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PERTURBED GROUND TRACK UNDER THE INFLUENCE OF J_2 AND LUNI-SOLAR FORCES

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ABSTRACT

A several types of forces are acting on the satellite. These forces are classified into conservative and non-conservative force. The main concern in the present research work is to studding the effect of conservative forces on the satellite orbital motion and represent this effect on satellite ground track. Where the ground tracks are the locus of points formed by the points on the Earth directly below a satellite as it travels in orbit. A mathematical model and a program code is designed using Matlab package to calculate the perturbed ground track under J_2 and luni-solar forces. Whereas the J_2 and Luni-Solar are a conservative forces, the secular variation is presented only in RAAN and ω. Otherwise the remaining orbital element is varies periodically. The perturbed ground track is calculated under the effect of J_2 and luni-solar forces. The perturbed position vectors for a satellite are converted to its corresponding latitudes and longitudes. The satellite's position in one revolution is displayed to represent where the satellite at the time desired.

Keywords: Satellite, oblateness, earth's gravitational force, luni-Solar force, ground Track.

1. INTRODUCTION

The studying and modeling perturbations are key disciplines in astrodynamics. We must consider the forces acting on the satellite. There are two types of forces causing the perturbative effects on a satellite:

1. Conservative forces (for example central-body and third-body gravitational).

2. Non-conservative forces (for example solar-radiation pressure, thrust, and drag).

Propagation concerns with the determination of the motion of a body over time. According to Newton's laws, the motion of a body depends on its initial state (i.e., its position and orientation at some known time) and the forces that act upon it over time. There are three types of orbit propagators:

1. Numerical Integration Propagators
2. Analytic Propagators
3. Semi-Analytic Propagators

Ground tracks are the locus of points formed by the points on the Earth directly below a satellite as it travels in orbit. To determine ground tracks for a satellite's orbit, we combine both of Kepler's routines with the conversion of a position vector to its sub-satellite point. This particular combination helps us in determine satellite orbit position and location relative to a ground site.

2. The equation of motion with perturbation

The Equation of motion for two-body of relative motion is

$$\ddot{\bar{r}} = -\frac{\mu}{r^3}\bar{r}\ , \qquad (2.1)$$

where

μ is Earth's gravitational parameter, μ = 398600.4418 km3/sec2, and
$\bar{r}$ is satellite position vector;
the perturbations that effects on the satellite can we classified into two types:

1. Gravitational perturbation: such as oblateness of Earth, N-body attraction and others.
2. Non-gravitational perturbation: such as atmospheric drag, solar radiation pressure, and others, then

$$a_p = a_{Gravitational} + a_{Non\text{-}gravitational}, \qquad (2.2)$$

where

a_p is the acceleration due to the summation of perturbing forces, then the acceleration due to perturbation is given by

$$\ddot{\vec{r}} = -\frac{\mu}{r^3}\vec{r} + \vec{a}_p . \qquad (2.3)$$

3. The Earth's gravitational force

The net result of the irregular shape of the Earth is to produce a variation in the gravitational acceleration that predicted using a point of mass distribution. An accurate model of the Earth can be obtained through the use of a series of spherical harmonics; which effectively represent a gravitational body as a series of mass centers, some more dominant than others, the most dominant term being that of a perfectly uniform sphere. The gravitational potential of the Earth U is defined by [11], [22] and [44].

$$U = \frac{\mu}{r}\sum_{l=0}^{\infty}\sum_{m=0}^{l}\left(\frac{R_\oplus}{r}\right)^l P_{l,m}[\sin\phi]\left\{C_{l,m}[\cos m\lambda] + S_{l,m}[\sin m\lambda]\right\}, \qquad (3.1)$$

where

$R_\oplus$ is Earth's mean equatorial radius, $R_\oplus$ = 6378.165 km,

ϕ is Geocentric latitude of the satellite,

λ is Geocentric longitude of the satellite,

$P_{l,m}$ is the associated Legendre polynomial of degree l and order m, and

$C_{l,m}$ & $S_{l,m}$ are Geopotential coefficients.

