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EXPERIMENTAL INVESTIGATION OF AN INCOMPRESSIBLE FLOW IN A LABYRINTH SEALS

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ABSTRACT

The present work is carried out to reduce the leakage through the labyrinth seal by optimizing the tooth profile and operating clearances.

The effects of pressure ratio, clearance, tooth width and height, and number of throttles upon the leakage through a straight-through labyrinth seal for incompressible flow were experimentally investigated. Measurements in a developed test section over wide geometric factors and flow conditions were carried out.

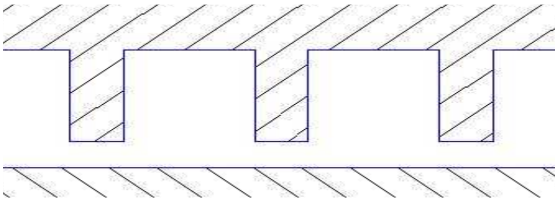
The experimental results indicated that the pressure ratio had significant effect on the flow coefficient. The flow coefficient decreased with increasing the pressure ratio. As the clearance increased the flow coefficient increased. However, the flow coefficient was increased as the clearance, became very small. The flow coefficient reached a maximum value at tooth width to clearance ratio of 2. The effect of tooth height on the flow coefficient was found negligible for tooth height to clearance ratio higher than 2. The flow coefficient decreased as the number of throttles increased, then attained constant value when the number of throttles became larger than seven. The observed trends of the results for incompressible fluid through straight-through labyrinth seals can be used to establish some general geometric constraints for minimizing the leakage.

هذا البحث تناول دراسة تجريبية لإيجاد الشكل والخلوص الأمثل لمانع اللابرنث العدل. تناولت الدراسة تأثير كل من نسبة الضغط عند المدخل الى الضغط عند المخرج ، الخلوص ، عدد الاسنان ، عرض السنة وارتفاعها وخطوتها على معامل التصرف . أجريت التجارب عند قيم مختلفة تتراوح بين 0.2 mm حتى 1.5 mm للخلوص ، 1.5 mm حتى 18 mm لعرض السنة ، 1.2 mm حتى 15 mm لارتفاع السنة وخطوتين مقدارهما 10 mm ، 20 mm ونسب ضغوط مختلفة تتراوح بين 0.1 حتى 0.9. نتائج التجارب أوضحت ان معامل التصرف يقل مع انخفاض قيمة الخلوص الى ان يصل الى أدنى قيمة له ثم يزداد مرة اخرى مع انخفاض الخلوص الى قيمة تقرب من الصفر. بينت نتنائج التجارب ايضا ان معامل التصرف يزداد بمعدل كبير للقيم الصغيرة لعرض السنة ويصل الى اقصى قيمة له ثم يقل تدريجيا مع زيادة عرض السنة حتى يصل الى قيمة ثابتة تقريبا للأسنان ذات العرض الكبير. أثبتت النتائج ان ارتفاع السنة ليس لة تأثير ملموس على معامل التصرف وذلك لقيم نسب ارتفاع السنة الى مقدار الخلوص للمانع أكبر من 2.0 . أظهرت النتائج ان زيادة عدد أسنان اللابرنث يقلل من معامل التصرف حتى عدد اسنان مقدارة سبعة وان زيادتها عن هدا العدد لا يؤثر على معامل التصرف. تم مقارنة النتائج مع باحثين اخرين ووجد توافق بينهما. من نتائج البحث يجب عند تصميم موانع اللابرنث العدلة تقليل الخلوص كل ما امكن مع الأخذ فى الاعتبار التمدد الحرارى ، عرض السنة يجب ان يكون صغيرا ويساوى تقريبا نصف خطوة السنة ، ارتفاع السنة لا يقل عن ضعف قيمة الخلوص.

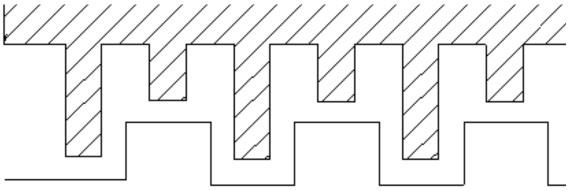
Keywords: Labyrinth seal, incompressible flow.

1. INTRODUCTION

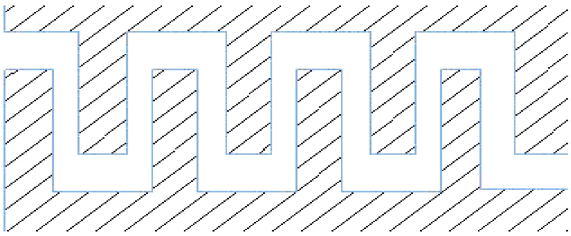
In the last decade, there is a growing demand for compressed air in the industry for various applications, increasing demand for thermal power (large capacity steam turbine) and high pressure pumps. Clearance plays a major role in the performance and reliability aspects of turbines, compressors and pumps. The performance of the turbo-machinery is directly related to the leakage rate through the seals. Labyrinth seal is a special type of seal, provided in order to prevent the leakage and widely used in turbo-machinery. Labyrinth seal is a non-contacting type seal that uses a tortuous path to minimize the fluid leakage. The pressure drop occurs at each labyrinth tooth as the medium is squeezed between the labyrinth tooth and the rotor. The leakage through the seal is directly related to the labyrinth profile and also the clearance between the rotor and the labyrinth tooth. Thus, understanding the influence of leakage through the labyrinth seal on rotor dynamics is of great practical significance to design and operation of machines. Toward this end, calculation of rotodynamic coefficients, which quantitatively characterize the influence of leakage through rotor dynamics, is pre-requisite. In Fig. 1 are diagrams of the most common types of labyrinth seals used today; straight-through, stepped, interlocking and radial. Fig. 2 indicates a typical application of labyrinth seals for preventing leakage in a centrifugal pump. In this particular pump, a straight-through labyrinth seal is used for sealing the inlet side of the impeller shaft and a redial labyrinth seal is utilized on the backside of the impeller. Typical flow in a straight-through labyrinth seal with three throttling is shown in Fig. 3.



