Arab J. Nucl. Sci. Appl., Vol. 53, 1, 156-162 (2020)

Arab Journal of Nuclear Sciences and Applications

ISSN 1110-0451 | Web site: ajnsa.journals.ekb.eg | (ESNSA)

The Saturation Properties of Nuclear Matter Using Different Three-Body Force at Zero and Finite Temperature

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Received 21st Apr. 2019
Accepted 12th Dec. 2019

The three-body force (3BF) has been used to modify the two body forces to achieve the empirical saturation points as well as study the ground state properties for symmetric nuclear matter using the Brueckner-Hartree-Fock approximation (BHF) with the Argonne AV18 potential at zero and finite temperature. Moreover, the energy per nucleon (E/A) as a function of nuclear density ρ is calculated. Furthermore, the correction of the two-body dependent potential (correction 1) is added to shift and improve the saturation properties of the nuclear matter ρ_o from 0.265 fm^{-3} to 0.149 fm^{-3} at E/A = -16.142 MeV towards to the empirical saturation point ρ_o = 0.16 fm^{-3}. Additionally, the pressure p for symmetric nuclear matter for zero-temperature T= 0 as a function of density ρ/ρ_o using the Argonne AV18 potential is calculated revealing a good agreement between our calculations and the experimental data. On other hand, more calculations for the pressure are added at different energies T= 4, 8, 12, 18 and 20 MeV. Also, the level-density parameter as a function of density ρ is calculated for the BHF approach. Moreover, the internal energy F for the symmetric nuclear matter as a function of density ρ using the Argonne AV18 potential for continuous choice at T= 4, 8, 12, 18 and 20 MeV for the BHF with and without 3BF is calculated.

Keywords: *Nuclear matter / Realistic potential / Saturation density /Binding energy*

Introduction

One of the big challenge in theoretical nuclear physics is the attempt to found the basic characteristics of nuclear systems to describe the realistic nucleon-nucleon interaction. The procedure of this attempt can typically be derived in two steps, in the first one, a model is deuteron. In the second step, the many-body problem of the interacting nucleons using a realistic nucleon-nucleon interaction needs to be solved. The simplest solution of this many chosen within which the pure nucleon-nucleon interaction is described which leading one to be guided by quantum chromodynamics [1]. Also, using an input Boson exchange model. i.e. a meson exchange model [2] and this leads to a microscopic description of the nucleon-nucleon interaction. On the other hand, a purely phenomenological approach, where two-particle spin isospin operators are used, each particle is multiplied by a local potential used such the Argonne AV18 potential [3]. All these models will be considered as a realistic description of nucleon-nucleon interaction by appropriate choice of model parameters, the nucleon-nucleon scattering phases for energies below the bion threshold are described. In addition, such realistic potentials also reproduce energy and other observables of the deuteron.

In the second step, the many-body problem of the interacting nucleons using a realistic nucleon-nucleon interaction needs to be solved. The simplest solution of this many-body problem is the Hartree-Fock approximation, in this approximation one can determine very well the ground state properties of the nuclei as well as its internal structure [4].

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DOI: 10.21608/ajnsa.2019.12162.1208

As long as one uses phenomenological potentials which have two or few adjustable parameters which are determined by fitting the empirical saturation density and energy of the cold symmetric nuclear matter in the BHF calculations [5], theses potentials are, however, unable to describe very well the data of nucleon-nucleon scattering. Such potentials are thus not realistic potentials designated. On the other hand, using the Haetree-Fock approximation with realistic potentials is completely inadequate, it leads to unbound systems [6]. The failure of the Hartree-Fock approximation has led to the development of a variety of techniques for correlation effects beyond the pure Hartree-Fock approximation, to be considered. These include, for example, the Brueckner development [7 and 8], the coupled cluster exponential S method [10 and 11], self-consistent calculation by using the green functions [12], variational method by using correlated basic functions [13 and 14] as well as recent development by using quantum Monte-Carlo techniques [15 and 16]. However, in addition to correlation effects, it is obvious that a relativistic treatment of the many-body problem is a necessary to improve the description of the ground state properties of nuclear systems.

