ERJ
Engineering Research Journal
Faculty of Engineering
Minoufiya University

NUMERICAL PREDICTION OF AIR-PARTICLES FLOW IN VERTICAL PIPES

Samy M. El-Behery, A. A. El-Haroun and M. R. Abuhegazy
Mechanical Power Engineering Department,
Faculty of Engineering, Menoufyia University,
Shebin El-Kom, Egypt.

ABSTRACT

This paper presents a steady state one-dimensional two-fluid model for gas-solid two-phase flow in a vertical raiser. The model is solved using conservative variable approach for the gas phase and fourth order Runge-Kutta method is used for the solid phase. The model predictions for pressure drop are compared with available experimental data and with Eulerian-Lagrangian predictions and a good agreement is obtained. The results indicate that the pressure drop increases as the solid mass flow rate, particle size and particles density increase. In addition, the model predictions for minimum pressure drop velocity are compared with experimental data from literature and the mean percentage error, MPE for minimum pressure drop velocity is -9.89%. It is found that the minimum pressure drop velocity increases as the solid mass flow, particle size and particle density increase while it decreases as the system total pressure increases.

تنبؤ عددي لسريان هواء وجسيمات صلبة في الأنابيب الرأسية

تعرض هذه الورقة دراسة نموذج عددي أحادي البعد لسريان غاز وجسيمات صلبة في الأنابيب الرأسية. تم حل النوذج العددي باستخدام طريقة conservative variables بالنسبة للغاز و طريقة Runge-Kutta من الدرجة الرابعة بالنسبة للجسيمات الصلبة. تم مقارنة النتائج النظرية التى تم الحصول عليها باستخدام النموذج المقترح مع نتائج معملية لآخرين وكذلك مع نتائج نظرية باستخدام نماذج حل أخرى وأوضحت تلك المقارنات توافقا جيداً. تم دراسة تأثير كل من معدل سريان الجسيمات الصلبة وقطر الجسيمات وكثافتها على الفقد في الضغط وأوضحت النتائج زيادة الفقد في الضغط كلما زادت تلك العوامل. تعتبر سرعة السريان التي يحدث عندها أقل فقد في الضغط من العوامل المهمة في تصميم أنظمة نقل الجسيمات الصلبة. تم مقارنة تنبؤ النموذج المقترح للسرعة التي يحدث عندها أقل فقد في الضغط مع نتائج معملية لعدد من الباحثين وأوضحت النتائج أن النموذج المقترح يتوقع هذه السرعة بنسبة خطأ متوسطة -٩.٨٩% وكذلك وجد أن السرعة التي يحدث عندها أقل انخفاض في الضغط تزداد كلما زاد معدل سريان الجسيمات الصلبة أو قطرها أو كثافتها وتنخفض كلما زاد الضغط الكلي للتشغيل.

Keywords: Gas-solid; Pneumatic conveying; Two-fluid model; Pressure drop

1- INTRODUCTION

Gas-solid flows systems are important in many industrial applications such as chemical processes, pneumatic conveying, drying, grains and metal powders. To improve these systems, it is important to understand the gas-solid flow regimes. This understanding is considered a great challenge due to the complexity of this flow. Many parameters including particle size, density and shape are still mysterious although many researches had been done in this field. The pressure drop in the system is a vital design issue. Zenz (1949) was the first one to suggest flow regime diagram (also called the state diagram) for pneumatic conveying systems. This diagram presents the pressure drop per unit length of the conveying pipe as a function of the superficial fluid velocity at a constant solid mass flow rate. The state diagram reflects a point of minimum pressure drop. The results of Zenz (1949) had been confirmed by many other investigators (see for example Narimatsu *et al.* 2007; Konno and Saito 1969; Singh 1982; Rautiainen *et al.* 1999; El-Behery *et al.* 2012, 2013 and many others). All of these investigations showed that the pressure drop increases as the solid mass flow rate increases. In addition, Narimatsu *et al.* (2007) investigated also the heat transfer coefficient using glass spheres and alumina particles transported in a vertical conveyor. They found that the maximum heat transfer coefficient occurs at the same velocity of minimum pressure drop. Therefore, it can be concluded that operating the vertical pneumatic conveyor at velocities close to the minimum pressure drop velocity reduces the consumed power and enhances the heat transfer rate.