Equation (3.1) describes the gravitational attraction resulting from the irregular distribution of the Earth's mass using a potential function. There are three types of spherical harmonic.

Zonal harmonic represented by l where

$$J_l = -C_{l,m}\ \forall_m = 0. \qquad (3.2)$$

The potential on longitude vanishes and the field is symmetrical about the polar axis. These are bands of latitude. For any $P_{l,m}[\sin\phi]$ there are l circles of latitude which $P_i = 0$, and hence ($l+1$) zones [11]. The strongest perturbation due to the Earth's shape is J_2. Where

J_2 = 0.0010826269, J_3 = - 0.000025323,
J_4 = - 0.000016204. (3.3)

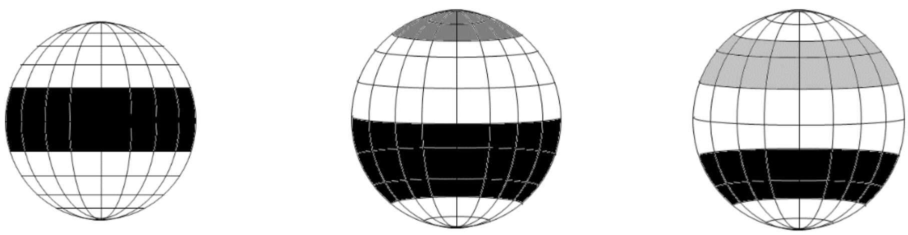
Sectorial harmonic represent bands of longitude where $l = m$. The polynomials $P_{l,m}[\sin\phi],\ \forall_\phi = \pm 90^\circ$. The sphere is divided into $2l$ sectors.

Tesseral harmonic which $l \neq m \neq 0$, the sphere is divided into a checkerboard array. The number of circles of latitude which $P_{l,m}[\sin\phi] = 0$ is equal to ($l - m$), whereas $C_{l,m}[\cos m\lambda] + S_{l,m}[\sin m\lambda]$ vanish along $2m$ meridians of longitude. These zero lines represent the center of the latitude and longitude bands. Figures (3.1, 3.2 and 3.3) are the various types of harmonic coefficients.

Now from equation (3.1), we use the gradient to determine the accelerations resulting from the central body. The gradient operation produces acceleration components along each axis. This is actually a special case for J_2. [3 and 10]

$$R_2 = \frac{\mu}{r}\left(\frac{R_\oplus}{r}\right)^2 P_{2,0}[\sin\phi]C_{2,0} . \qquad (3.4)$$

Using equation (3.2)



J_2 J_3 J_4

Figure (Error! No text of specified style in document.**-1): Zonal harmonics**

$$R_2 = -\frac{\mu J_2}{r}\left(\frac{R_\oplus}{r}\right)^2 P_{2,0}[\sin\phi]. \qquad (3.5)$$

