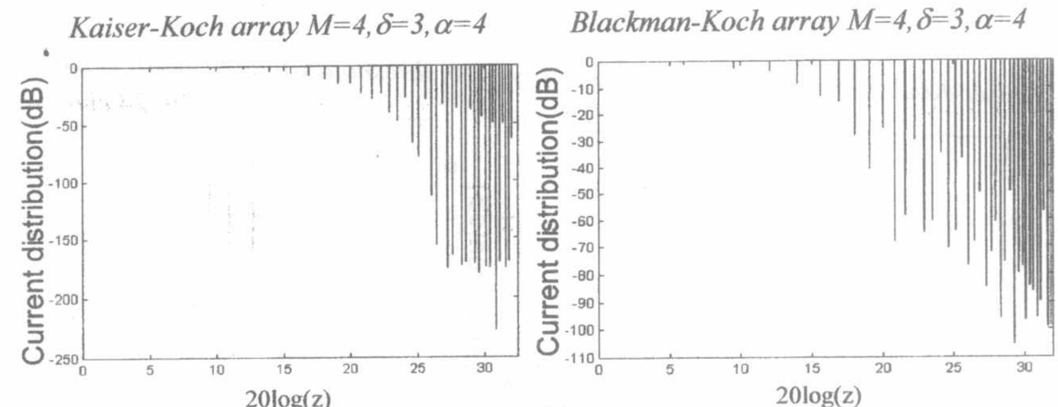


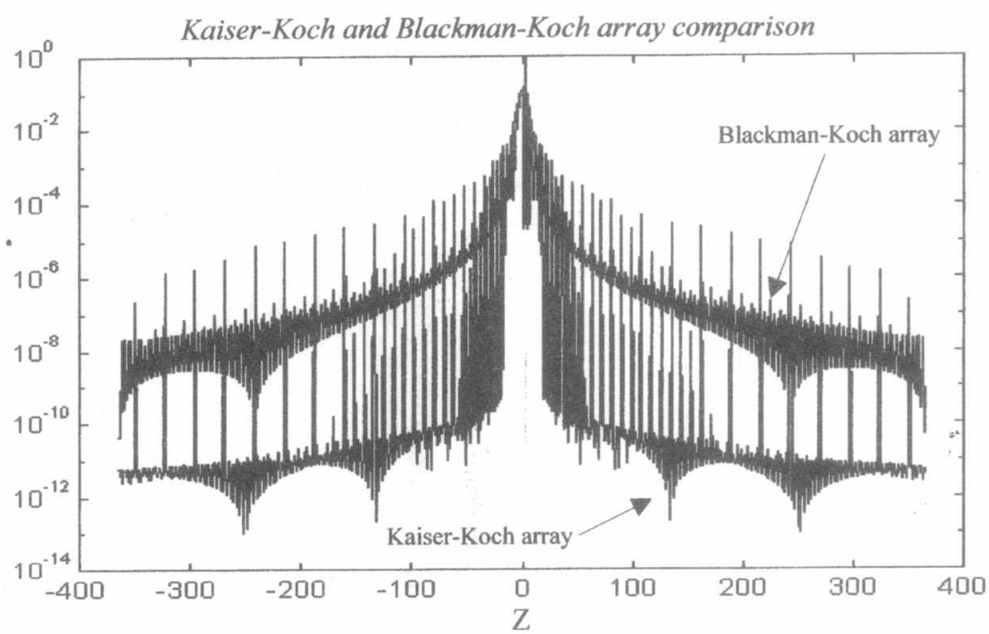
Fig.3 RHS is the current distribution in logarithmic scale for its corresponding LHS Kaiser-Koch pattern



(a)



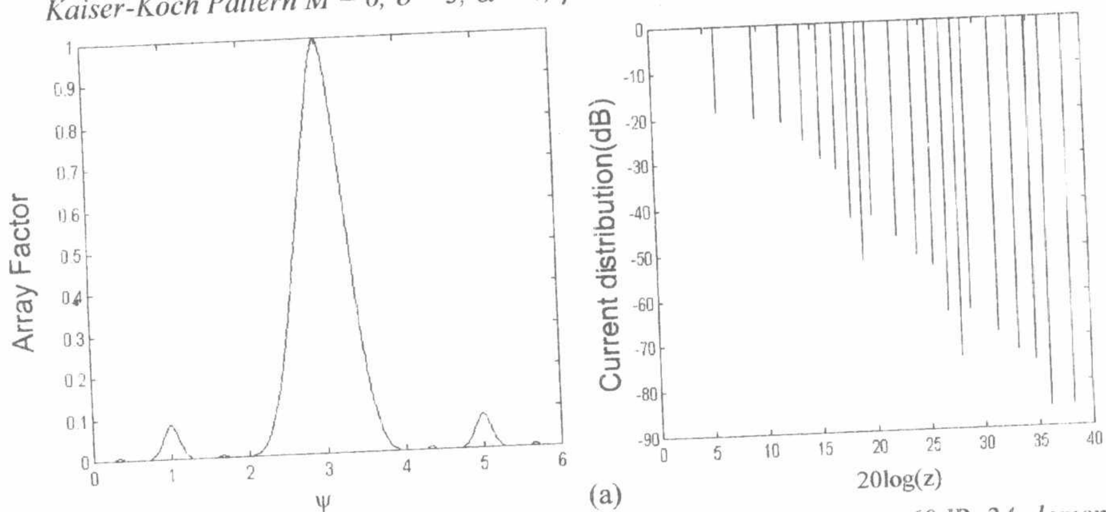
(b)



(c)

