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Performance Enhancement of a Long Range Missile Using H_∞ Robust Control Design

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Abstract

A distinguishing feature of aerospace applications is the large envelope of operation in which the process is usually highly nonlinear and has different characteristics from one operating condition to another. Therefore, the objective of the present work is to design an autopilot that can cope with modeling errors, plant parameters variations, external disturbances and unmodeled dynamics, besides providing good performance and high stability. To achieve this objective, an adequate nonlinear mathematical model representing the dynamical behavior of the missile was derived (previous work) from which the linearized model for the underlying missile is obtained. A robust flight control system or autopilot is designed, for precise tracking, using (1) classical control design technique (previous work) and (2) Mixed sensitivity H_∞ control design technique, the contribution of this paper. The paper presents, briefly, different issues in robust control highlighting the robustness to different sources of uncertainty for the purpose of achieving good tracking and disturbance rejection, and preserving the system internal stability. Then it summarized some of the controller design techniques including classical and advanced methods such as mixed sensitivity H_∞. Then, the structure of the underlying missile control system with the performance requirements imposed on it is developed. Finally, the design trials and analysis of the flight control system are carried out using the above techniques with the objective to satisfy the performance requirements including good tracking and disturbance rejection in presence of unmodeled dynamics. The H_∞ approach has good robustness compared to the classical. However, the H_∞ controller has higher order. Therefore, this technique needs to be investigated more with the system, giving more attention to the weight selection and its order, the crucial point in its design.

Keywords: Guidance and Control, Robust Control, Optimal Control, Polynomial Techniques

1 Robust Control and Uncertainty within Missiles

The objective of a control system is to alter the dynamical behavior of physical process so that the response from the controlled system, more nearly, satisfies the user's requirement. Any linear control law can only provide the required closed loop response at the expense of permitting the occurrence of some unsatisfactory features in the response to disturbance. This disturbance can be extraneous to the missile, such as atmospheric turbulence, or introduced by the flight control system itself through sensor noise. If the linear control system is designed for reducing the effect of unwanted inputs, the desired dynamic performance of the closed loop system to command inputs is unavoidably impaired. Even if some compromise can be achieved, such a solution can only be used within a most restricted region of the missile's flight envelope. Consequently, when a missile is required to fly on some particular mission, through the extreme regions of flight envelope, one of the following two ways must be pursued. First, the nonlinear system is linearized around different operating points, then for every operating point a controller is designed. All the controllers are then brought into operation successively as the system passes through conditions where the corresponding models are approximately valid. That is, a gain schedule is designed such that the missile can be controlled over the entire flight envelope. Second, a nominal operating point is chosen for which a robust controller is

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to be designed. This controller must satisfy stability and performance robustness when it is applied to the process (missile) at different operating conditions within the limits of its robustness margin. Designing a robust controller can be achieved using either conventional control or modern control techniques.

Conventional or classical control methods are being regarded as those appropriate to time–invariant, linear single–input–single–output (SISO) systems. The conventional controllers are designed to satisfy specified requirements for steady–state error, transient response, stability margins or closed loop pole locations. The essence of classical design was successive loop closure guided by a good deal of intuition and experience that assisted in selecting the control system structure. Optimal control is one particular branch of modern or advanced control, which provides the best possible performance from its class when it responds to some particular input. Linear optimal control is a special sort of optimal control, in which the plant to be controlled is assumed linear, and the controller is constrained to be a linear function. Linear controllers are achieved by working with quadratic performance indices. Such methods that achieve linear optimal control are termed Linear-Quadratic (LQ) methods. In addition, there is an approach called H_∞ in which the performance index is specified such that the infinite norm of a certain transfer within the system is satisfied.

A distinguishing feature of a guided missile is the very wide range of operation characterized by time varying nonlinear dynamics system. Therefore, mathematical modeling can never exactly describe this system, and consequently there are unmodeled dynamics, which considered sources of uncertainty in this system. Linearization and separation of missile equations of motion into two uncoupled sets, longitudinal and lateral equations of motion, represent errors in evaluating the mathematical model because they are based on some assumptions. There are parameters in the model, which are only known approximately or simply in error such as aerodynamic data from wind–tunnel tests. These model uncertainties represent the differences between the actual physical system, missile, and the mathematical model. There are other sources of uncertainty such as wind gusts, sensors measurement noises, and the implementation inaccuracy where the implemented controller may differ from the one obtained by solving the synthesis problem. In this case, one may include uncertainty to allow for controller implementation inaccuracy. Therefore, the controller, to be designed and implemented within the missile control system, should be insensitive to model uncertainties and able to suppress disturbances and noises over the whole envelope of operation, i.e. should be robust.