The effect of particle size on the pressure drop was investigated by El-Behery *et al*. (2012), Xiao-ping *et al.* (2009), Nieuwland *et al.* (1979), Hariu and Molstad (1949) and Plasynski *et al.* (1994). The common conclusion of those studies was the increase of pressure drop with the increase in particle size. On the other hand, Chung *et al.* (2001) found that the pressure drop decreases as the particle size increases. Narimatsu and Ferreira (2001) found that the particle

size is more significant at low conveying velocities.
Henthorn *et al.* (2005) investigated the effect of Reynolds number, mass loading, and particle shape and size on pressure drop in a vertical gas-solids pneumatic conveying line. They found that the pressure drop increases as the particle diameter increases at Reynolds number of 15,000, while at Reynolds number of 20,400 the pressure drop first decreases as particle size increases, and then increases with increasing particle size.
Generally, gas-solid flow can be modeled using either Eulerian-Eulerian or Eulerian-Lagrangian approaches. Detailed description of these approaches can be found in the comprehensive reviews by Enwald *et al.* (1996), Gouesbet and Berlemont (1998) and Balachandar and Eaton (2010). Realized and sophisticated Eulerian-Eulerian and Eulerian-Lagrangian models are used when detailed local flow parameters and variables are required at microscopic level (see El-Behery *et al.* 2012, 2013; Jamaleddine and Ray 2011; Patro *et al.* 2014). However, many industrial applications such as on-line control and automation require a robust model capable of predicting macroscopic variables (Narimatsu *et al.* 2007). The one-dimensional model based on the two-fluid theory is one such model. Despite the huge simplified assumptions employed in the one-dimensional two-fluid models, they capable of predicting the flow variable at macroscopic level with fair accuracy (see Narimatsu *et al.* 2007; Nieuwland *et al.* 1997; Arastoopour and Gidaspow 1979; Solsvik *et al.* 2012, 2014).
The present paper investigates the effect of different operating parameters on the gas-solid pressure drop in vertical pipes. The analysis is carried out for gas-solid flow in a vertical pipe using one-dimensional steady-state model based on two-fluid theory. The model is capable of modeling compressible gas-solid with heat transfer. However, the results presented in the current paper concentrate on the pressure drop in isothermal gas-solid flow.

2- MATHEMATICAL MODELING

To formulate the suggested model, a quasi-one dimensional situation has been considered. This model is concerned with two-phase flow of gas and particles through a vertical pipe under the following assumptions:

- The flow is one-dimensional and steady.
- The particles are spherical in shape.
- All particles interaction is ignored in the model.

This implies that any momentum transfer between particles is negligible compared with the momentum transfer between the particles and the gas stream.

- The model assumes that the solid will be conveyed as discrete particles.

2.1 Governing Equations

Based on the preceding assumptions the governing equations for the gas and dispersed phases are derived according to the basic laws of fluid mechanics as follows:
The mass balance equation for the gas phase can be written as:

$$\frac{d}{dx}(\alpha_g \rho_g u_g A) = S_{mass} \tag{1}$$

whereas the momentum equation for the gas phase can be expressed as:

$$\frac{d}{dx}(\alpha_g \rho_g u_g^2 A) = -A\frac{dp}{dx} + \alpha_g \rho_g g A - F_{wg} + S_{mom} \tag{2}$$

The total energy equation for the gas phase can be formulated as:

$$\frac{d}{dx}[\alpha_g \rho_g u_g A(H_g + 0.5u_g^2)] = Q_{wall} + S_{energy} \tag{3}$$

where, S_{mass}, S_{mom} and S_{energy} are mass, momentum and energy coupling source/sink terms, respectively. In the current study the mass transfer between faces as well as the heat transfer through the pipe wall are neglected (i.e. $Q_{wall} = S_{mass} = 0.0$)
The equation of motion for a particle in a gas is given by:

$$\frac{du_p^2}{dx} = \frac{2\beta}{\rho_p \alpha_p}(u_g - u_p) - 2g(1 - \rho_g/\rho_p) - 4 f_p u_p |u_p|/D \tag{4}$$

where, β represents the inter-phase momentum transfer coefficient due to drag. According to Gidaspow (1994) the factor β is calculated by:

$$\beta = \begin{cases} 150\dfrac{\alpha_p^2 \mu_g}{\alpha_g d_p^2} + 1.75\dfrac{\alpha_p \rho_g}{d_p^2}|u_g - u_p| & \alpha_g < 0.8 \\ \dfrac{3}{4}C_d \dfrac{\alpha_p \rho_g}{d_p}|u_g - u_p|\alpha_g^{-1.65} & \alpha_g \ge 0.8 \end{cases}$$