The nuclear equation of state (EOS) is a very important tool to interpret the nucleus-nucleus collision. To establish the EOS it needs mainly two tools, the first is the nuclear force model such as non-relativistic potential model, relativistic mean field model or finite-range Thomas Fermi model. And the second one is a numerical technique, we might choose liquid droplet approach, Thomas Fermi approximation or BHF approximation. The two tools can be merged to establish the equation of state. In case of experimental studies of the equation of state, one of the main motivations of study of nuclear collisions in the range of relativistic energies is the experimental determination of the equation of state of symmetric nuclear matter. The first exploration of this range was at the BEVALAC via bombarding energies of about 0.2 till 2 *GeV*/nucleon, some models like Hydrodynamical and cascade models predict the existence of a compression phase and the total density reaches to 2-3 times nuclear density for incident energies of 0.5 till 1 *GeV*/nucleon [16, 17], the temperature of this system during this phase can be become as high as 50-100 *MeV* [16].

The equilibrium density of the ideal symmetric nuclear matter found to be $\rho_o = 0.17$ nucleons/fm^3 which corresponds to the energy per nucleon E/A = -16 MeV [7], where ρ represents the saturation density and E/A represents the saturation energy. For discussing the ground state properties of nuclear matter we have used the most realistic nucleon-nucleon force Argonne AV18 [3]. Besides, it is constructed by a set of two-body operators, which arise naturally in meson exchange processes, its form factor are partly phenomenological except the one-pion exchange.

In the present work, the energy per nucleon as a function of the density E/A and the pressure p are computed by using the Argonne AV18 potential [3], in the frame work of BHF approach added to density term eq.(9) compared with the eight terms with different powers of the densities [17], the saturation density of nuclear matter is found $\rho_o =$ 20 fm^{-3} and its saturation energy per nucleon E/A = -16 *MeV*. Also the pressure of symmetric nuclear matter p at $T =$ 4, 8, 12, 18 and 20 *MeV* have been calculated by using T^2 approximation [16]. The second part of this work the BHF as well as the Beth-Goldstone equation were investigated. The third part shows the theoretical results comparing with the available experimental data. Conclusion is given in section 4.

The theoretical model

The Brueckner-Hartree-Fock approximation depends on the BGE for the ground state energy, the time-dependent Hamiltonian operator of a system consists of A identical nucleons can be split as the following:

$$\hat{H} = \sum_{i=1}^{A} t_i + \sum_{i=j}^{A} v_{ij} = \hat{H}_o + \hat{V} \quad (1)$$

Where the first term denotes the sum over the kinetic energy of all particles, the second term represents the sum over the two-particles interaction for a realistic nucleon-nucleon interaction. The splitting of the Hamiltonian operator $\hat{H}$ into the two parts H_o and V takes place according to:

$$\hat{H}_o = \sum_{i=1}^{A} (t_i + u_i) \quad (2)$$

$$\hat{V} = \sum_{i=j}^{A} u_{ij} + \sum_{i=j}^{A} u_i \quad (3)$$

Where the single particle potential $U = \sum u_i$.

The Brueckner-Bethe-Goldstone (BBG) equation of the interaction between two nucleons can be described via the G matrix:

$$G = V + V\frac{Q}{\omega + H_o + i\eta} G \qquad (4)$$

Where V denotes the nucleon-nucleon interaction, η is infinitesimal small number and ω is the starting energy which represents the sum of the single particle energy of the interacting nucleons and it can be written as:

$$\omega = e(k) + e(\acute{k}) \qquad (5)$$

And the single particle energy $e(k)$ can be written as:

$$e(k) = \frac{k^2}{2m} + \sum_{\alpha<F} \langle k\alpha \left| G\left(\omega = e_k^{BHF} + e_\alpha^{BHF}\right) \right| k\alpha\rangle_A \qquad (6)$$

Where A denotes an anti-symmetrization of the matrix elements, and Q denotes the Pauli operator which can be written as [18]:

$$\langle (\grave{L}s)\hat{J} | Q(K,k) | (Ls)\rangle = \sum_{m_s m_L} \langle LM_L SM_s | JM_J\rangle \langle \hat{J}M_J | \grave{L}M_L SM_s\rangle \langle \grave{L}M_L | Q(K,k) | LM_L\rangle \qquad (7)$$

Via the BHF approach from eq. (6) we can evaluate the binding energy per nucleon as:

$$\frac{E}{A} = \frac{3}{5}\frac{K_f^2}{2M} + \frac{1}{2\rho} Re \sum_{k\leq k_F} \langle k\acute{k} | G(e(k) + e(\acute{k})) | k\acute{k}\rangle_a \qquad (8)$$

Where k_F is the Fermi momentum.