In Robust control there are some issues such as: robustness, command tracking, disturbance rejection, measurement noise attenuation and internal stability that has to be taken into consideration during design. Robustness of the controller includes both stability robustness and performance robustness. The stability robustness is the ability to provide internal stability in spite of modeling errors due to high–frequency unmodeled dynamics and plant parameter variations, while the performance robustness is the ability to guarantee acceptable performance even when the system is subjected to disturbances and measurement noises. In addition the designed controllers must satisfy the performance requirements imposed on disturbance rejection, such as how fast it will be rejected. The other issue is the command tracking which is considered one of the most fundamental problems in missile control. That is, the missile has to follow or track the reference or guidance command signal very accurately.

2 Controller Design Techniques

2.1 Classical Control Design

The controller determines the deviation of actual output from the desired one yielding a control signal, which tries to reduce this deviation to zero. The manner in which the automatic controller produces the control signal is called the control action. From the basic control actions that commonly used in industrial automatic controllers are: proportional, integral, proportional–plus–integral, proportional–plus–derivative, and proportional–plus–integral–plus–derivative control actions. Understanding the basic characteristics of these control actions is necessary in order to select the most suitable one to the underlying application design. The control signal at the output of the controller is simply related to its input by a proportional constant k_p, integral constant K_I, and the derivative term K_d. This combination of the proportional, integral and derivative control actions has the advantages of each of the three individual control actions. The design problem involves the determination of values of real constants K_p, K_D and K_I, so that the performance of the system meets the design requirements. Determination of such values depends on a trade–off approach, using some rules of thumb as in the literature. There are many heuristic rules for tuning the PID-controller, such as the transient response method and the ultimate– sensitivity method due to Ziegler and Nichols [2, 12]. All of these tuning rules can be used to obtain the starting guess for the unknown parameters. Then, fine-tuning is to be carried out for satisfying the different performance requirements.

2.2 Advanced Control Techniques

There are various advanced control techniques such as the linear quadratic gaussian (LQG), linear quadratic regulator (LQR), generalized LQG (GLQG), H_∞ and generalized H_∞ (GH_∞) [7, 8, 9, 10]. The GH_∞ design philosophy is mechanized to the underlying system. For this theory or design approach, the GLQG is considered to be the base for its derivation [9]. In the GLQG controller, the plant–structure is considered in a more general form that contains colored input disturbance and measurement noise Fig. 1.

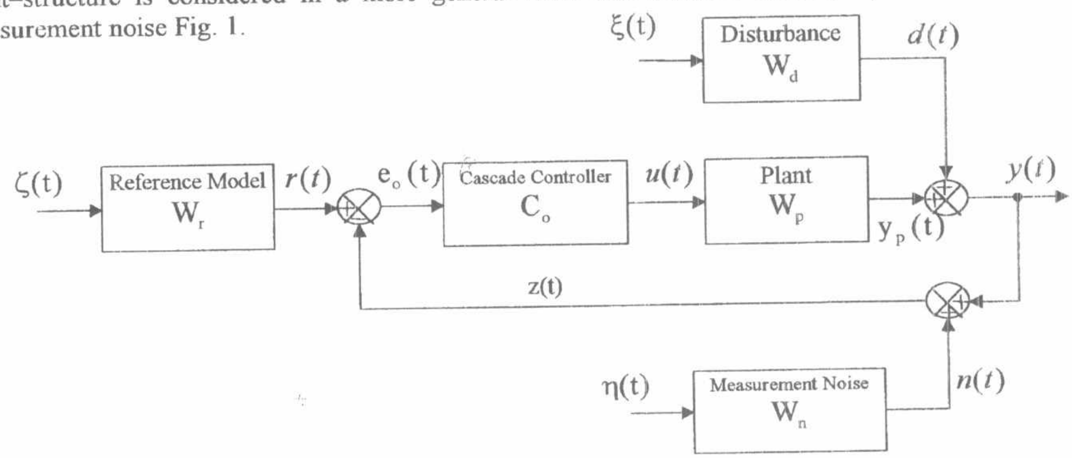

Fig. 1: Feedback Control System with input dist., Meas. noise and Ref.

This general plant structure could be broken down easily to any special case that might exist in reality, so the GLQG controller can be applied to a wide range of industrial processes and gives optimal rejection of measurable load disturbances. The cost function used includes dynamic weighting element allowing integral action to be modified [7, 8]. In this system the high–frequency (HF) disturbances n (t) affect the observed system output, represented by the signal y (t). The different signals can be represented by the coprime polynomials [9, 12, 15, 16, 17] as follows: the plant $W_p = A^{-1}B$, input disturbance $W_d = A_d^{-1}C_d$, the measurement noise $W_n = A_n^{-1}C_n$, the reference $W_r = A_r^{-1}E_r$, and the controller $C_o = C_{od}^{-1}C_{on}$. The signals $\{\zeta(t), \xi(t), \eta(t)$ are white gaussian signals with zero means and unity variances.