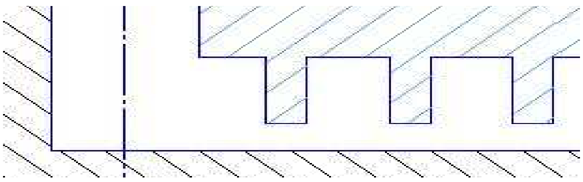
(a) Geometry of the straight- through labyrinth seal



(b) Geometry of the stepped- through labyrinth seal



(c) Geometry of the interlocking- through labyrinth seal.



(d) Geometry of the radial- through labyrinth seal

Fig.1 Common types of labyrinth seals

The basic mechanism for preventing leakage is as follows: A portion of the high pressure head of a fluid entering a seal is converted into kinetic energy by flowing through a small constriction. A large portion of this kinetic energy is then dissipated by small scale turbulence-viscosity interaction in a chamber which follows the flow constriction. Subsequently, a portion of the pressure in this chamber is converted to kinetic energy when the flow progresses to the next chamber downstream. In this chamber a large portion of the kinetic energy is again dissipated. This process continues in the seal until the fluid finally exhausts through the last constriction. Many attempts have been made in describing the leakage losses through labyrinth seals. These attempts can be classified according to whether the fluid is compressible or incompressible. The studies for compressible flow have been widely reported. References [1-26] are representative of the majority of the date for compressible flow through labyrinth seals found in the literature. However, there is not a great deal of data available in the literature for leakage of incompressible fluids through labyrinth seals [27-37]. Selvaraji et al. [12] used CFD as a tool to optimize the design cofiguration. The optimization is carried out on different design configurations of labyrinth seal by comparing the deviation in leakage rates. Effect of rotor speed, width of seal and pressure ratio on air leakage rate was also investigated. A set of labyrinth seals has been designed based on the above optimization and tested in the compressor. The results have been compared with the CFD prediction. The advantages with CFD are that the effects of turbulence and friction, which required an empirically determined correction factor in the theoretical analysis, are accounted automatically by the solver. Thus, for any specified labyrinth seal configuration with number of throttling, geometry and clearance, CFD can give a better solution. Also, it was observed that the effect of rotation on air leakage can be considered as negligible. Investigations for possible profiles and their leakage effects are compared. The optimized clearance is obtained from the thermo-structural analysis of housing and seal based on the boundary temperatures from heat transfer analysis.

Numerous studies have been made of rotodynamic coefficients associated with leakage air flow through labyrinth seals. Recentally, various seal geometries, e.g., stepped labyrinth seal and honeycomb seal, were numerically investigated by Kleynhans [13], Yucel and Kazakia [14] and Dursun [15]. Cross-coupled stiffness coefficients of stepped labyrinth seal predicted by Yucel [16] showed relatively good agreement with the experimental data, Kwanka [17]. Sealing parts have been specially designed and placed in turbomachinery [18] to suppress the leakage flow from the high pressure to low pressure regions, which otherwise deteriorates the engine efficiency. In practice, the available labyrinth seals [19, 20 and 21] are resulted from the compromise between the complexity of teeth arrangement and technologies of manufacturing and fabrication. No doubt that in-depth understanding of the influence of the teeth arrangement on the leakage flow is of great significance. The tip clearance between the rotor and the casing [22 and 23], the sealing clearance between the hooked stator and the shaft [24], labyrinth seal between the casing and the shaft has received widespread attention. Wnag Wei-zhe and LIU Ying-zheng [25], investigated numerically the leakage flow through the interlocking seal and the stepped seal. The major concern of the study was directed toward influence of the teeth arrangement on the leakage fluid flow. Based on the one-dimensional control volume method, the numerical results showed favorable agreement with the experimental measurements and by employing CFD and K- ε turbulence model, the results showed that the stepped seal better sealing performance than the interlocking seal. Anikeev et al [32] investigated the unsteady flow structure, the distribution and fluctuations of pressure on the body of the labyrinth. The results are compared with the data of a physical experiment. The flow in channels of the labyrinth seal type with one or two throttling stages was calculated on the interval of Reynolds numbers from $1.9.10^4$ to $4.5.10^5$. A mathematical model of calculating rotodynamic coefficients which quantitatively represent influence of the leakage steam flow through labyrinth seals on the rotor dynamics was proposed by Wang et al [26]. Rotodynamic coefficients associated with the leakage steam flow through a straight- through labyrinth seal were calculated at the same condition and compared with that of the air flow. The incompressible flow in a labyrinth seal is computed using the '$\kappa-\varepsilon$' turbulence model with a pressure-velocity computer code in order to explain leakage phenomena against the mean pressure gradient, Stoff [33]. The flow is axisymmetric between a rotating shaft and an enclosing cylinder at rest. The main stream in circumferential direction induces a secondary mean flow vortex pattern inside annular cavities on the surface of the shaft. The domain of interest is one such cavity of an enlarged model of a labyrinth seal, where the finite difference result of a computer program is compared with measurements obtained by a back-scattering laser-Doppler anemometer at a cavity Reynolds number of 3×10^4 and a Taylor number of $1 \cdot 2 \times 10^4$. The turbulent kinetic energy and the turbulence dissipation rate are verified experimentally for a comparison with the result of the turbulence model.

Therefore, the present work undertaken to study incompressible flow through a straight-through labyrinth seal. It is an attempt to clearify the effects of pressure ratio, number of throttles, tooth width and height and clearance upon the non-dimensional flow rate (flow coefficient). Rotational effects have not been considered, since previous investigations [29, 31 and 38] have indicated that the effects of rotation speed on the flow coefficient can be neglected.