In order to study the properties of nuclei and nuclear matter, two-body density dependent potentials have been used.

The second one is the Mansour three body force (correction 2) is defined as [19]:

$$V(\vec{r_1}, \vec{r_2}) = \sum_{i=1}^{4} t_i (1 + x_i \rho_\sigma) \rho^{\alpha i}\, \delta(\vec{r_1} - \vec{r_2}) \qquad (9)$$

Where r_1 and r_2 are the positions vectors of the nucleon (1) and nucleon (2) respectively, and $\alpha_i = (\frac{1}{3}, \frac{2}{3}, \frac{1}{2}$ and 1) , this correction has been studied previously [20,21,22,23 and 24], t_i and x_i represent the parameters of the interaction, ρ_σ is the spin exchange operator and ρ is the density, the four values of x_i listed in [25], in addition, we have obtained these values via fitting the experimental symmetry energy as well as the values of t_i.

The first correction (correction 1) [25] is calculated from the following equation:

$$\frac{E}{A} = \frac{3}{8} t_o\, \rho + \frac{1}{16} t_3\, \rho^{1+\alpha} \qquad (10)$$

The values of the two parameters t_o and t_3 are adjusted to reproduce the biding energy per nucleon (E/A) and the saturation density of nuclear matter ρ_o where t_o = -325.902 fm^{-3} and t_3 = 4837.6 fm^{-3} and α represents a fixed parameter (typically, $\alpha = 0.5$).

Results and discussion

The energy per nucleon E/A in MeV as a function of density for symmetric nuclear matter is shown in Figure (1). Besides, we have used the Brueckner-Hartree-Fock approximation in the calculations with and without the contribution of the 3BF. The calculations have been performed using the Argonne AV18 potential [3]. It's obvious that, there are substantial region in which the slope of the energy per nucleon (E/A) is negative at low densities, then the (E/A) should increase as density increases. The results of BHF using the Argonne AV18 potential are plotted as solid curve. In addition, the 3BF of the same calculations are plotted as dashed curve. The correction 1 which added to the results of BHF plotted is shown as dotted curve and the Mansour's body force correction with the BHF values is plotted with solid-dashed curve.

It's obvious from Figure (1) that the contributions of the corr.(1) from [25] when added to the values of the BHF participate to improve the saturation point to become $\rho_o = 0.149\, fm^{-3}$ at $E/A =$ -16.142 MeV close to the empirical one $\rho_o = 0.16\, fm^{-3}$ at $E/A = -16$ MeV. In case of comparing the results of Mansour three body force correction [17 and 19] and the results of corr.1 Of [25] using the same potential AV18, we have found that the Mansour three body force correction is very close to the calculations of the 3BF.

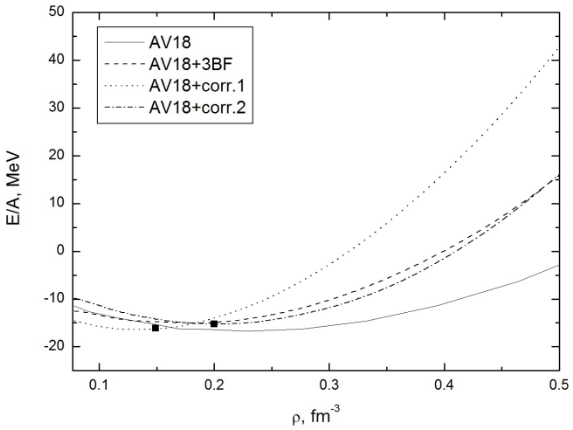

Figure (1) The binding energy per nucleon E/A as a function of density symmetric nuclear matter ρ for two-body forces BHF (solid curve), the 3BF (dashed curve), corr. 1 (dotted curve) and corr. 2 (solid-dashed curve) with Argonne AV18 potential

The reason for this comes from the number of the parameters in each correction, for corr. 1 we have used only two parameters, t_o = -325.902 fm^{-3} and $t_3 =$ 4837.6 fm^{-3}. While in Mansour three body force (corr. 2) more parameters have been added in eq.(9) for choosing $\alpha_i = (\frac{1}{3}, \frac{2}{3}, \frac{1}{2}$ and 1) then fitting the parameters t_i and x_i, the results of these fitting parameters are listed in table 1 in [16].