2.2.1 GH_∞ Controller

The H_∞ norm of a stable scalar transfer function $f(s)$ is simply the peak value of its absolute value $|f(j\omega)|$ as a function of frequency, that is $\|f(s)\|_\infty \stackrel{\Delta}{=} \max|f(j\omega)|$. The H_∞ is a design method which aims to press down the peak of the selected transfer function. Our objective is that the output (y) of the system G_p will track the reference signal (r) by designing the controller G_c which has, as its input, the tracking error. This problem can be thought of as minimizing the H norm of the transfer function from r to (r-y) under the constraint of internal stability to prevent undesirable performance. This transfer function is known as the sensitivity function and given by:

$$S_n = (I + W_p C_o)^{-1} \qquad (1)$$

Although the tracking error signal is assumed to be unknown, it has a limited frequency spectrum, because it is impossible to track signals of very high frequency reasonably well. On the other hand, since in general there are bandwidth limitations on the actuators and sensors and due to problem in implementing controllers with a large bandwidth, there will be a certain bandwidth for the system to track signals. The GH_∞ is a design technique that can be tailored to the underlying system based on the GLQG and overcome most of these problems. The parameters in quadratic cost index are to be selected such that the closed–loop system frequency response is shaped in a desired form leading to what is called loop shaping. That is, the stability problem may be regarded as one of shaping the system loop transmission so that it avoids critical points in the s-plane. Therefore the loop shaping design in this section is based on GH_∞ robust stabilization. In addition, the controller C_o guarantees the robustness of the stability property with maximum stability margin. To obtain good design of a particular system, the weighting functions P_c and F_c are to be selected properly, where P_c costs the error signal e (t), while F_c costs the control effort signal u (t). The cost function weights P_c and F_c provide the mechanism by which the sensitivity function S_n, the complementary sensitivity function T_n and the control sensitivity function M_n can be modified and shaped [2, 3, 4, 5]. The control law provides a guarantee of stability and good performance in presence of model uncertainties. The bound for model uncertainty provides one method of defining the form of dynamic weights in GH_∞ cost–function. According to the type of weighting choice (P_c has high gain at low frequency while F_c has high gain at high frequency) the closed–loop designs will have good performance and disturbance rejection properties.

The optimal control problem requires the definition of control law structure and cost function to be minimized. The control law is defined as follows:

$$u(t) = C_o\, e_o(t) \qquad (2)$$

where e_o is the tracking error defined as the difference between the reference signal r(t) and the measured actual output z(t). The cost function or performance index to be minimized has the following form:

$$\begin{aligned} J_{GH_\infty} &= \|\phi_{\psi\psi}\|_\infty \\ &= \sup|\phi_{\psi\psi}| \\ &= \sup|P_c\phi_{ee}P_c^* + F_c\phi_{uu}F_c^* + P_c\phi_{eu}F_c^* + F_c\phi_{ue}P_c^*| \qquad (3) \\ \psi &= P_c \cdot e_o(t) + F_c \cdot u(t) \end{aligned}$$

where, Ψ is a weighted signal composed of the tracking error $e_o(t)$ and the control signal u(t) with the weight elements P_c and F_c as rational transfer functions and $\phi_{\psi\psi}$ is the spectral density of the weighted signal. The weighting elements P_c and F_c are defined as follow: $P_c = P_{cd}^{-1}P_{cn}$ and $F_c = F_{cd}^{-1}F_{cn}$. Where, P_{cd} and F_{cd} are strictly Schur polynomials with $P_{cd}(0) = F_{cd}(0) = 1$ and F_{cn} might be given with delay [10]. The GH_∞ optimal controller, which is the solution to the control law, is given by the following theorem [9]:

Theorem: The proof can be found in [9, 11, 12]

The controller design can be simplified by neglecting the measurement noise in system structure i.e. $C_n = 0$. In addition, the external signal–filters (W_r, W_d) are assumed to be asymptotically stable and the plant, the reference and the input disturbance denominator polynomials are assumed equal i.e. $A = A_r = A_d$. Thus, the GH_∞ optimal controller transfer function can be simplified and given by:

$$C_o = \frac{GF_{cd}}{HP_{cd}} \tag{4}$$

Which requires the solution of two spectral factorizations and two diophantine equations as follows:

(1) Spectral factors (D_c, D_f):

The strictly Schur spectral factors D_c and D_f which will be used in solving the diophantine equations are defined as follows:

$$D_c^* D_c = (P_{cn} F_{cd} B - F_{cn} P_{cd} A)^* (P_{cn} F_{cd} B - F_{cn} P_{cd} A) \tag{5}$$

$$D_f^* D_f = E_r^* E_r + C_d^* C_d \tag{6}$$

(2) The diophantine equations:

The polynomials G and H are obtained from the minimal degree solution $\{F, G, H\}$ with respect to F of the coupled diophantine equations;

$$F^- A\lambda P_{cd} + L_2 G = P_{cn} D_f F_s^- \tag{7}$$

$$F^- B\lambda F_{cd} - L_2 H = F_{cn} D_f F_s^- \tag{8}$$

where F_s^- is strictly Schur [2-5,7,12,16,17] and satisfies the relation $F_s^- F_s^{-*} = F^- F^{-*}$ and $L = L_1 L_2 = P_{cn} F_{cd} B - F_{cn} P_{cd} A$. The polynomial L_1 is a schur one while L_2 is non-schur.

2.2.2 Weighting Function Selection

Before attempting a controller design, control and error weighting functions, which reflect the frequency and time domain requirements must be selected. A good feedback design for a particular system is obtained by selection of the frequency dependent weighting functions P_c and F_c, where P_c costs the error signal e (t), while F_c costs the control signal u (t). Frequency shaping of P_c and F_c allows e(t) and/or u(t) to be weighted more in particular frequency ranges. That is, at low frequency the system is required to be insensitive to disturbances, while at high frequency it is required to filter out unwanted signals, like measurement noise. It is clear that $|S_n|$ must have a small value at low frequency and $|T_n|$ must have a small value at high frequency. It may be demonstrated that weighting e (t) and u (t) is equivalent to weighting S_n and M_n, respectively.

Toward the objective of weight selection, there are some rules and guidelines from the experience and literature [2-5, 9]. A simple method of selecting the weighting function using parameters that specify the corner frequency, the gain and the integral action that might be included is to parameterize it in the following from:

$$P_c = \frac{\gamma_1(1-\beta_1 s)}{(1-\alpha_1 s)} \quad \text{and} \quad F_c = \frac{\gamma_2(1-\beta_2 s)}{(1+\alpha_2 s)} \tag{9}$$

Where, the scalars $\gamma_1, \beta_1, \alpha_1, \gamma_2, \beta_2$ and α_2 are the tuning parameters for adjusting the system performance. It is often desirable to introduce integral action to the controller to improve the low-frequency performance of the control system including tracking and low frequency or steady-state disturbance rejection.

3 Missile and Actuator Dynamics

The missile modes can be divided into two categories: one includes modes that involve the rotational degrees of freedom and known as the short period, roll and Dutch–roll modes. The second category includes modes that involve the translational degrees of freedom which known as the phugoid and spiral modes. The responsiveness of a missile to maneuvering commands is determined by the speed of the rotational modes. The frequencies of these modes tend to be high. Therefore, it is necessary to design an autopilot system to control these modes, and to provide the missile with a particular type of response to the control inputs. One of the main channels constituting the autopilot system is the pitch channel, the general structure of which is shown in Fig. 2.

It can be seen that there are four principle elements: the missile dynamics, the sensor dynamics, the actuator dynamics and the controller or autopilot. The missile dynamics are described by a set of nonlinear differential equations whose coefficients are time varying and stochastic. Therefore, to design and analyze the performance of an autopilot, a state space representation or transfer function representing these dynamics should be available. Toward this objective the set of equations was linearized, in a previous work [1], using the perturbation around trim conditions among the whole envelope of missile flight. Then, the Laplace transform is applied to the linearized equations yielding the following transfer functions:

$$G_m = G_\delta^\vartheta = \begin{cases} \dfrac{2.47998s + 1.31762}{s^3 + 0.573s^2 + 0.0699s - 0.0211} & \text{for } t = 2\,[\text{sec}] \\ \dfrac{2.658s + 0.53415}{s^3 + 0.69055s^2 + 11.9562s - 0.02128} & \text{for } t = 30\,[\text{sec}] \\ \dfrac{2.495s + 0.2306}{s^3 + 1.11516s^2 + 0.02s - 0.00092} & \text{for } t = 60\,[\text{sec}] \end{cases} \tag{10}$$