The drag coefficient, C_d is calculated by:

$$C_d = \begin{cases} \dfrac{24}{\mathrm{Re}_p}(1 + 0.15\mathrm{Re}_p^{0.687}) & \mathrm{Re}_p < 1000 \\ 0.44 & \mathrm{Re}_p \ge 1000 \end{cases}$$

where, $\mathrm{Re}_p = \rho_g d_p |u_g - u_p|/\mu_g$ is the particle Reynolds number.
The equation for particle temperature, assuming uniform temperature throughout the particle, can be written as:

$$u_p m_p C_{p_p} \frac{dT_p}{dx} = \pi d_p^2 h(T_g - T_p) \tag{5}$$

2.2 Coupling between phases

An important concept in the analysis of two-phase gas-solid flow is to consider the mutual effect between the two phases.

The number of particles per unit volume, N_p, can be expressed as:

$$N_p = 6\alpha_d / (\pi d_p^3) \tag{6}$$

The momentum coupling source term due to the reverse effect of particles can be written as:

$$S_{mom} = -N_p A m_p \beta (u_g - u_p) / (\rho_p \alpha_p) \tag{7}$$

The energy coupling source term for the total energy equation evolves convective heat transfer and the work due to particle drag (first and second terms on the right hand side, respectively) (Hamed 2005).

$$S_{energy} = -N_p A h \pi d_p^2 (T_g - T_p) + S_{mom} u_p \tag{8}$$

2.3 Friction force

The friction force per unit length between the pipe wall and the gas phase is estimated by,

$$F_{wg} = 0.5\pi D f \rho_g (\alpha_g u_g)^2 \tag{9}$$

The total friction factor, f is the sum of gas and particles friction factors which is expressed as given by Han *et al.* (2000) as:

$$f = f_g + \frac{\lambda}{1+\lambda} f_p \tag{10}$$

The gas friction factor, f_g can be calculated from the well-known Blasius formula while, the friction factor between particles and the wall of the pipe can be calculated as given by Han et al. (2000) as:

$$f_p = 1.0503 Fr_p^{-1.831} \tag{11}$$

where, $Fr_p = u_p / \sqrt{g d_p}$ is the particle Froude number.

2.4 Heat transfer

The convective heat transfer coefficient h is calculated from Nusselt number, $Nu = h d_p / k_g$ which is expressed as a function of Reynolds number Re_p.

Various empirical correlations can be used to calculate the heat transfer coefficient. El-Behery *et al.* (2009) compared different correlations for heat transfer coefficient and they found that Baeyens *et al.* (1995) correlation produces the most accurate results. This correlation can be written as:

$$Nu = 0.15 Re_p \tag{12}$$

2.5 Supplementary Equations

In order to solve the above set of equations the following supplementary equations are required.

The volume fraction equation:

$$\alpha_g + \alpha_p = 1 \tag{13}$$

Density of gas stream:

$$\rho_g = p / (R_g T_g) \tag{14}$$

3- SOLUTION PROCEDURE

The system of equations 1 to 5 is solved numerically using the conservative variable formulation for the gas phase with the help of auxiliary and supplementary equations (Crowe *et al.* 1998). The fourth order Runge-Kutta method is used for the dispersed phase. The conservative variable formulation is a cell by cell iterative procedure in which the gas phase variables are specified at the cell inlet and are sought at the cell exit. The average values of the gas phase variables are then used to calculate the solid phase velocity and temperature. The source terms and void fraction are then evaluated and new flow variables at the cell exit can be calculated. The procedure is continued until the gas velocity no longer changes with continued iteration. Once the solution is obtained for one cell, the exit conditions are taken as start condition for the adjacent cell and the procedure is repeated.