We have calculated the pressure p as a function of density for symmetric nuclear matter at $T = 0$ from the following formula:

$$p(\rho) = \rho^2 \frac{\partial(E/A)(\rho)}{\partial\rho} \qquad (12)$$

Figure (2) shows the results of pressure with four calculations, The results of BHF using the Argonne AV18 potential are plotted as solid curve, the 3BF of the same calculations are plotted as dashed curve, the corr.(1) which is added to the results of BHF is plotted as dotted curve and the corr.(2) with the BHF values is plotted with solid-dashed curve.

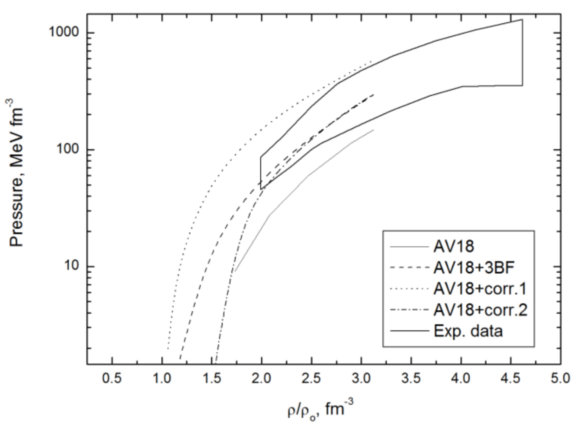

Figure (2) The pressure p for symmetric nuclear matter for zero-temperature $T = 0$ as a function of density ρ/ρ_o using the Argonne AV18 potential, the results are compared with the experimental data from [27]

It's obvious that, the pressure should increase as the density of symmetric nuclear matter increases, also, we have found that the discrepancies between all calculations with or without the contributions of the two corrections are negligible specially at low densities but at high densities, these discrepancies will be noticeable, in case of comparison with the available experimental data we have found that the 3BF and the corr. (2) very close to the experimental data.

Figures (3, 4, 5, 6 and 7) show the calculations of the pressure p at different temperatures $T =$ 4, 8, 12, 18 and 20 *MeV*, we have used the T^2 approximation [17] to carry out the following formula:

$$p_T = p_{T=0} + \frac{T^2}{9}\frac{2m^*}{\hbar^2}\left(\frac{3\pi^2}{2}\right)^{1/3}\rho^{1/3} \qquad (13)$$

We can see from the Figures (3, 4, 5, 6, 7 and 8) that the pressure as a function of density nuclear matter ρ decreases by increasing the ρ then increases by increasing the density.

In the present paper, we have used two methods in order to calculate the thermodynamics properties of the ground state