Determine the associated Legendre

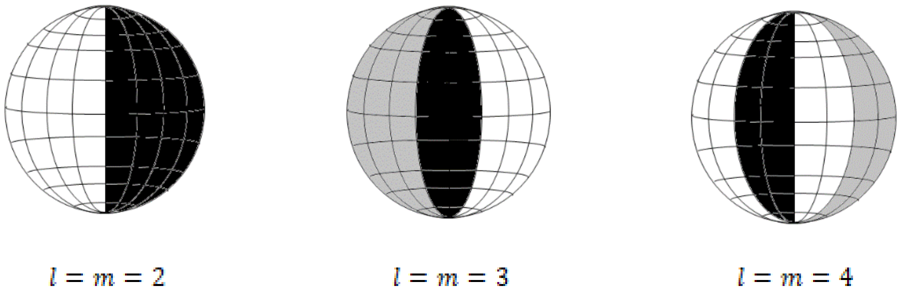


Figure (3.1): Zonal harmonics

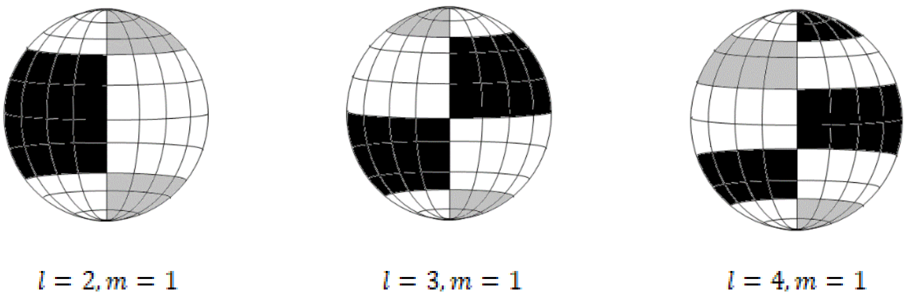


Figure (3.2): Tesseral harmonics

function for $P_{2,0}[\sin\phi]$ [11].

$$P_{2,0}[\sin\phi] = 0.5[3\sin^2(\phi) - 1], \qquad (3.6)$$

by a substitute in equation (3.5), then

$$R_2 = -\frac{3\mu J_2}{2r}\left(\frac{R_\oplus}{r}\right)^2\left[\sin^2\phi - \frac{1}{3}\right]. \qquad (3.7)$$

Let

$$\sin\phi = r_k / r. \qquad (3.8)$$

Substituting in equation (3.7)

$$R_2 = -\frac{3\mu J_2 R_\oplus^2}{2r^3}\left(\frac{r_k}{r}\right)^2 + \frac{\mu J_2 R_\oplus^2}{2r^3}. \qquad (3.9)$$

Differentiate equation (3.9) to get

$$\frac{\partial R_2}{\partial r_l} = -\frac{3\mu J_2 R_\oplus^2 r_k^2}{2r^3}\left(-\frac{5(2r_l)}{2r^7}\right) + \frac{\mu J_2 R_\oplus^2}{2}\left(-\frac{3(2r_l)}{2r^5}\right). \qquad (3.10)$$

Simplify to

$$\frac{\partial R_2}{\partial r_l} = -\frac{3\mu J_2 R_\oplus^2 r_k}{2r^5}\left(-\frac{5(r_k^2)}{r^2} + 1\right). \qquad (3.11)$$

Similarly, we obtain to $\frac{\partial R_2}{\partial r_l}$ and $\frac{\partial R_2}{\partial r_k}$, the accelerations component due to J_2 are

$$a_l = \frac{\partial R_2}{\partial r_l} = -\frac{3J_2 R_\oplus^2 r_k}{2r^5}\left(1 - \frac{5(r_k^2)}{r^2}\right), \qquad (3.12)$$

$$a_J = \frac{\partial R_2}{\partial r_J} = -\frac{3J_2 R_\oplus^2 r_J}{2r^5}\left(1 - \frac{5(r_k^2)}{r^2}\right), \qquad (3.13)$$

$$a_k = \frac{\partial R_2}{\partial r_k} = -\frac{3J_2 R_\oplus^2 r_k}{2r^5}\left(3 - \frac{5(r_k^2)}{r^2}\right). \qquad (3.14)$$

4. The Luni-Solar perturbation

The other bodies, such as the Sun or Moon, have a greater effect on satellites in higher altitude orbits. Because the cause of perturbations from the Sun and the Moon is the gravitational attraction; which is conservative.

let the third body denoted by 3 and assume the mass of the satellite is negligible. The general form of the equation of motion for the three-body system is [11]

$$\ddot{\vec{r}}_{\oplus sat} = -\frac{\mu_{\oplus}\,\vec{r}_{\oplus sat}}{r^3_{\oplus sat}} + \mu_3\left(\frac{\vec{r}_{sat3}}{r^3_{sat3}} - \frac{\vec{r}_{\oplus 3}}{r^3_{\oplus 3}}\right), \quad (4.1)$$

where

$\oplus$ is the subscript denoted to the Earth, and

sat is the subscript denoted to the artificial satellite.