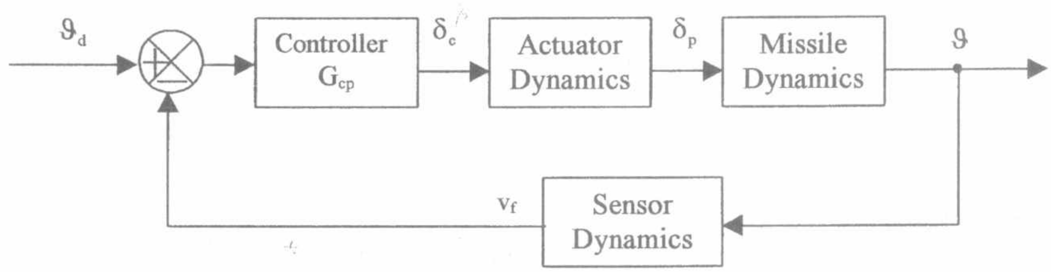

Fig 2: The general structure of Pitch channel

The inertial measuring sensor act as transducers in the missile flight control systems, in that it measures the motion variables and produces output voltages or currents, which correspond to these motion variables. For simplicity, the gyro is taken as a constant gain value $G_{gp} = 2.7\,[\text{volt/degree}]$. The actuator system was represented by three elements: a power amplifier, a servomotor and a feed back element. The actuator dynamics with loading is represented by the following transfer function [1]:

$$G_{actp} = \frac{92.4}{0.000075s^2 + 0.0058s + 1} \tag{11}$$

4 Pitch Channel Compensator

The control system is designed to perform a specific task such that the performance specifications are satisfied. These specifications are generally related to transient and frequency response such as overshoot, speed of response, phase margin and gain margin. Some performance specifications concerning the aerospace applications and have to be satisfied by the autopilot are summarized as follows [1,5]: rise time $\leq 0.5\,[\text{sec}]$, maximum peak overshoot $\leq 5\%$ and reject 50% of the disturbance within 1.5 [sec] and 95% within 4 [sec]. Toward this objective, the classical design technique using PI/PD/PID was designed in a previous work [1] and will be compared with advanced control techniques (GH_∞) in the next subsections.

4.1 Classical Design

A classical control had been designed in a previous work [1] for the pitch channel as follows:

$$G_{cp} = \frac{0.0129(1 + 0.225s)}{(1 + 0.0086s + 0.0004466s^2)} \tag{12}$$

Then, the transient and frequency responses for the obtained compensated system are shown in Fig. 3, using the MATLAB. Form these figures, it is clear that the design satisfied the system performance with overshoot (25%), rise time 0.35 [sec], settling time 1.5 [sec], phase margin 55.4 [deg], gain margin 10 [dB], bandwidth 4 [rad/sec] and corner frequency 7.3 [rad/sec].

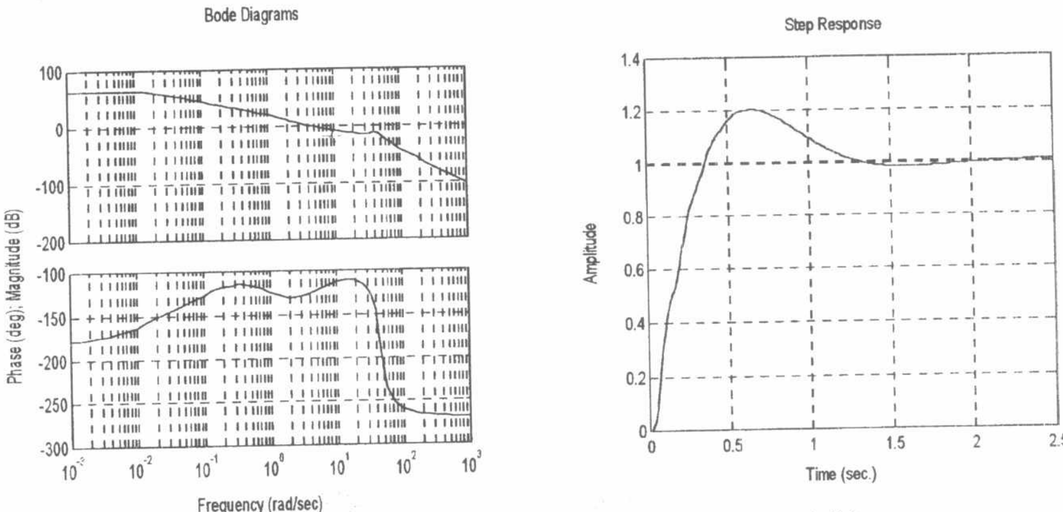

Fig 3: Classical system design (a) Bode diagram and (b) Step response

4.2 GH_∞ Controller Design

For the design and analysis trials using this philosophy, the operating point system dynamics at flight time t_f = 60[sec] is considered as the nominal operating condition. Then, the cost function weights are selected using different trials according to Eqs (9), yielding the transient and frequency responses as shown in Fig. 4 using the MATLAB. In one of the trials, the cost function weights are selected as follows:

$$P_c = \frac{(100s + 0.32)}{(7000s + 1)} \quad \text{and} \quad F_c = \frac{(50s + 0.032)}{(350s + 1)} \tag{13}$$