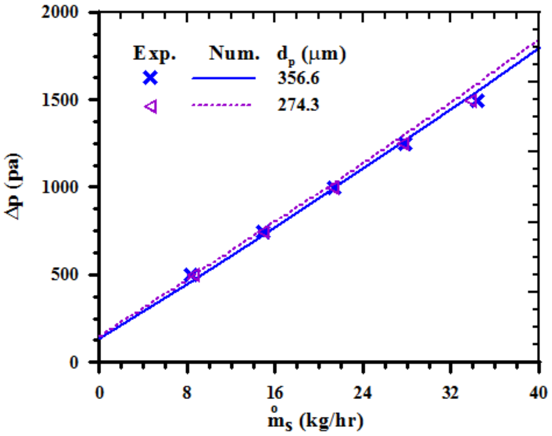
4. MODEL VALIDATION

The models predictions are verified using experimental data reported by Hariuand and Molstad (1949) and Henthorn *et al.* (2005). The experimental data of Hariu and Molstad (1949) includes the pressure drop in the acceleration region while the data of Henthorn *et al.* (2005) were for fully developed flow. The effect of solid mass flow rate on the total pressure drop at different particles diameters and gas velocities is presented in Fig. 1. The figure indicates that the pressure drop increases linearly with solid mass flow rate in both the predicted results and measurements. It can be seen also that the particles size has a small effect at these flow conditions. The figure shows also good agreement between predicted and measured pressure drop.

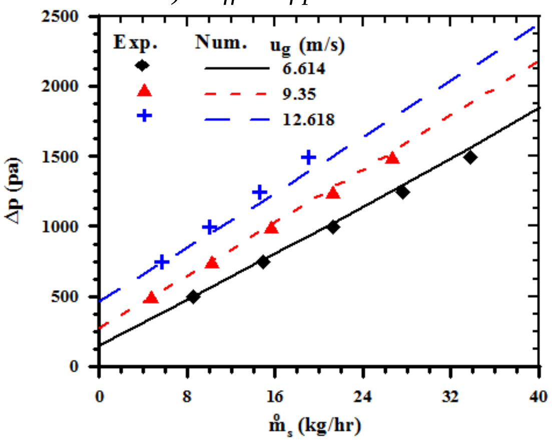
In the second test case, the effect of particle diameter on the ratio between two-phase and single phase pressure drops is shown in Fig. 2. The Eulerian-Lagrangian predictions of El-Behery *et al.* (2011) are also included for comparison. Despite the simplifying assumptions of the one-dimensional model, it predicts the ratio between two-phase and single phase pressure drops fairly good as compared with the realized Eulerian-Lagrangian model. In addition, the model predicts the linear variation of pressure drop with mass loading ratio very well. In general, the model accuracy is quite acceptable and can be used for pressure drop predictions in vertical pneumatic conveying.

5- RESULTS AND DISCUSSION

A parametric study on the effects of gas velocity, solid mass flow rate, particle size and density on the pressure gradient in the fully developed flow was carried out, as shown in Figs. 3-5. In general, the results indicate that the pressure gradient decreases with increasing gas velocity up to certain value then the pressure drop increases. The point of minimum pressure gradient appears because the contributions of weight force, gas-to-particles friction force and mixture friction force with pipe wall change with conveying velocity. The dense phase pneumatic conveying is considered to occur at velocities less than the minimum pressure drop velocity, U_{mp}. In this range of conveying velocities the solid concentration increases and the contribution of weight and gas-to-particles friction forces in the total pressure drop is dominated. The dilute pneumatic conveying is assumed to be at velocities higher than U_{mp}. In this range of conveying velocities the solid concentration is low and the contribution of mixture friction force with pipe wall is dominated. The standard Zenz diagram is presented in Fig. 3. It can be seen from this figure that the pressure gradient increases as the solid mass flow rate increases. This can be attributed to the increase of solid holdup and weight force as the solid mass flow rate increases. In addition, the minimum pressure drop velocity increases with solid mass flow rate which is in agreement with the finding of Zenz (1949) and Rautiainen *et al.* (1999).