The first term of equation (4.1) is the two-body acceleration of the Earth acting on the satellite. The second term has two parts (direct and indirect effect) and it represents the perturbation.

5. Variation of the parameter (*VOP*)

Lagrange and Gauss both developed *VOP* methods to analyze perturbations. Lagrange's technique works for conservative accelerations. Gauss's technique works for non-conservative accelerations.

The *VOP* equations of motion are a system of first-order differential equations that describe the rates of change for the time-varying elements. The gauss's *VOP* uses the specific force components resolved in the satellite coordinate system RSW [8, 9 and 11]. It's expressed as

$$\frac{da}{dt} = \frac{2}{n\sqrt{1-e^2}}\left(e\sin\nu\,F_R + \frac{p}{r}\,r\,F_S\right), \quad (5.1)$$

$$\frac{de}{dt} = \frac{\sqrt{1-e^2}}{na}\left\{\sin\nu\,F_R + \left(\cos\nu + \frac{e+\cos\nu}{1+e\cos\nu}\right)F_S\right\}, \quad (5.2)$$

$$\frac{dI}{dt} = \frac{r\cos u}{n\,a^2\sqrt{1-e^2}}\,F_W\,, \quad (5.3)$$

$$\frac{d\Omega}{dt} = \frac{r\sin u}{n\,a^2\sin\nu\sqrt{1-e^2}}\,F_W\,, \quad (5.4)$$

$$\frac{d\omega}{dt} = \frac{\sqrt{1-e^2}}{n\,a\,e}\left\{-\cos\nu\,F_R + \sin\nu\left(1+\frac{r}{p}\right)F_S\right\} - \frac{r\cot I\sin u}{h}\,F_W, \quad (5.5)$$

$$\frac{dM_0}{dt} = \frac{1}{n\,e\,a^2}\left\{(p\cos\nu - 2er)F_R - (p+r)\sin\nu\,F_S\right\} - \frac{dn}{dt}(t-t_0) \quad (5.6)$$

If the disturbing function *R* is known, we can use the Gaussian form of each force component.

The acceleration components of the disturbing force are

$$F_R = \frac{\partial R}{\partial r}\,, \quad (5.7)$$

$$F_S = \frac{1}{r}\frac{\partial R}{\partial u}\,, \quad (5.8)$$

$$F_W = \frac{1}{r\sin u}\frac{\partial R}{\partial I}\,. \quad (5.9)$$

Using equation (3.1), then

$$F_R = \frac{\partial R}{\partial r}\,, \quad (5.10)$$

$$F_S = \frac{1}{r}\frac{\partial R}{\partial u}\,, \quad (5.11)$$

$$F_W = \frac{1}{r\sin u}\frac{\partial R}{\partial I}\,. \quad (5.12)$$

zonal harmonics cause secular variation in three orbital elements, right ascension of ascending nod Ω the argument of perigee ω, and mean anomaly *M* [6 and 12].

The secular rate of change of nod Ω is given by

$$\dot{\Omega}_{\sec} = -\frac{3n\,J_2\,R_{\oplus}^2}{2\,p^2}\cos I\,, \quad (5.13)$$

where

$R_{\oplus}$ is the radius of the Earth,
n is the mean motion,
p is the semi-parameter,
I is the inclination.

An analytical solution to determine the change in the node over time is

$$\Omega = \Omega_0 + \dot{\Omega}\,\Delta t\,, \quad (5.14)$$

where Ω_0 is the initial value of the node.

The secular rate of change of argument of perigee ω is

$$\dot{\omega}_{\sec} = -\frac{3n\, J_2\, R_\oplus^2}{4\, p^2}(4 - 5\cos^2 I) \text{ ,(5.15)}$$

An analytical solution to determine the change in the argument of perigee over time is

$$\omega = \omega_0 + \dot{\omega}\Delta t \, , \tag{5.16}$$

where ω_0 is the initial value of the argument of perigee.