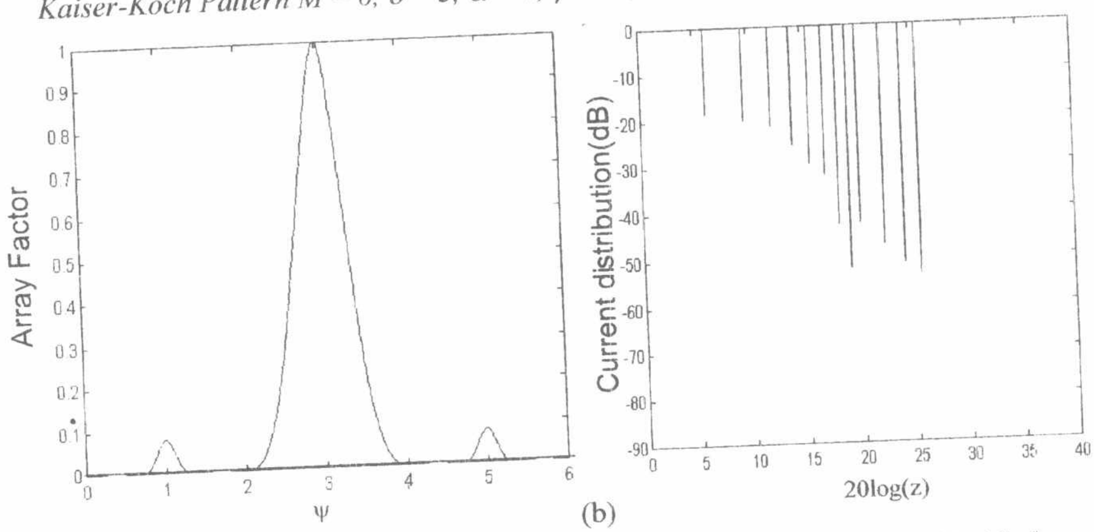
Fig.4 A comparison between (a) the Kaiser-Koch and Blackman-Koch array Factors, constructed from M=4, δ=3, α=4, γ=20. (b) the Kaiser-Koch and Blackman-Koch current distributions. (c) Current distribution for the Kaiser-Koch and Blackman-Koch array logarithmic scale.

Kaiser-Koch Pattern $M = 6$, $\delta = 3$, $\alpha = 4$, $\gamma = 10$, threshold value = -90dB, 40 element



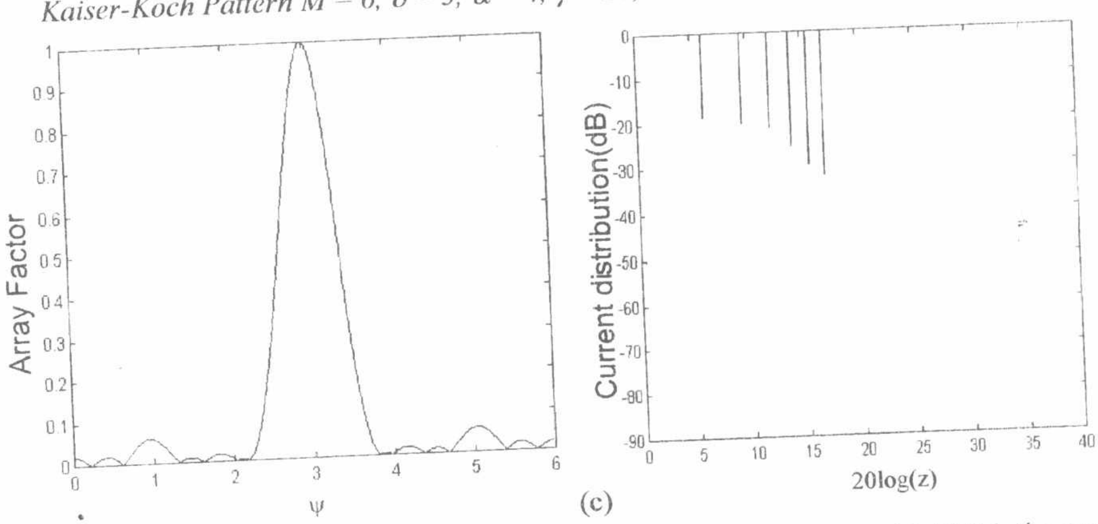
(a)

Kaiser-Koch Pattern $M = 6$, $\delta = 3$, $\alpha = 4$, $\gamma = 10$, threshold value = -60dB, 24 element



(b)

Kaiser-Koch Pattern $M = 6$, $\delta = 3$, $\alpha = 4$, $\gamma = 10$, threshold value = -40dB, 12 element



(c)

Fig.5 The Kaiser-Koch array Factor, constructed from M=6, δ=3, α=4, γ=10. RHS is the current distribution for its corresponding LHS pattern after element reduction due to applied threshold

8. Conclusion

Both analytical and numerical results on the Kaiser-Koch pattern have been presented. All of them describe a multiband behavior of the fractal array factor. This behavior is consistent from the directivity, lobe structure, SLR and radiation patterns point of view. From fig.5, it is clear that –90dB is the threshold level beyond which the pattern is distorted. A Koch pattern designed by using a Kaiser window generator can be conformed with a 40 element structure at a –90dB threshold, resulting in an array factor that would operate at five bands. If the threshold is reduced to a value of –130dB, the pattern is conformed with only 92 elements (using Kaiser generator) instead of 256 elements (using Blackman generator) operating at five bands through a whole 81:1 frequency range. The main advantage of using Kaiser windows is that the pattern may easily be altered through the Kaiser independent parameter γ. This flexibility parameter does not only allow the control of the mainlobe width but also the current distribution and its side lobe level.

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