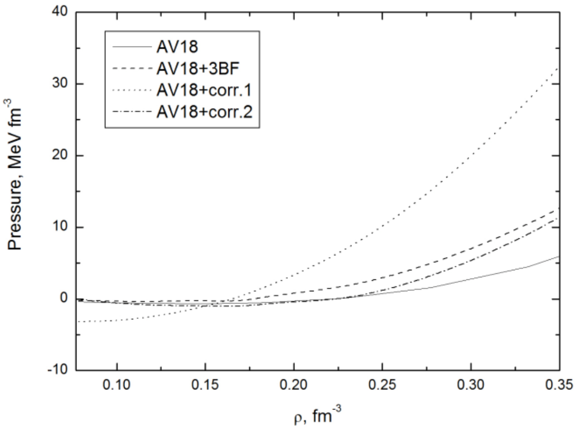

Figure (3) same as Figure (2), but for $T = 4$

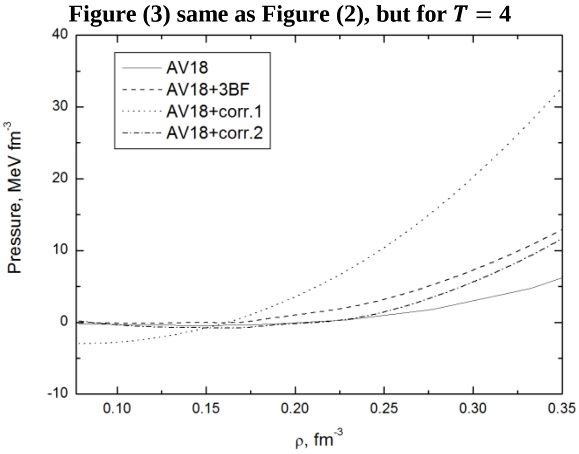

Figure (4) The same as Figure 2, but for $T = 8$

of the symmetric nuclear matte. The first method depends on the calculations of the *G*-matrix elements with $T = 0$ and the second

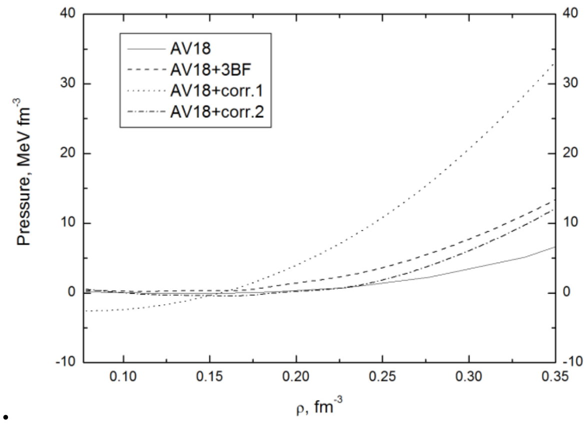

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Figure (5) same as Figure 2, but for $T = 12$

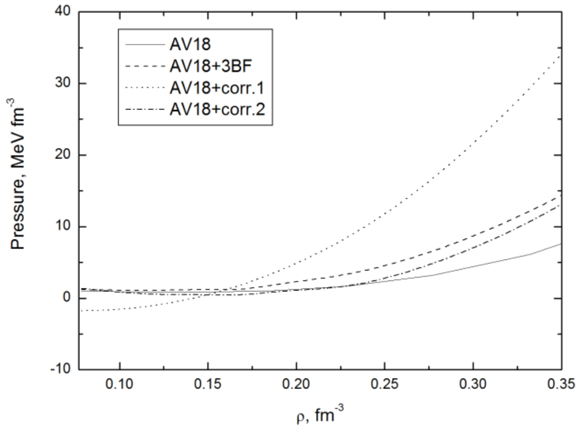

Figure (6) same as Figure 2, but for $T = 18$

method based on the internal energy of the system F which is defined as

$$F = E - TS_T \quad (14)$$

Where E represents the energy per nucleon (E/A) at $T = 0$, this internal energy is calculated using the effective mass $m*$ and the entropy of the system S_T as a function of temperature T. At low temperatures $E \rightarrow E_{T_o} + aT^2$ and $S_T = 2aT$, where a represents the level-density parameter introduced as a function of density [17], it can be defined by:

$$a(\rho) = \left(\frac{2m^*(\rho)}{\hbar^2}\right) \left(\frac{3\pi^2}{2}\right)^{\frac{1}{3}} \rho^{-2/3} \quad (15)$$