An analytical solution to determine the change in the mean anomaly M over time is

$$M = M_0 + n\, \Delta t, \tag{5.17}$$

where

M_0 is the initial value of mean anomaly.

The secular rate of change of M_0 is

$$\dot{M}_0 = -\frac{3n\, J_2\, R_\oplus^2 \sqrt{1-e^2}}{4\, p^2}(-2 + 3\sin^2 I)\, . \tag{5.18}$$

Now we will reproduce the *VOP* equations of motion under Luni-solar force. These expressions show the complexity of analytically modeling for third-body perturbations. In the first, we needed to the direction cosines for the third body. The direction cosines, *A*, *B* and *C*

$$A = \cos(I_3)\ \sin(u_3)\ \sin(\Omega\text{-}\Omega_3) + \cos(\Omega\text{-}\Omega_3)\ \cos(u_3), \tag{5.19}$$

$$B = \cos(I)\ [\cos(I_3)\ \sin(u_3)\ \cos(\Omega\text{-}\Omega_3) - \sin(\Omega\text{-}\Omega_3)\ \cos(u_3)] + \sin(I)\ \sin(I_3)\ \sin(u_3), \tag{5.20}$$

$$C = \sin(I)\ [-\cos(I_3)\ \sin(u_3)\ \cos(\Omega\text{-}\Omega_3) + \sin(\Omega\text{-}\Omega_3)\ \cos(u_3)] + \cos(I)\ \sin(I_3)\ \sin(u_3), \tag{5.21}$$

The secular and periodic (short and long periodic) rates of change (*deg./day*) of the elements [1] are

$$\dot{a} = 0\, , \tag{5.22}$$

$$\dot{r}_p = -a\, \dot{e}\, , \tag{5.23}$$

$$\dot{e} = -\frac{15\mu_3\, e\sqrt{1-e^2}}{4\, n\, r_3^3}\left[2\, AB\cos(2\omega) - (A^2 - B^2)\sin(2\omega)\right] \, , \tag{5.24}$$

$$\dot{I} = \frac{3\mu_3\, C}{4\, n\, r_3^3\sqrt{1-e^2}}\left\{A\left[2 + 3e^2 + 5e^2\cos(2\omega)\right] + 5e^2\, B\sin(2\omega)\right\}, \tag{5.25}$$

$$\dot{\Omega} = \frac{3\mu_3\, C}{4\, n\, r_3^3 \sin\nu\sqrt{1-e^2}}\left\{B\left[2 + 3e^2 - 5e^2\cos(2\omega)\right] + 5e^2\, A\sin(2\omega)\right\}, \tag{5.26}$$

$$\dot{\omega} = \frac{3\mu_3\sqrt{1-e^2}}{2\, n\, r_3^3}\left\{5AB\sin(2\omega) + \frac{5}{2}\left(A^2 - B^2\right)\cos(2\omega) - 1 + \frac{3\left(A^2 - B^2\right)}{2}\right\} - \dot{\Omega}\cos I + \frac{15\, \mu_3\, a[A\cos\omega + B\sin\omega]}{4\, e\, n\, r_3^4}\left\{1 - \frac{5}{2}\left(A^2 - B^2\right)\right\}, \tag{5.27}$$

For small eccentricities, the second-order terms become noticeable for the argument of perigee. The only secular rate of changes will be in the node, the perigee, and the mean anomaly at epoch. For a circular orbit, we obtain the secular rate of change of nod Ω is

$$\dot{\Omega}_{\sec} = -\frac{3\mu_3\left[2 - 3\sin^2 I_3\right]\left(2 + 3e^2\right)}{16\, n\, r_3^3\sqrt{1-e^2}}\cos I\, , \tag{5.28}$$

The secular rate of change of argument of perigee ω is

$$\dot{\omega}_{\sec} = -\frac{3\mu_3\left[2 - 3\sin^2 I_3\right]}{16\, n\, r_3^3\sqrt{1-e^2}}\left(4 + e^2 - 5\sin^2 I\right). \tag{5.29}$$