Using these weights and the GH_∞ programs [6] with the underlying system, the obtained controller is:

$$G_{cm} = \frac{0.10109\,S^6 + 7.9653\,S^5 + 1364.1496\,S^4 + 2823.7512\,S^3 + 261.2115\,S^2 + 1.4221S + 0.0019966}{S^6 + 84.6543\,S^5 + 13973.5241\,S^4 + 108321.6238\,S^3 + 10213.2125\,S^2 + 29.7093\,S + 0.0040361}$$

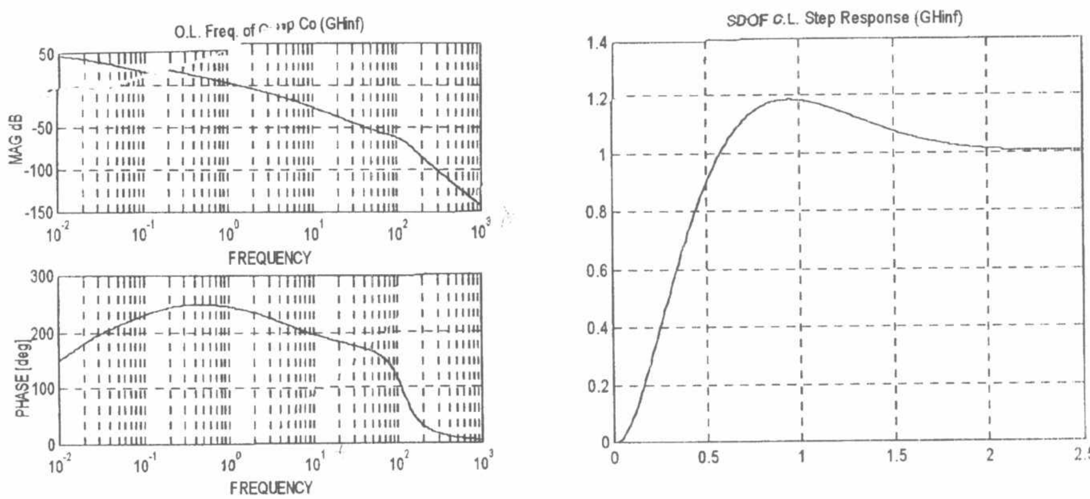

Fig 4: GH_∞ controller design (a) Bode diagram and (b) Step response

Form these figures, it is clear that the autopilot design satisfied the system performance with overshoot (18%), rise time 0.51[sec], settling time 0.95[sec], phase margin 53.3[deg], gain margin 33.23 [dB], bandwidth 1.5[rad/sec] and corner frequency 3.1 [rad/sec]. For easy comparison, the previous results can be summarized in a tabulated form as follows:

	Overshoot [%]	Rise time [sec]	Settling Time [sec]	B.W. [rad/sec]	Phase margin [deg]	Gain margin [dB]
Classic	25	0.35	1.5	4	55.4	10
GH_{∞_1}	18	0.51	0.95	1.5	53.3	33.2

From this table, it is clear that the GH_∞ controller has a smaller overshoot, settles faster and has better stability margins than the classical controller.

5 Robustness of the Designed Autopilot

Both the controllers, the classical and the GH_∞, are subjected to disturbance for justifying their capability in rejecting its effects upon the system response and the control effort. In addition, measurement noise figures are considered to represent those originated within a measuring device. Toward this justification, the system is modeled using the SIMULINK with MATLAB, to evaluate the system performance within environments close to the real.

5.1 Disturbance rejection

For evaluating the disturbance rejection property, different forms and levels are injected at the system output. Among these forms is the step disturbance to which the system response is shown in Fig. 5. From these figures, it is clear that the GH_∞ controller (autopilot) rejects the disturbance corrupting the system output faster than the classical one and settles its response to zero value. The GH_∞ controller rejects 50% of the disturbance within 1.3 [sec] while the classical one rejects 50% of the disturbance within 1.7 [sec]. In addition, the GH_∞ controller rejects 95% of the disturbance within 3 [sec] while the classical one rejects 95% of the disturbance within 5 [sec], with somehow oscillatory profile. The peak undershoot in case of GH_∞ is smaller (-0.8) than that obtained with the classical controller (-1).