Then the internal energy can be written by

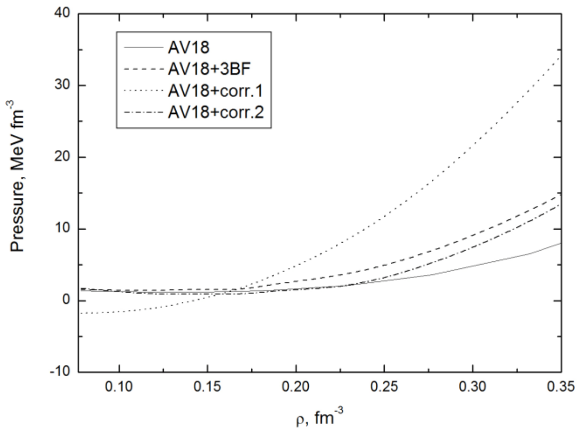

Figure (7) The same as Figure 2, but for $T = 20$

$$F = E + aT^2 - 2aT^2 = E - a(\rho)T^2 \quad (16)$$

From equations (15) and (16) we get:

$$F = E_{T=0} - \frac{T^2}{6} \left(\frac{2m^*(\rho)}{\hbar^2}\right) \left(\frac{3\pi^2}{2}\right)^{\frac{1}{3}} \rho^{-2/3} \quad (17)$$

It's convenient to mention that in case of zero-range forces we can use the same expressions [26]. Figure (8) shows the level-density parameter a as a function of the density ρ for the Argonne AV18 potential [3] with the 3BF and the two added corrections. It is clear that the level-density parameter decreases by increasing the density ρ.

In Figures (9,10,11,12 and 13) we have plotted the internal energy F for symmetric nuclear matter in MeV as a function of density ρ at different temperatures $T = 4$, 8, 12, 18 and 20 MeV. The curves represent the BHF approach using the Argonne AV18 potential [3] with 3BF and the two added corrections. We can observe from these figures that the internal energy decreases at low densities then it increases by increasing the density ρ. For more details, one can see more attempts for studying the BHF calculations at finite temperatures in [27,28,29 and 30].

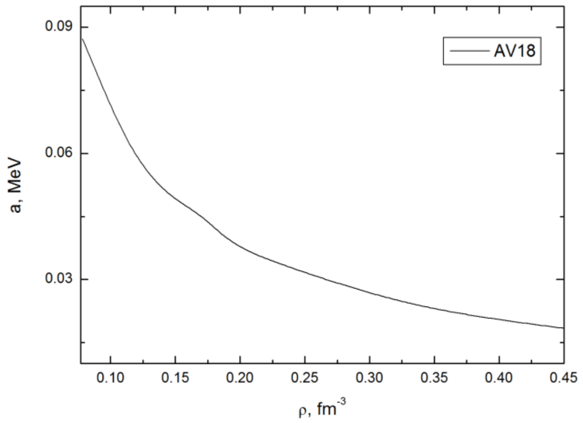

Figure (8) The level-density parameter a for the BHF calculations as a function of the density using the Argonne AV18 potential

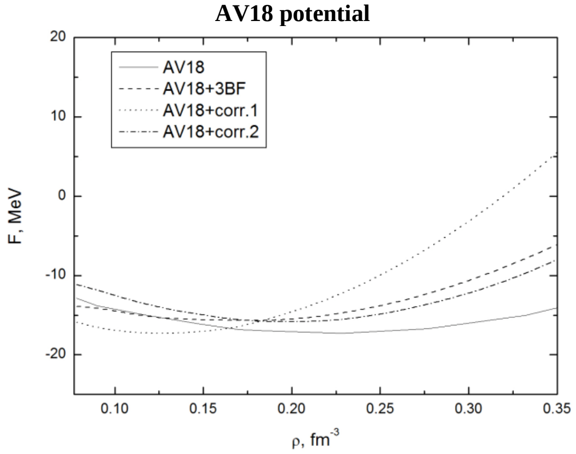

Figure (9) The internal energy for symmetric nuclear matter as a function of density using the Argonne AV18 potential for continuous choice at $T = 4$ for two-body forces BHF (solid curve), the 3BF (dashed curve), corr. 1 (dotted curve) and corr. 2 (sold-dashed curve)