Smith's equations [2] that include terms in e_2 are

$$\dot{\Omega}_{\sec} = -\frac{3\mu_3\left[2 - 3\sin^2 I_3\right]\left(1 - e^2\right)^2\left(1 + 5e^2\right)}{8\, n\, r_3^3\sqrt{\left(1 - e_3^2\right)^2}}\cos I\, , \tag{5.30}$$

$$\dot{\omega}_{\sec} = \frac{3\mu_3\left[2 - 3\sin^2 I_3\right]\left(1 - e^2\right)^2}{16\, n\, r_3^3\sqrt{1-e^2}}\left(4 + 5e^2\left(3 - \frac{7}{2}\sin^2 I\right) - 5\sin^2 I\right)\cdot \tag{5.31}$$

6. The Ground tracks

To convert a position vector for a satellite to the corresponding latitude and longitude (is the core technique in determining ground tracks). We have two ways to do this transformation: one is iterative and another is analytical [5]. We find the right ascension directly from the Cartesian position vector. Let the equatorial projection of the satellite's position vector be

$$r_{\delta sat} = \sqrt{r_I^2 + r_J^2}\ , \tag{6.1}$$

We find the right ascension through sine and cosine expressions

$$\sin\delta = \frac{r_I}{r_{\delta sat}}\ , \tag{6.2.1}$$

$$\cos\delta = \frac{r_J}{r_{\delta sat}}\ . \tag{6.2.2}$$

The difficult part of finding the geodetic latitude is that it usually requires iteration. To determine a starting value for the iteration, we can use the position vector as a rough guess because the declination and geocentric latitude are equal [7]. Thus,

$$\sin\delta = \frac{r_{k\,sat}}{r}\ , \tag{6.3}$$

Now we find an expression for geodetic latitude ϕ_{gd}, we now have the satellite coordinates and not the site coordinates. Assume $\phi_{gd} = \delta$. The sine and cosine expressions [11] are given by

$$\sin\phi_{gd} = \frac{r_k}{S_\oplus + h_{ellp}}\ , \tag{6.4.1}$$

$$\cos\phi_{gd} = \frac{r_\delta}{C_\oplus + h_{ellp}}\ . \tag{6.4.2}$$

Solving the sine expression for h_{ellp} gives us

$$h_{ellp} = \frac{r_k}{\sin\phi_{gd}} - S_\oplus\ , \tag{6.5}$$

the tangent expression is

$$\tan\phi_{gd} = \frac{\sin\phi_{gd}}{\cos\phi_{gd}} = \frac{r_k\left(C_\oplus + h_{ellp}\right)}{r_\delta\left(S_\oplus + h_{ellp}\right)}\ . \tag{6.6}$$

Substitute h_{ellp} using equation (6.5)

$$\tan\phi_{gd} = \frac{r_k\left(C_\oplus + \dfrac{r_k}{\sin\phi_{gd}} - S_\oplus\right)}{r_\delta\left(S_\oplus + \dfrac{r_k}{\sin\phi_{gd}} - S_\oplus\right)}\ , \tag{6.6}$$

where $C_\oplus$ denoted by

$$C_\oplus = \frac{S_\oplus}{1 - e_\oplus^2}\ , \tag{6.7}$$

Substitute equation (6.7) into equation (6.6)

$$\tan\phi_{gd} = \frac{r_k\left(\dfrac{S_\oplus}{1 - e_\oplus^2} + \dfrac{r_k}{\sin\phi_{gd}} - S_\oplus\right)\sin\phi_{gd}}{r_k\, r_\delta}\ , \tag{6.8}$$

$$\tan\phi_{gd} = \frac{r_k\,(1 - e_\oplus^2) + S_\oplus \sin\phi_{gd} - S_\oplus \sin\phi_{gd}\,(1 - e_\oplus^2)}{r_\delta\,(1 - e_\oplus^2)}\ , \tag{6.9}$$

$$\tan\phi_{gd} = \frac{r_k\,(1 - e_\oplus^2) + S_\oplus\, e_\oplus^2 \sin\phi_{gd}}{r_\delta\,(1 - e_\oplus^2)}\ . \tag{6.10}$$