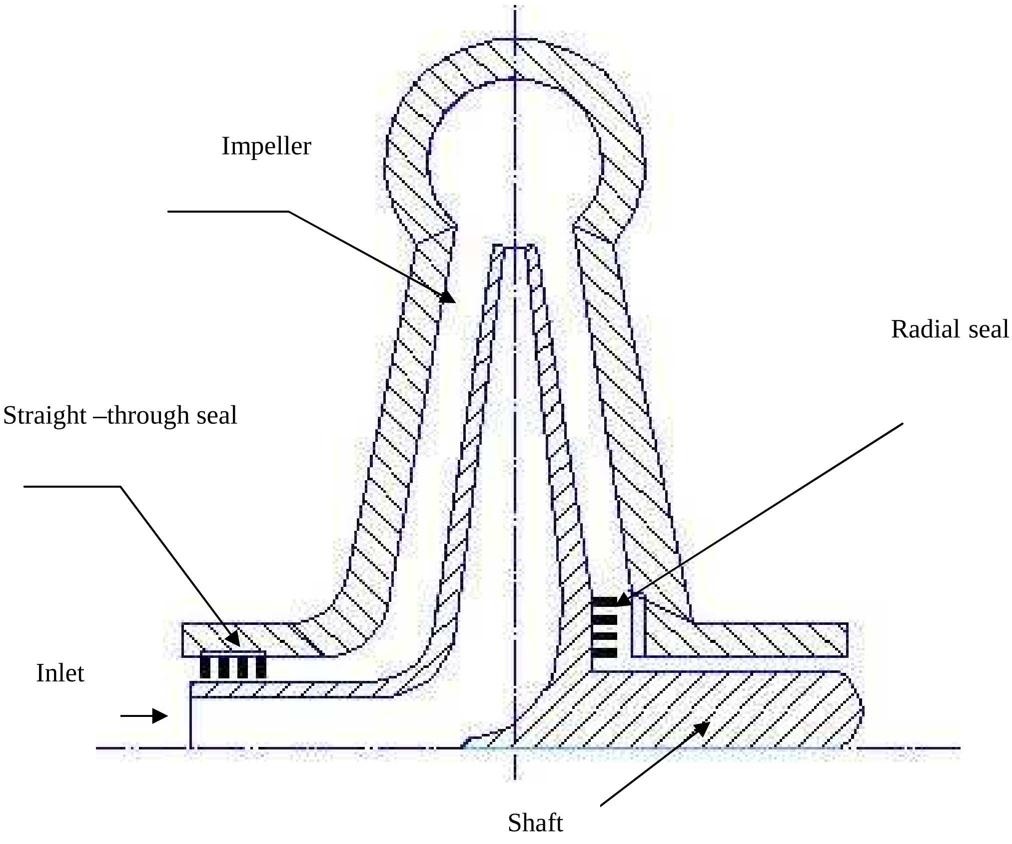


Fig. 2 Typical application of a labyrinth seal

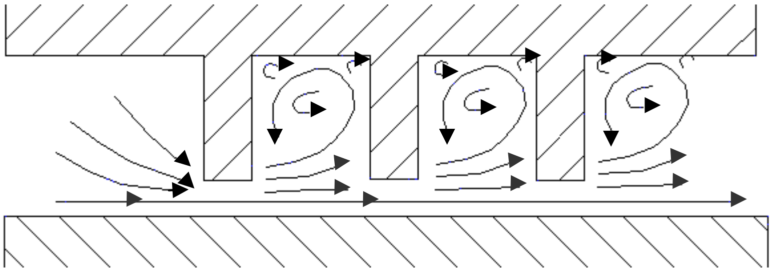


Fig. 3 Typical flow in a straight – through labyrinth seal

2. EXPERIMENTAL SET UP AND MEASUREMENT TECHNIQUES

Convential straight-through seal configurations were used. Because relative to the other types of labyrinth seals, straight-through seals are easier to design and less expansive to manufacture. In addition, in straight-through seals, the rotor shaft can be installed and removed from the seal by sliding it in or out; staggered seals require a split casing to allow for assembly or disassembly. These advantages are offset by the fact that straight-through seals, in general, do not perform as well as more advanced labyrinth seal design in preventing leakage losses.

Figure 4 shows the geometrical configuration of the labyrinth. The number of throttles was varied from 2 to 12. The clearance, pitch and height were varied. Each tooth/cavity combination was machined onto a separate piece. In this manner, the pieces could be stacked such that seals with different number of throttles and various ratios of geometric parameters could be assembled.

The range of variation of geometric parameters was; 0.2 mm to 1.5 mm for clearance, 1.5 to 18 mm for width; and 1.2 to 15 mm for height. Two values of pitch of 10 mm and 20 mm were used.

A test section was designed for labyrinth arrangement. In designing the test section, care has been taken to achieve operation conditions similar to those under normal conditions and to avoid any geometrical deviations. Large side window of Perspex for the test section was made for visual observations.

The test section was mounted with the water tunnel at Menoufiya University. The overall layout of the experimental facility is shown in Fig. 5.

Water from the laboratory was used as working fluid. The water was pumped from the downstream tank of the water tunnel to upstream tank then to the test section by a 44 kW, 120 m^3/hr, and 76.5 m head centrifugal pump. There is a bypass line for returning excess water to the downstream tank. The pressure in the test section could be varied by a pressure vessel connected with the upstream tank and supplied with a compressed air from a compressor. The water pressure at the inlet of the test section was varied between 0-8 bar. A valve at the exit of the test section was used to select the back pressure such that ratios of the outlet to inlet pressure could be varied from 0.1 to 0.9. During the operation the water temperature was kept at approximately 32^{O}C. The water flowing through the test section would increase a maximum of 1°C during several hours of operation. Since water cooling coils were installed within the downstream tank. The water pressures at the outlet and inlet of the test section were measured using pressure transducers of model Philips; the linearity error + 0.3% of span. The flow rate of water through the seal was measured by two meters. The first was an electromagnetic flow-meter mounted at the outlet of the test section. The second was the indication of pressure drop through a calibrated nozzle at the inlet of the test section on a mercury manometer. The accuracy of the electromagnetic flow meter was ± 0.2% from the measured quantity of the flow rate, while the accuracy of flow rate as measured by the pressure drop through the nozzle was ± 0.5% from the measured flow rate. For the present experiments, the leakage data from the straight-through labyrinth seals tested have been non-dimensionlized according to the development of equation suggested by Egli's [2] and Benvenuti [10] where the flow coefficient (α) has been defined as:

$$\alpha = \frac{m^{\bullet}}{\varepsilon\ A}\sqrt{V_0 / P_0\left[z + \ln\left(\frac{P_0}{P_f}\right)\right]} \Big/ \sqrt{\left[1 - \left(\frac{P_f}{P_0}\right)^2\right]} \qquad (1)$$

The derivation of this equation is presented in Appendix A.