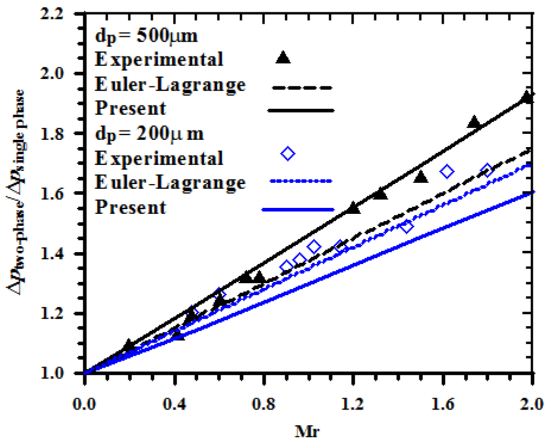
a) Effect of particle size



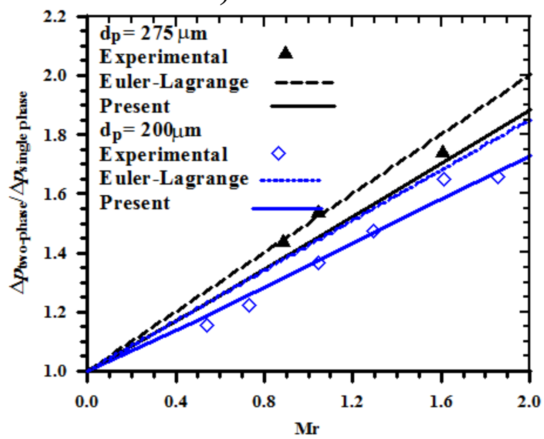
b) Effect of gas velocity

Fig. 1. *Comparisons between predicted pressure drop and experimental data of Hariu and Molstad (1949)*

The effect of particle size on the pressure gradient in vertical pneumatic conveying is shown in Fig. 4. The particle size has two contra effects on the particle's drag force. Increasing the particle size results in a higher particle mass which increases the slip velocity between the two phases. Therefore, the drag force increases to overcome weight force. On the other hand, for constant mass loading ratio, as the particle size decreases the number of particles increases and the total surface area increases as a result. Therefore, the total drag force increases as the particle size decreases. It can be seen from the results presented in Fig. 4 that the pressure gradient increases as the particles size increases. Since, the contribution of particle drag and weight force is more significant in dense phase, the pressure gradient increases significantly in the dense phase. Similar observation was reported by Mastellone and Arena (1999) and Narimatsu and Ferreira (2001). The figure shows also that the minimum pressure drop velocity increases as the particle size increases. This can be attributed to the increase in particles terminal velocity as the particle size increases (Mastellone and Arena 1999)



a) Re = 20400



b) Re = 15100

Fig. 2. *Comparisons between predicted pressure drop and measured data of Henthorn et al. (2005) and Eulerian-Lagrangian simulations of El-Behery et al. (2011).*

Figure 5 depicts the effect of particle density on the pressure drop. The particles density has similar effects to the particles size. Thus, the particle mass increases as the particles density increases. On the other hand, the number of particles and the total surface area increase as the particle density decreases. This figure indicates that the pressure gradient increases as the particle density increases and being more significant at low conveying velocities (dense phase). The minimum pressure drop velocity increases as the particle density increases due to the increase in terminal velocity. In general, it can be concluded that the particle properties (size and density) are more significant in dense phase pneumatic conveying due to the higher solid concentration.

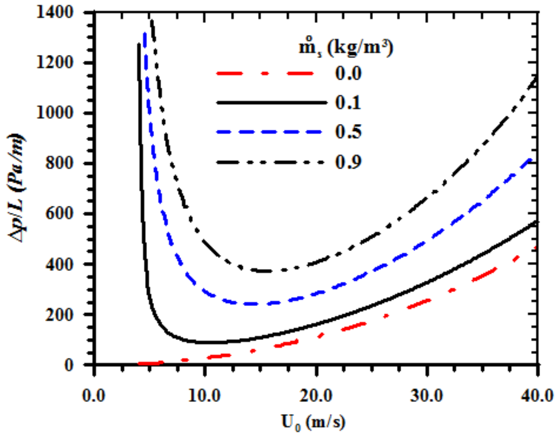


Fig. 3. *Effect of solid mass flow rate on the pressure gradient (d_p = 500 µm, ρ_p = 2500 kg/m^3).*

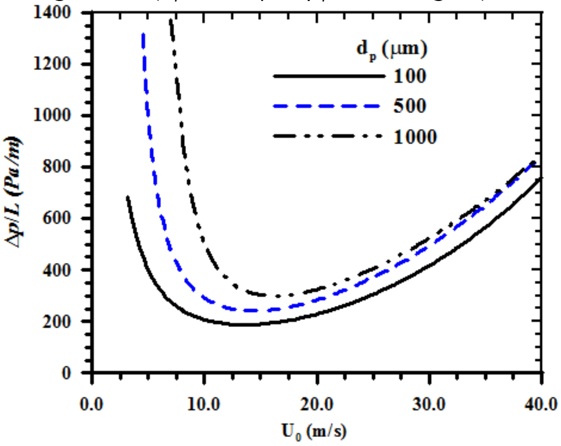


Fig. 4. *Effect of particle diameter on the pressure gradient ($\overset{o}{m}_s$ = 0.5 kg/s, ρ_p = 2500 kg/m^3)*