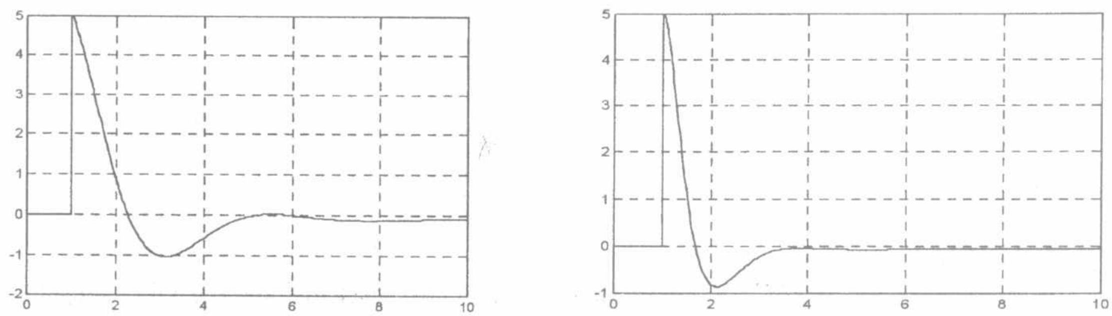

Fig.5: Step disturbance response of (a) classical and (b) GH_∞ design

5.2 Measurement Noise Attenuation

This criterion is validated through observing the system output and the control signal in response to the injected measurement noise. The system output responses to the white measurement noise are shown in Fig. 6.

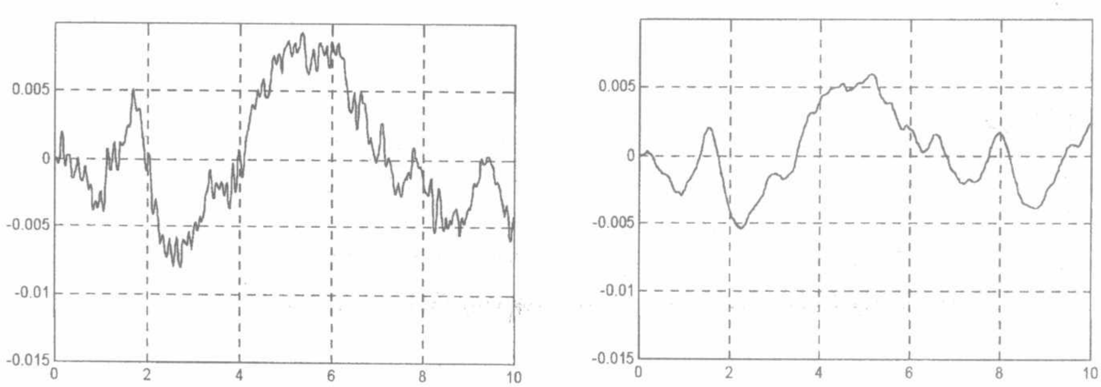

Fig.6: Measurement noise response of (a) classical and (b) GH_∞ design

These obtained results showed that the GH_∞ attenuates the measurement noise more than the classical can do. In addition, the GH_∞ has a filtering property as clear from the smoothing of the output response.

5.3 Unmodeled Dynamics

For autopilot design the system model at certain operating point is considered as the nominal one. Therefore, the system models at other operating points have dynamics that might not be exist in the nominal one which known as unmodelled dynamics. Consequently, the robustness of the autopilot designed with the nominal model is essential to guarantee its capability to overcome the effect of unmodeled dynamics. That is, the designed autopilot performs robustly with the system at other operating conditions. To justify this property, the designed autopilot (either classical or GH_∞) is implemented with different operating points models and trhe responses are observed in Fig. 10. It is clear that the GH_∞ autopilot is more robust than the classical one, especially at the operating conditions corresponding to flight times $t_f = 2[\text{sec}]$ and $t_f = 60[\text{sec}]$. However, both approaches give nearly close responses in case of flight times $t_f = 30[\text{sec}]$. Thus, it is necessary to revise this model and its assumptions in addition to looking forward for more robust GH_∞ autopilot upon more careful selection of the cost function weights.