The carry-over coefficient (ε), found in equation (1), is an empirical coefficient which is included to account for the effects of kinetic energy carry-over in straight-through seals. Carry-over is defined as the phenomenon whereby a portion of the kinetic energy of the flow is not completely dissipated as it enters a cavity constriction to the succeeding cavity. Hodkinson [3] has proposed the following equation for the carry-over coefficient:

$$\varepsilon = 1 / \sqrt{1 - \left(\left[(z-1).\left(\frac{C}{m}\right)\right] / \left[z\left(\frac{C}{m}\right) + 0.02\right]\right)} \qquad (2)$$

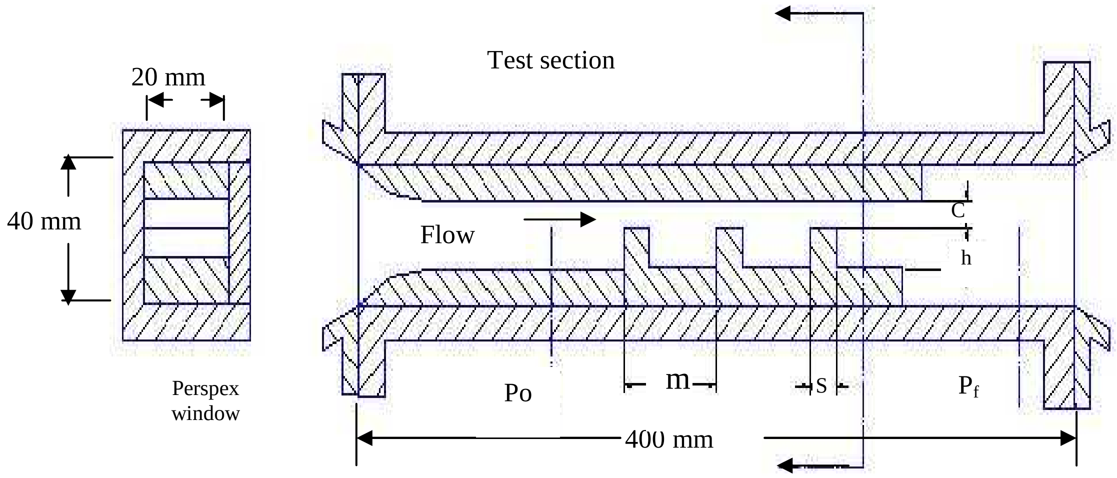


Fig. 4 Configuration of straight –through seal and its location in the test section

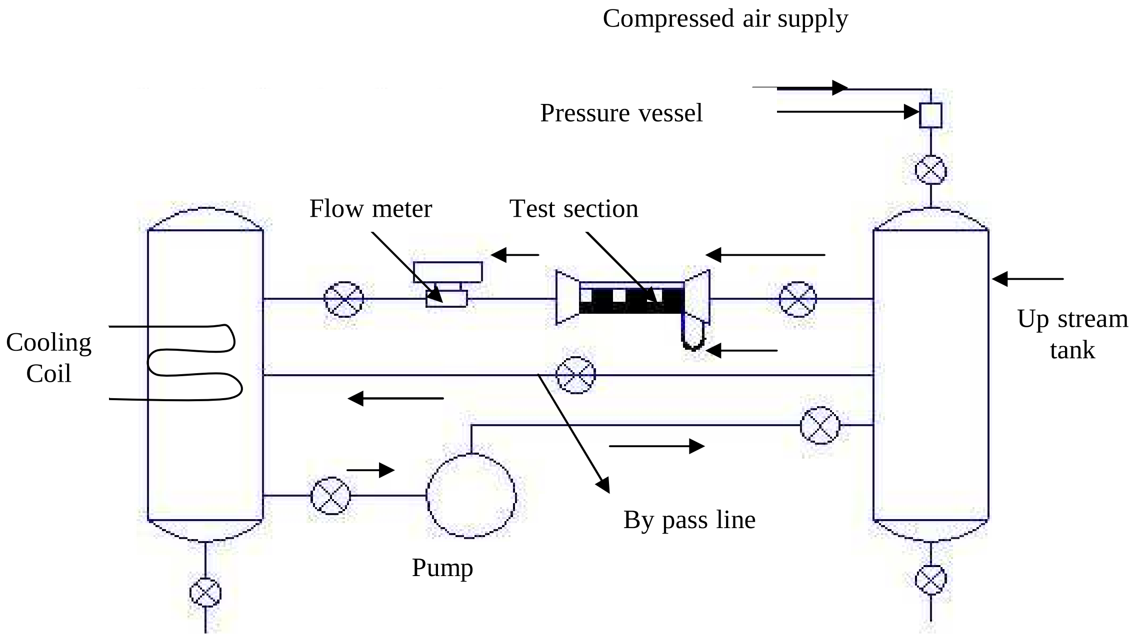


Fig. 5 Overall layout of the test rig

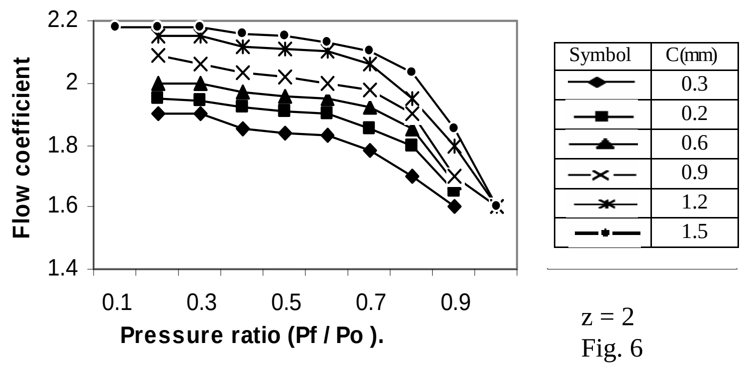
3. EXPERIMENTAL RESULTS AND ANALYSIS

From the measurements made during the experimental program, plots of the flow coefficient dependence upon pressure ratio, clearance to pitch ratio, tooth width to clearance ratio, tooth width to pitch ratio, tooth height to clearance ratio and number of throttles for incompressible flow through straight-through labyrinth seals were obtained. A wide range of these geometric parameters and flow conditions were tested to observe their effects on the flow coefficient.