According to Plasynski *et al.* (1994), the velocity corresponding to the minimum pressure drop, U_{mp} for the system is generally the most sought-after information when designing a system. Many correlations have been reported in the literature for the minimum pressure drop velocity. However, none of these correlations can be applied to all systems. For instance, when Narimatsu and Ferreira (2001) applied the correlation developed by Rizk (1986) to their experimental results they found high discrepancies between measured and estimated values. They developed a new correlation for the minimum drop velocity using their experimental results for glass particles of different diameters. However, they reported that the new correlation did not provide good predictions for polypropylene particles. Another example can be found in Plasynski *et al.* (1994). When they applied the correlation developed by Knowlton and Bachovchin (1975) they found that it cannot fit to experimental data and a new correlation was developed.

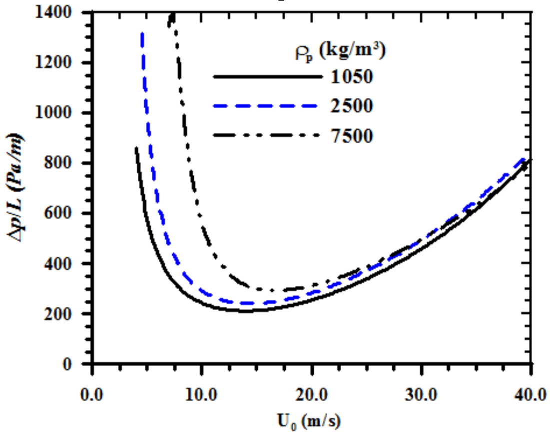


Fig. 5. *Effect of particle density on the pressure gradient ($\overset{o}{m}_s$ = 0.5 kg/s, d_p = 500 µm)*

Therefore, it is concluded that the validation of the present model for predicting the velocity corresponding to the minimum pressure drop is necessary. Several test cases by different investigators at different operating conditions are compared with the present predictions. Table 1 lists the condition for these test cases. Figure 6 presents a comparison between predicted and measured velocity at minimum pressu.re drop. It can be seen from this figure that most of the predictions lays in error range of ±30%. Despite this error is relatively high, it is quite acceptable in gas-solid flows where many uncontrolled parameters act. The mean percentage error, MPE was found to be -9.89%. The MPE is calculated by:

$$MPE = \frac{\sum_{1}^{N}\left[\left(U_{mp_{meas}} - U_{mp_{est}}\right)/U_{mp_{meas}}\right]}{N} \times 100 \qquad (15)$$

where, $U_{mp_{meas}}$ and $U_{mp_{est}}$ are the measured and estimated velocities at minimum pressure drop, respectively, and N is the number of data point.

Figure 7 shows comparison between predicted minimum pressure drop velocity and measured values of Mok *et al.* (1989). The figure indicates that the minimum pressure drop velocity increases as the solid mass flow rate increases, in agreement with many authors such as Rautiainen *et al.* (1999) and

Rizk (1986). In addition, the figure shows that the agreement between predicted and measured minimum pressure drop velocity is fairly good.

The effect of total (operating) pressure on the velocity at minimum pressure drop is presented in Fig. 8. It can be seen from this figure that the minimum pressure drop velocity decreases as the total pressure increases. This can be attributed to the increase of the gas density as the total pressure increases which in turn increases the pressure drop due to friction with pipe wall. The figure shows also that the agreement between the present predictions and measured value is acceptable.

An important parameter in gas-solid is the particle properties (size and density). The effects of these parameters are presented in Figs. 9 and 10. It can be seen from these figures that the minimum pressure drop velocity increases as particle size or particle density increase. This can be attributed to the increase of solid holdup and the increase of weight force as a result.

5- CONCLUSIONS

The gas-solid flow in vertical pneumatic conveyor was numerically simulated using a steady state one-dimensional two-fluid model. It was found that the pressure drop increases as the solid mass flow rate, particle diameter and particle density increase. The transition from dense phase to dilute phase occurs at the point of minimum pressure drop. The velocity at minimum pressure drop increases as the solid mass flow rate, particle diameter and particle density increase and decreases as the system total pressure increases.