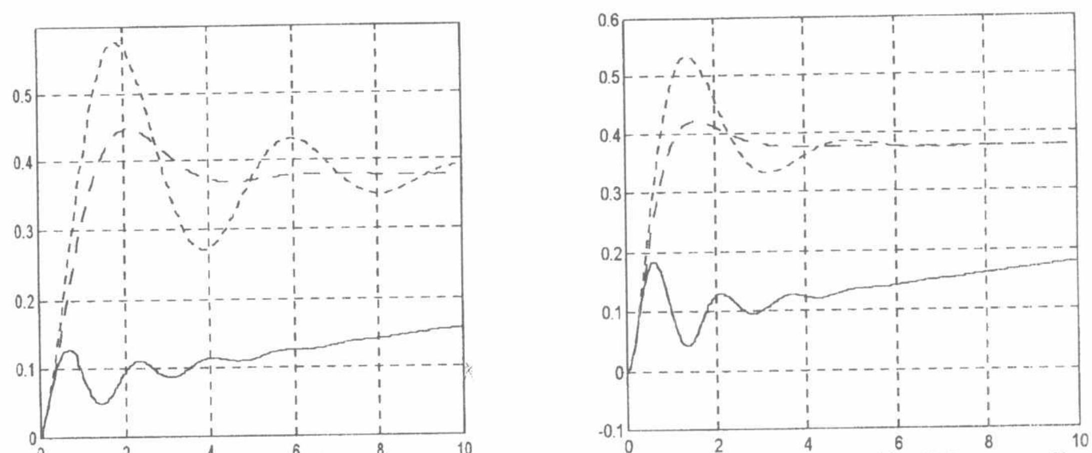

Fig. 7: Step responses at different operating points using (a) classical and (b) GH_∞ autopilot

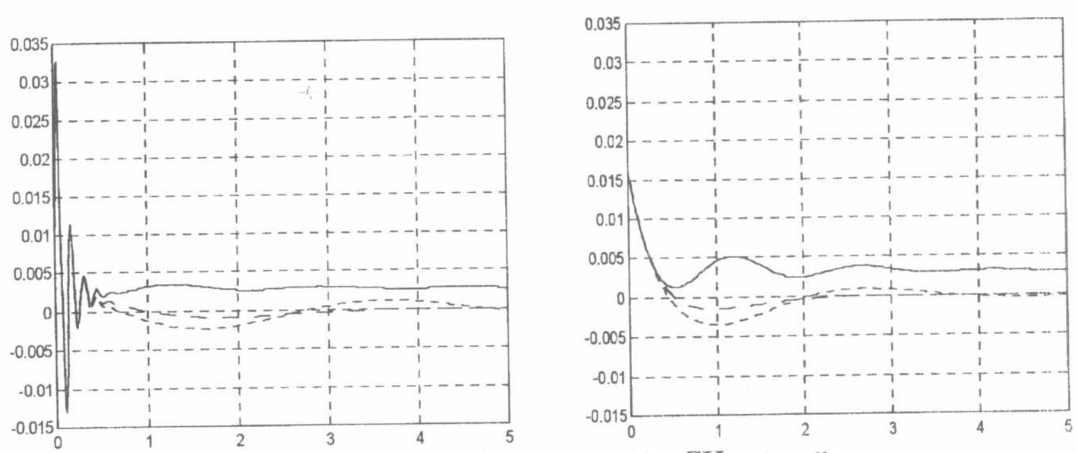

Fig. 8: Actuating signal using (a) classical and (b) GH_∞ controller

The actuating signal, Fig. 8, shows oscillatory profile during the transient period in case of classical design in addition to larger values than those obtained with the GH_∞ design. This problem/disadvantage reduces the life time of the missile and increases the required control effort.

Conclusions

This paper presented, briefly, different issues in robust control and summarized some of the controller design techniques including classical and advanced methods such as generalized H_∞ (GH_∞). The weight selection in advanced controller design techniques in addition to the controller structure are clarified. The obtained results showed that the GH_∞ controller has a smaller overshoot, settles faster and has better stability margins than the classical controller. In addition, the GH_∞ autopilot rejects the disturbance corrupting the system output faster than the classical one and settles its response to zero value. These obtained results showed that the GH_∞ attenuates the measurement noise more than the classical can do. In addition, the GH_∞ has a filtering property as clear from the smoothing of the output response. That is, the GH_∞ autopilot is more robust than the classical one with this system, especially at the operating conditions corresponding to flight times $t_f = 2\,[\text{sec}]$ and $t_f = 60\,[\text{sec}]$. The actuating signal shows oscillatory profile during the transient period in case of classical design in addition to larger values than those obtained with the GH_∞ design. This problem/disadvantage reduces the life time of the missile and increases the required control effort. However, both approaches give nearly close responses in case of flight times $t_f = 30\,[\text{sec}]$. Thus, it is necessary to revise this model and its assumptions in addition to look forward for more robust GH_∞ design upon more careful selection of the cost function weights.

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