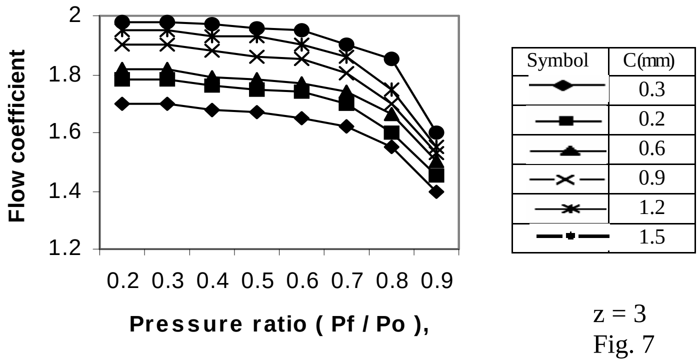
3.1. Pressure Ratio Effects

Figs, 6-11 show the effects of pressure ratio upon the flow coefficient at various geometric factors. From these Figures it is noted that the flow coefficient decreased with increasing pressure ratio. The pressure ratio has a significant effect on the flow coefficient. It is noted that as the pressure ratio approaches one a sharp decrease in the flow coefficient is observed. When the pressure drop across the seal which acts as the driving force for the leakage approaches zero, the mass flow rate of the seal will also approach zero. This decrease in the leakage is reflected by the sharp decrease in the flow coefficient.

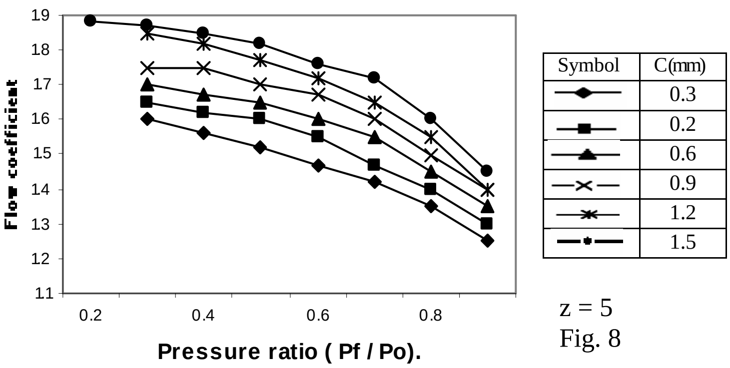
Figures 6 to 11 show the variation of flow coeff-icient with pressure ratio for a given number of thr-ottles and various clearances. These figures are re-presentative of the curves found in all the present tests.



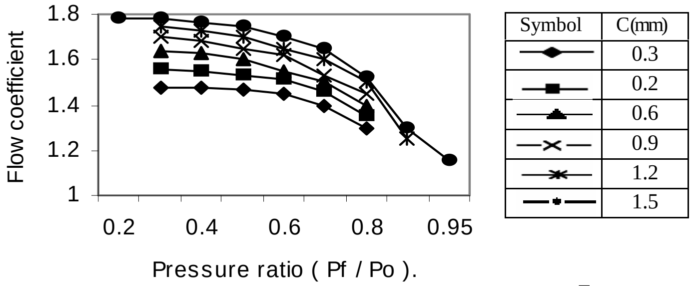
z = 2
Fig. 6



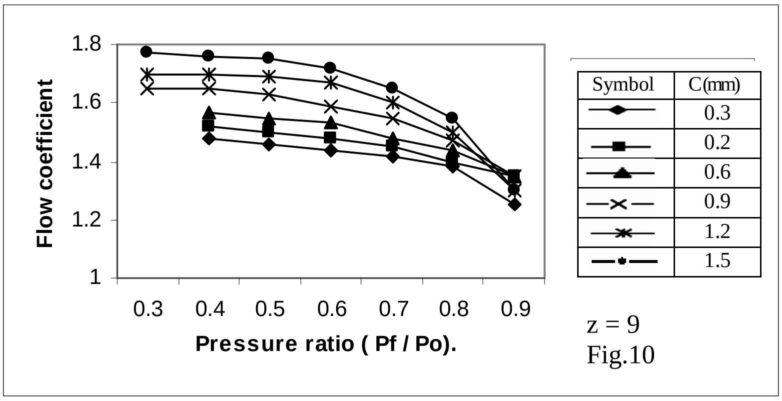
z = 3
Fig. 7



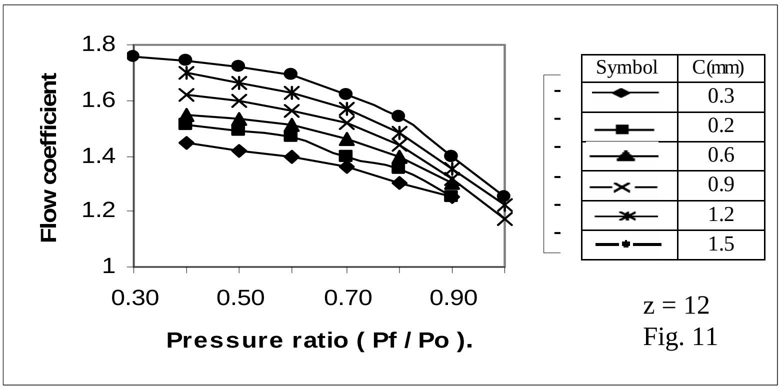
z = 5
Fig. 8



z = 7
Fig. 9



z = 9
Fig.10



z = 12
Fig. 11

Figs. 6-11 Variation of flow coefficient with pressure ratio for a given number of throttles and various clearances, (m=10 mm, s = 6 mm, h = 5 mm)