Table 1 Geometric and operating conditions for minimum pressure drop velocity test cases

Author(s)	Particle material	Particle density ρ_p (kg/m^3)	Particle diameter dp (μm)	Total pressure P_t (kPa)	Solid mass flow rate, $\dot{m}$ (kg/s)	Pipe diameter, D (mm)
Zenz (1949)	Rape seeds	1089	1676.4	101	0.0295 – 0.219	44.4
	Glass	2483	586.74	101	0.0204-0.197	44.4
	Sand	2643	929.46	101	0.01-0.241	44.4
	Salt	2099	167.64	101	0.017-0.108	44.4
Plasynski *et al.* (1994)	Glass	2400	79	101 – 4238	0.0067-0.0183	25.4
	Glass	2400	545	101 – 4238	0.0067-0.0183	25.4
	Coal	1200	89	101-2170	0.0033-0.0833	25.4
	Coal	1200	505	101-4238	0.0033-0.00833	25.4
Narimatsu and Ferreira (2001)	Glass	2500	1000	101	0.041-0.132	53.4
	Glass	2500	2050	101	0.068-0.115	53.4
	Glass	2500	1850	101	0.068-0.119	53.4
	Glass	2500	3680	101	0.037-0.144	53.4
	Polypropylene	935	3680	101	0.006-0.037	53.4
Mok *et al.* (1989)	Sand	2620	210	101	0.011-0.089	20
Costa et al. (2004)	glass	2503	1000	101	0.523	81.4
	glass	2503	1700	101	0.507	81.4
	glass	2503	2850	101	0.861	81.4
	glass	2503	1000	101	0.049-0.78	104.8
	glass	2503	1700	101	0.334-0.869	104.8
	glass	2503	2850	101	1.004	104.8
	glass	2503	1000	101	0.735	147
	glass	2503	1700	101	0.869	147
	glass	2503	2850	101	0.938	147

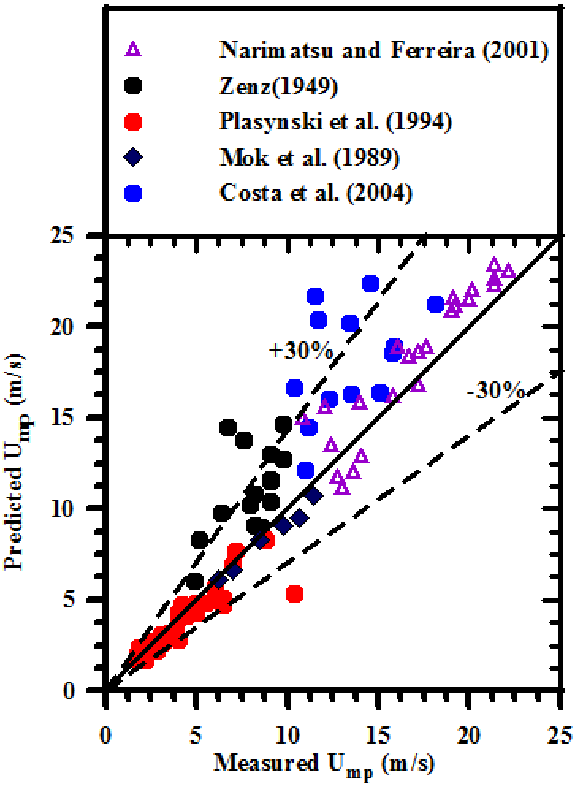


Fig. 6. *Comparison between predicted and measured minimum pressure drop velocity.*

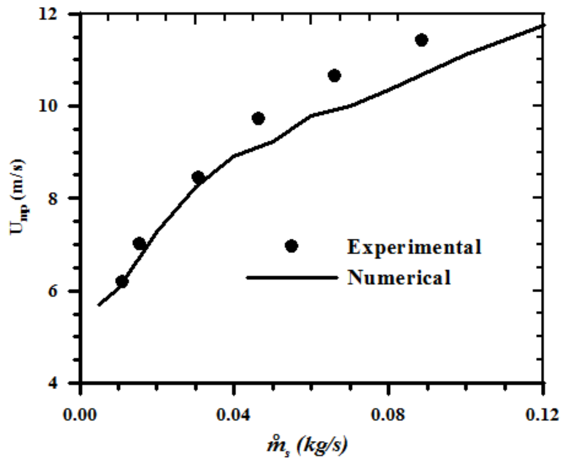


Fig. 7*. Effect of solid mass flow rate on the minimum pressure drop velocity: comparison with experimental data of Mok et al. (1989).*