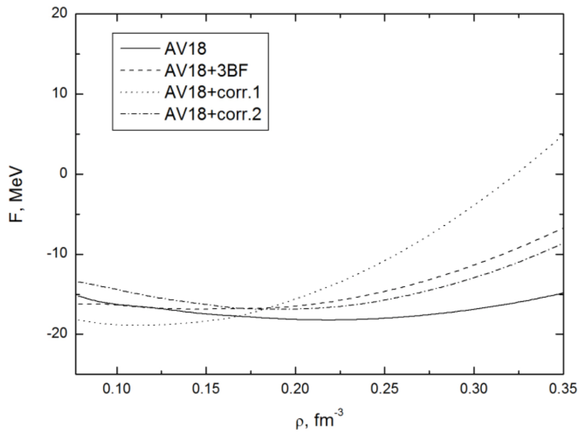

Figure (10) same as Figure 9, but for $T = 8$

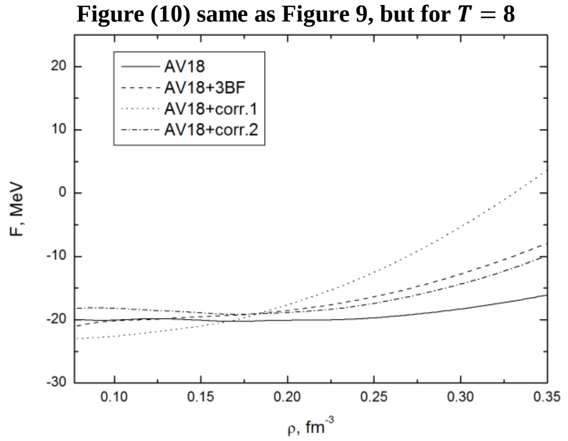

Figure (11) same as Figure (9), but for $T = 12$

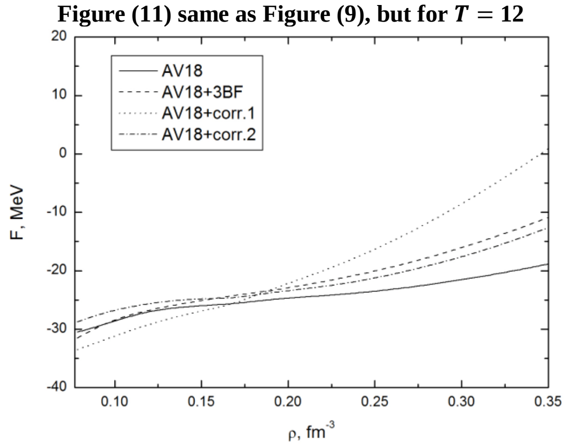

Figure (12) same as Figure (9), but for $T = 18$

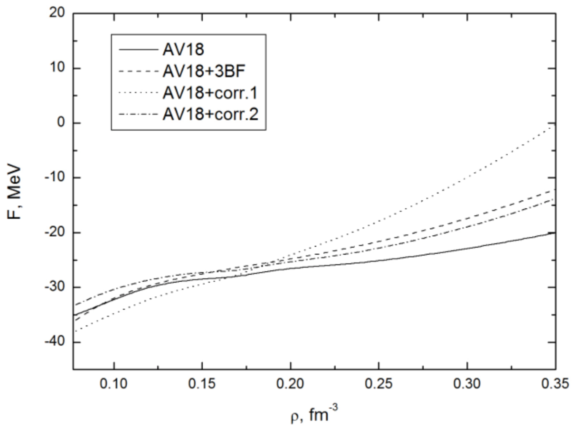

Figure (13) same as Figure 9, but for $T = 20$

Conclusions

It's important to study the three-body force to modulate the two-body forces to achieve the empirical saturation points as well as study the ground state properties of the symmetric nuclear matter at zero and finite temperature. The main goal in the present work is to calculate the correction parameters [25] and add its results to the results of the two-body force to achieve the 3BF results. Also, comparing our results for the BHF approach with Mansour three body force correction [17 and 19] to study the properties of the symmetric nuclear matter at zero and finite temperature by using the same nucleon-nucleon interaction Argonne AV18 potential [3]. In case of comparing the results of the two corrections, we have found that the results of Mansour three body force (corr.2) is very close more than (corr.1) to the calculations of the 3BF, this is as a result of the number of parameters in each correction. In the corr.1 two parameters t_o an t_3 are used while in Mansour three body force correction eight parameters were added to eq.(9).

Acknowledgement

The author expresses his deepest gratitude to Prof. Dr. Hesham Mansour, professor of theoretical nuclear physics, physics department, faculty of science, Cairo university, for his continuous encouragement and helpful discussions throughout all steps of this work

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