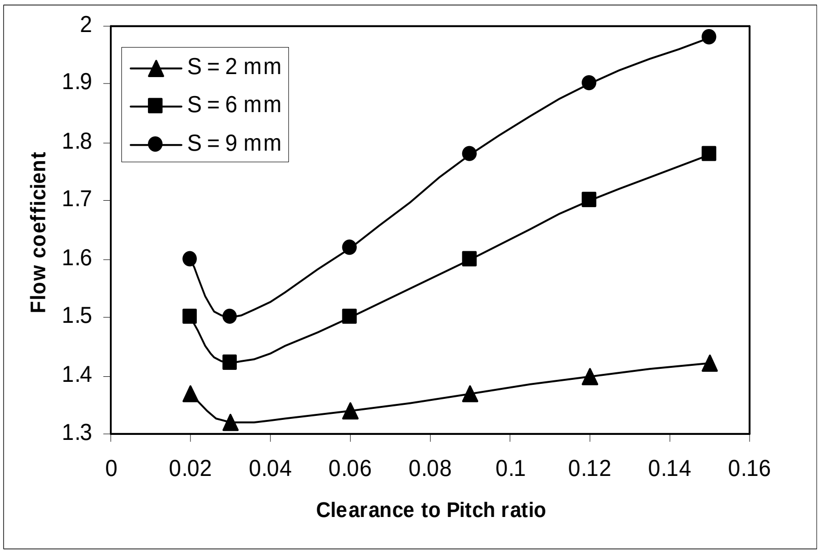


Fig. 12 Flow coefficient verses clearance to pitch ratio for three different widths.

(Pf / Po = 0.5, z = 9, h = 9 mm and m = 10 mm)

Figures 6-11 indicate the effects of clearance on the flow coefficient as the pressure ratio varies. These Figures show that increasing the clearance results in an increase in the flow coefficient. Noting the relationship from equation (1) among the mass flow rate, the flow coefficient, the carry-over coefficient and the area, an increase in clearance will cause the mass flow rate to increase proportional to the increase in the leakage area times the increase in the flow coefficient times the increase in the carry-over coefficient.

As the clearance increase, the flow coefficient increases, precipitating a rapid increase in the mass flow rate. The mass flow rate is also increased due to the increase in carry-over coefficient as the clearance increases. In addition, Figs. 6-11 indicate that the flow coefficient is increased as the clearance becomes very small or approaches zero. This implies that decreasing the clearance will not decrease the flow coefficient under all conditions, and at very small clearances decreasing the clearance can cause the flow coefficient to increase as shown in Fig. 12.

Figure 12 is a plot of the flow coefficient as a function of variable clearance for different tooth widths. This Figure shows that seals with wide teeth, the flow are influenced by the clearance along larger portion of the seal, than in the case of a seal with narrow teeth. In addition, Fig.12 indicates that at relatively small clearances the flow coefficient increases with decreasing the clearance.

The reason for increasing the flow coefficient with decreasing the clearance can be only explained from the fact that the friction coefficient decreases with decreasing the clearance for relatively small clearances. This decrease in the friction coefficient will partially offset the decrease in the leakage area and could limit further decrease in the leakage rate. Thus from the present findings there is sufficient evidence to suggest that the flow regime is within the transition region. Relatively small clearances yield comparatively small Reynolds numbers even when high pressure ratios are considered. The Reynolds numbers for small clearances in the present work were ranged from 300 to 450 compared with 400 from the Nikitin and Ipatov [30] results of the transition for laminar to turbulent in straight-through labyrinth seals with incompressible fluids. The decrease in the friction coefficient with decreasing the clearance in the transition region of the flow is supported by Witting et al [11] results which showed that the friction coefficient sharply decreased as the Reynolds number approached the transition zone (i.e. into laminar region), it sharply increased in the transition zone then it gradually decreased into the turbulent[1] region.

Based on these results, decreasing the leakage can be accomplished by decreasing the clearance, as this will cause *a* decrease in the leakage area, the carry-over coefficient, and, generally the flow coefficient. As the clearance approaches zero, however the flow coefficient will tend to increase, which will partially offset the decrease in the leakage area and carry-over coefficient and could limit further decreases in the leakage rate.

3.3 Tooth Width Effects

The effect upon the flow coefficient of altering the tooth width is shown in Fig. 13. This Figure indicates that as tooth width increases from a value close to zero, the flow coefficient rises sharply, reaching a maximum value at tooth width to clearance ratio of 2. As the tooth width continues to increase beyond this point, the flow coefficient decreases, gradually approaching a constant value for wide teeth. The reason for increasing the flow coefficient with increasing tooth width/clearance is not obvious. Nevertheless, it can be attributed to the following. For small widths, the flow condition is broadly similar to the free turbulence flow in which the effect of viscosity is negligible since the solid boundaries are almost absent. Moreover, variation in the chamber width will lead to a variation in the turbulence structure within the chamber. The combination of these two factors may cause the observed behavior.

At values of tooth width/clearance ratio higher than two there is a tendency for the flow coefficient to increase as tooth width/clearance increases. This is because increasing the tooth width increases the friction coefficient which amounts to reduced flow rate.

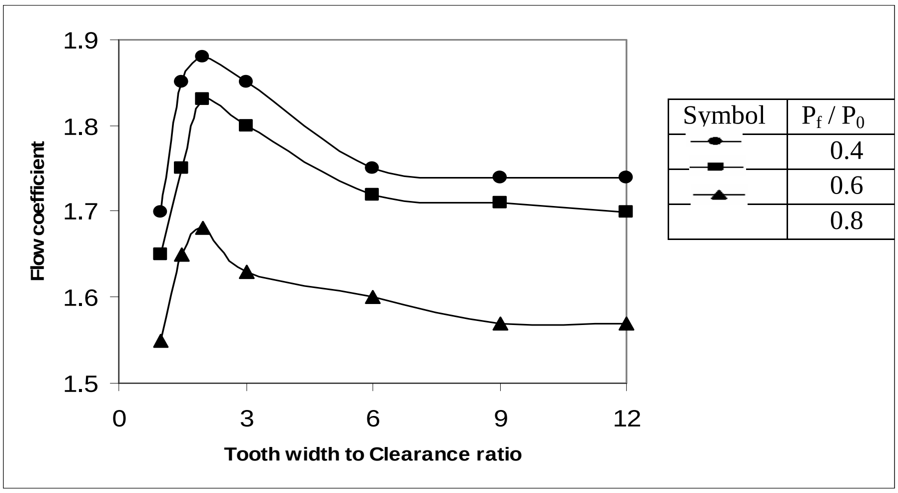


Fig. 13 Flow coefficient verses tooth width to clearance ratio at various pressure ratios. (c = 1.5 mm, z = 5, h = 5 mm and m = 20 mm)

3.4 Tooth Height Effects

Figure 14 shows the dependency of the flow coefficient upon the tooth height to clearance ratio. This figure shows that the effects of tooth height on the flow coefficient are negligible for tooth height to clearance ratios greater than 2. For tooth height to clearance ratios greater than 2, the flow coefficient/ is relatively constant. In addition, Fig. 14 demonstrates a slight increase as the tooth height to clearance ratio approaches a value of 1.