NOMENCLATURE

A pipe cross-sectional area (m^2)
C_p specific heat (J/kg.K)
C_d drag coefficient
D pipe diameter (m)
dp particle diameter (μm)
f friction coefficient
g gravity acceleration (m/s^2)
H enthalpy (J/kg)
h heat transfer coefficient ($W/m^2.K$)
k thermal conductivity (W/m.s)
mp mass of single particle (kg)
Mr ratio of mass flow rate of particles to the mass flow rate of gas
P total gas pressure (N/m^2)
R gas constant (J/kg.K)
T temperature (K)
u velocity (m/s)
x distance along the pipe (m)
α void fraction
λ ratio of mass flow rate of particles to the total mass flow rate
ρ density (kg/m^3)
μ viscosity (kg/m.s)

Subscripts

p particle
g gas phase

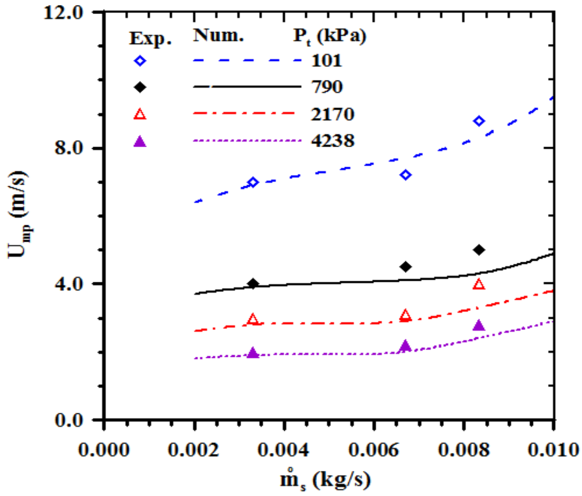


Fig. *8. Effect of total operating pressure on the minimum pressure drop velocity: comparison with experimental data of Plasynski et al. (1994).*

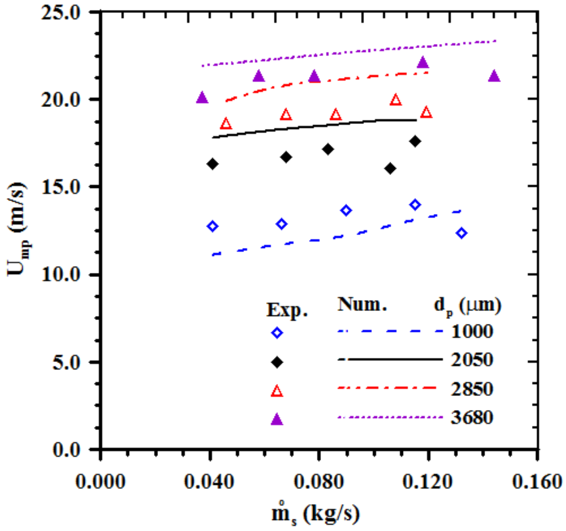


Fig. 9*. Effect of particle diameter on the minimum pressure drop velocity: comparison with experimental data of Narimatsu and Ferreira (2001)*

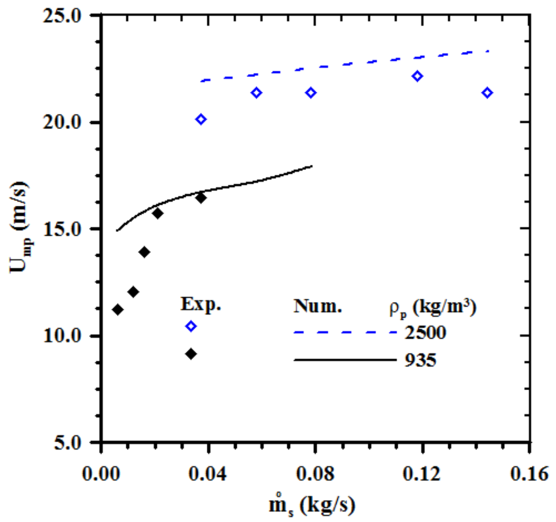


Fig. 10. *Effect of particle density on the minimum pressure drop velocity: comparison with experimental data of Narimatsu and Ferreira (2001)*

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