Using equation (6.7), then

$$\tan\phi_{gd} = \frac{(1 - e_\oplus^2)\left[r_k + C_\oplus\, e_\oplus^2 \sin\phi_{gd}\right]}{r_\delta\,(1 - e_\oplus^2)}\ , \tag{6.11}$$

$$\tan\phi_{gd} = \frac{r_k + C_\oplus\, e_\oplus^2 \sin\phi_{gd}}{r_\delta\,(1 - e_\oplus^2)}\ . \tag{6.12}$$

7. RESULTING AND CONCLUSION

In this section, a computer simulation has been developed to the equation of perturbed orbital motion due to spherical zonal harmonics J_2 and Luni-Solar forces using the Matlab program. The perturbed ground track under J_2 and Luni-Solar was calculated after 5 days for China sat 2D, Molniya 3-31, and Egypt sat A satellites. The two line elements [13] are

China sat 2D

1 43920U 19001A 19011.36550613 .00000967 13253-5 10000-3 0 9995

2 43920 27.1061 4.7362 7309322 179.7744 160.1328 2.28097794 25

Molniya 3-31

1 17328U 87008A 19002.75529764 .00000161 00000-0 00000+0 0 9998

2 17328 63.9674 259.4577 6796076 265.7427
20.5276 2.00665848230580

Egypt sat A

1 44047U 19008A 19055.20707225 -
.00003965 00000-0 -62322-3 0 9995
2 44047 98.0166 121.3798 0003071 71.7230
288.4358 14.72075970 373

Figure (7.1) shows one revolution of perturbed ground track for China sat 2D after 5 days.

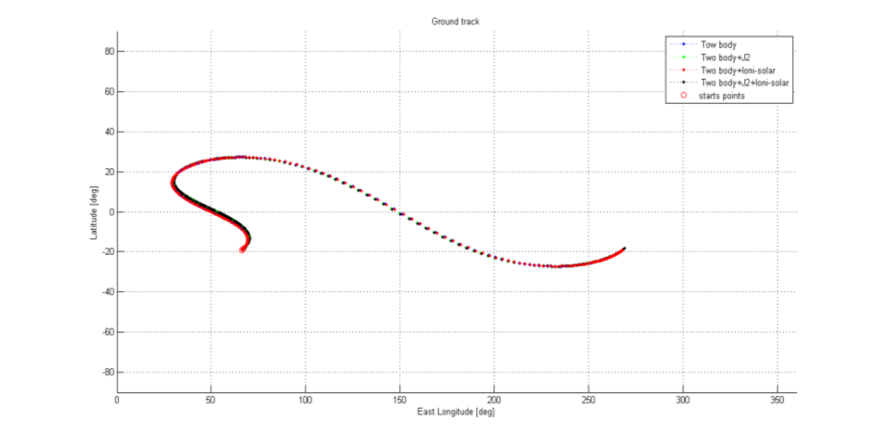


Figure (7.1): Perturbed ground track for China sat 2D.

Figure (7.2) shows one revolution of perturbed ground track for Molniya 3-31 after 5 days.