Consequently, the only constraint on tooth height in order to minimize leakage is that the height be at least equal to twice the clearance.

3.5 Number of Throttles Effects

Figure 15 shows the dependency of the flow coefficient upon the number of throttles with various pressure ratios. This Figure shows that for a given pressure ratio, the flow coefficient decreased as the number of throttles increased. From equation (1) the mass leakage rate is proportional to the flow coeff-icient times the carry-over coefficient divided by $\sqrt{Z}$. The value of the carry-over coefficient will always be less than or equal to $\sqrt{Z}$, as shown in equation (2); consequently, decreasing the flow coefficient by increasing the number of throttles will always result in a decrease in the leakage rate. In addition increasing the number of throttles increases the friction coefficient which results in a decrease in the flow rate. Figure 15 also shows that as the number of throttles becomes larger than seven, the flow coeffi-cient approaches a constant value.

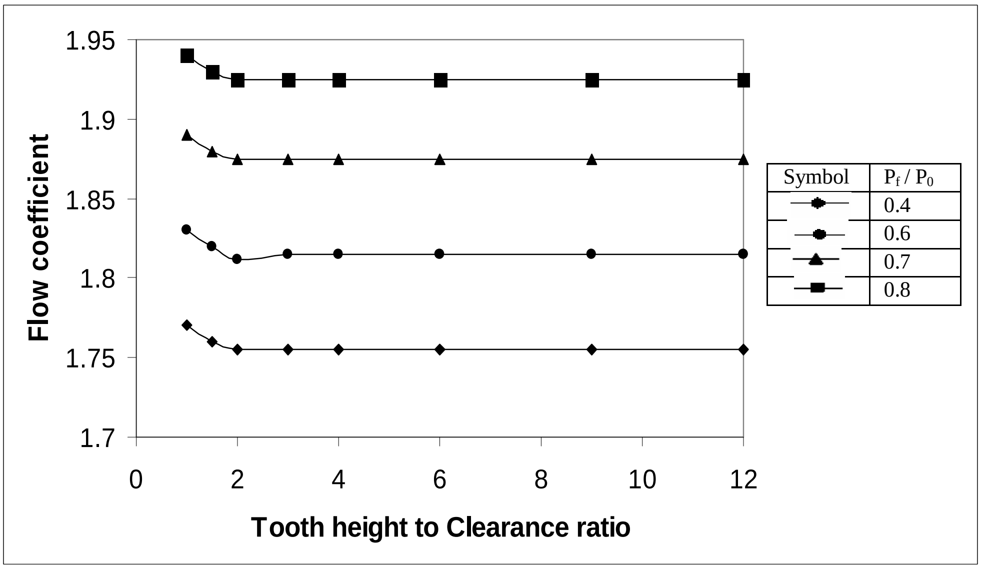


Fig. 14 Flow coefficient verses tooth height to clearance ratio (z = 3, c = 1.2 mm, s = 6 mm, and m = 10 mm)

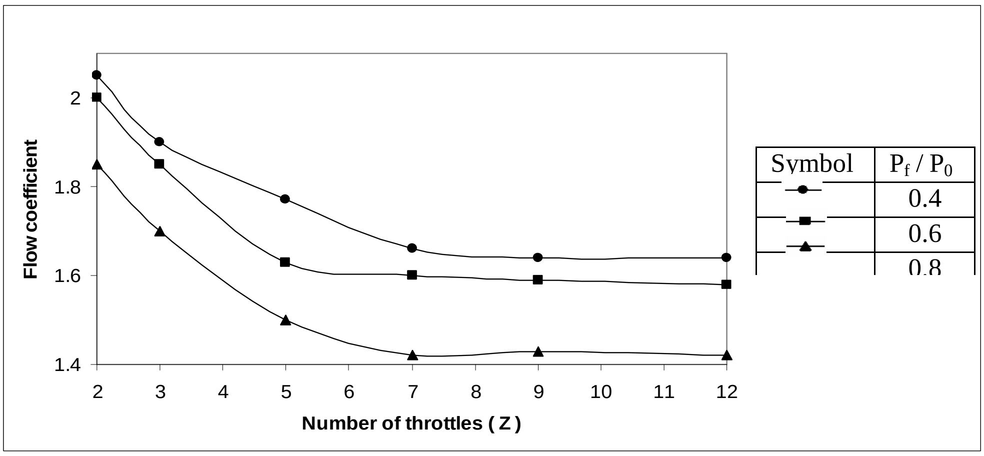


Fig. 15 Flow coefficient verses number of throttles at various pressure ratios (c = 0.9 mm, m= 10 mm, s = 6 mm, h = 5 mm).

4. COMPARISON OF RESULTS WITH OTHER INVESTIGATIONS

The lack of quantitative agreement between labyrinth seals investigators is not surprising and essentially can be traced back to differences in flow types and conditions fluid properties, test devices and method of analyzing the results. Therefore any quantitative comparison between the present results and other investigators results is not realistic. However, a qualitative comparison could be achieved.