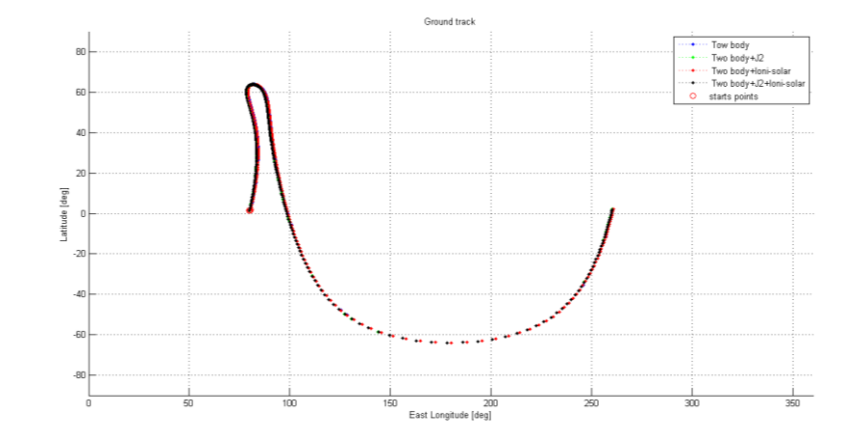


Figure (7.2): Perturbed ground track for Molniya 3-31

Figure (7.3) shows one revolution of perturbed ground track for Egypt sat A after 5 days.

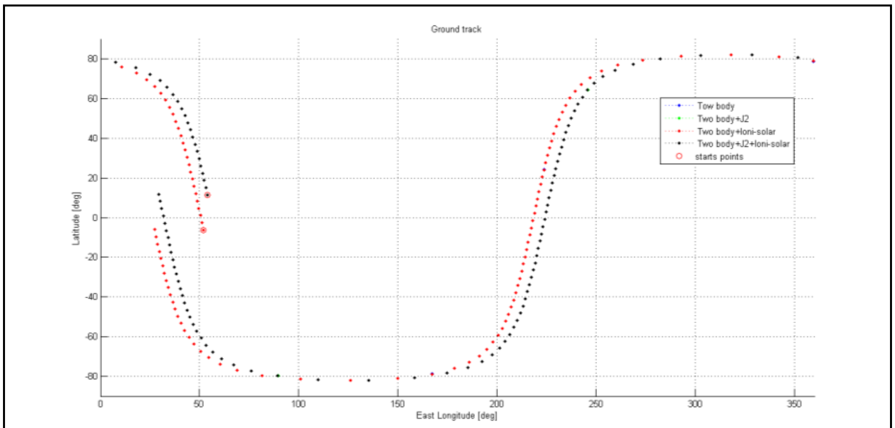


Figure (7.3): Perturbed ground track for Egypt sat A

The perturbed ground track is calculated under the effect of J_2 and luni-solar forces. The perturbed position vectors for a satellite is converted to the corresponding latitude and longitude. As expected the strongest perturbation due to the J_2 acting on the nearest satellite to the Earth as shown Figure (7.1) to Figure (7.3). The satellite's positions in one revolution are displayed to represent where the satellite at the time desired.

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[13] www.space-track.org.

الملخص العربي :

هناك عدة أنواع من القوى التى تؤثر علي القمر الصناعي ، حيث تصنف هذه القوى الي قوتان الأولى قوة محافظة والثانية قوى غير محافظة . الهدف الرئيسي في هذه الورقة البحثية هو دراسة تأثير القوة المحافظة علي الحركة المدارية للأقمار الصناعية وإيجاد هذا التأثير علي المسار الأرضى للأقمار الصناعية ، حيث تكون المسارات الأرضية هي موضع النقاط التي تشكلها النقاط على الأرض مباشرة أسفل القمر الصناعي اثناء انتقاله في المدار . ثم تصميم نموذج رياضي ورمز باستخدام برنامج حزمة Matlab وذلك لحساب المسار الأرضى المضطرب تحت تأثير J_2 وقوة جذب القمر – الشمس luni-solar .

في حين أن J_2 وقوة جذب القمر – الشمس هي قوة محافظة ، فقد تم إيجاد اختلاف تراكمي فقط في كل من العنصرين Ω و ω ، أما باقي عناصر المدار فإنها تختلف بشكل دوري .

يتم حساب المسار الأرضى المضطرب تحت تأثير J_2 وقى جذب القمر – الشمس . يتم تحويل متجهات الموقع المضطربة للقمر الصناعي الي المسار الأرضى (أي خطوط الطول والعرض المقابلة) . يتم تمثيل موقع القمر الصناعي في دورة واحدة عند أي وقت مطلوب .