In the present investigation it was reported that the flow coefficient decreases with decreasing the clearance, reaching a minimum value and then decreasing with further decreasing of the clearance for all number of the clearance for all number of throttles tested. This general trend is qualitatively in agreement with the published data reported by References [8, 31 and 39]. They did not attempt to explain the reason for this phenomenon in their analysis of their data. Rao and Sidheswar [6], Dobek [40], Vermes [41] and Jeris [42] showed that the leakage was decreased by decreasing the seal clearance. Mayer and Lowrie [9] stated that the leakage increases as the seal clearance decreases This trend is somewhat contrary to the trend reported herein. Kearton tests [43], showed that as the clearance decreases the flow coefficient also decreases, except at small clear-dances, where the flow coefficient appears to maintain a constant value. His data for small clearances must be of questionable quality, since it has been shown by the present and previous investigations [8, 31 and 39], that the flow coefficient increases as the clearance approaches zero.

It has been noted in the present investigation that as the tooth width increases the flow coefficient rises sharply, reaching a maximum value and then decreases gradually approaching a constant value for wide teeth. Data collected by investigators [42, 43 and 44] observed the same behavior of the flow coefficient for various tooth widths, but they could offer no explanation as to its cause. However, Yamada [31] found the flow coefficient to be independent of the tooth width. This trend is similar to that occurs for large tooth widths. Egli's data [2] showed that as the tooth width increases the flow coefficient increases. This is the same trend that occurs for small tooth widths.

In the present investigation it was clear that the flow the flow coefficient remained constant for various tooth heights. Yamada [31] and Jerie [42] tests showed similar trend.

The present tests showed that the flow coefficient decreases with increasing the number of throttles and as the number of throttles becomes large, the flow coefficient approaches a constant value. Similar trend was reported by Nikitin and Ipatov [30]. However, Banckert's data [45] indicated that the flow coefficient is independent of the number of throttles, This trend is similar to that occurs for large number of throttles. Rao and Sidheswar[6]and Meyer and Lowrie[9]tests showed that the leakage was dec-reased by increasing the number of throttles. This is similar to the trend that observes for seal with small number of throttles.

5. CONCLUSIONS

Based on the observed trends of the experimental results reported in the present paper for incompressible flow through straight-through labyrinth seals, the following general constraints for minimizing the leakage can be made.

Clearance in straight-through labyrinth seals should be made as small as possible for minimum leakage. When selecting a clearance, however, allowance must be made for thermal expansion and shaft misalignment.

A narrow tooth width is preferable to wider tooth width. The tooth width should always be less than one-half the value of the pitch.

The only constraint on tooth height is that it be at least twice the value of the clearance. The leakage will remain relatively constant for tooth height/clearance ratios greater than two.

For a seal of fixed overall length, there is a trade-off between choosing the pitch and the number of throttles. As the number of throttles increases, the pitch will decrease, causing the carry-over coefficient to increase. This increase leakage, thereby offsetting some, if not all of the decrease in the leakage achieved by decreasing the flow coefficient. Solving for the optimum number of throttles and pitch to minimize leakage for a specific seal requires an iterative technique.

NOMENCLATURE

C = clearance between tooth and sealing surface in seal (mm).

h = height of tooth (mm)

m = pitch of teeth (mm)

$\dot{m}$ = mass flow rate (m^3/sec)

s = width of tooth (mm)

u = fluid velocity (m/sec)

v = specific volume (m^3/kg)

A = leakage area (m^2)

L = total length of seal (mm)

P = pressure (kN/m^2)

z = number of throttles

α = flow coefficient

β = pressure ratio, (p_f/p_o)

γ = polytrophic index

ε = carry-over coefficient

Subscripts:

f = final value

o = initial value

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APPENDIX (A)

The development of Egli's [2] equation for leakage in labyrinth seals is as follows:

The velocity of a jet extending adiabatically through a throttling from pressure P_1 to pressure P_2 is given by:

$$\frac{U^2}{2g} = \int_{P_1}^{P} V\,dp \qquad (1A)$$

By substituting

$$VP^{\frac{1}{\gamma}} = constant \qquad (2A)$$

into equation 1A and then integrating yields:

$$\frac{U^2}{2g} = P_1 V_1 \left(\frac{\gamma}{\gamma-1}\right)\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right] \qquad (3A)$$

The term (P_1 / P_2) can be expanded into a series, if higher order terms are neglected, the result is:

$$\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 1 + \left(\frac{\gamma-1}{\gamma}\right)\frac{\Delta P}{P_1} \tag{4A}$$

Substituting equation (4A) into equation (3A) yields:

$$\frac{U^2}{2g} = V_1 \Delta P \tag{5A}$$

The equation for mass conservation in a jet is given by:

$$\frac{\dot{m}}{A} = \frac{u}{V_2} \tag{6A}$$

When V_2 can be found from equation (2A) as:

$$V_2 = \left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}} V_1 \tag{7A}$$

The term $\left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}}$ can be expanded using a series, and after neglecting higher order terms, the value can be substituted back into equation (7A) to yield:

$$V_2 = V_1\left(1-\gamma\left(\frac{\Delta P}{P_1}\right)\right) \tag{8A}$$

Combining equations (5A), (6A) and (8A) results in:

$$\left(\frac{\dot{m}}{A}\right)^2 = (-2gP_1\Delta P)/P_1V_1\left(1-\frac{2}{\gamma}\left(\frac{\Delta P}{P_1}\right)\right) \tag{9A}$$

If PV = constant, then (9A) can be rewritten as:

$$\left(\frac{\dot{m}}{A}\right)^2 = (-2gP_1\Delta P)/P_0V_0\left(1-\frac{2}{\gamma}\left(\frac{\Delta P}{P_1}\right)\right) \tag{10A}$$

Dividing through equation (10A) by Δx, rearranging, and letting $\frac{\Delta P}{\Delta X}$, yields:

$$\left[\left(\frac{\dot{m}}{A}\right)^2 \cdot \frac{1}{\Delta x}\right] - \left[\left(\frac{\dot{m}}{A}\right)^2 \cdot \frac{2}{\gamma}\frac{1}{P_1}\right]\frac{dp}{dx} = \frac{-2g}{P_0V_0}P_1\frac{dp}{dx} \tag{11A}$$

Integrating over the length of the seal results in:

$$\left(\frac{\dot{m}}{A}\right)^2 \frac{1}{\Delta x}\int_{x_0}^{x_f} dx - \left(\frac{\dot{m}}{A}\right)^2 \frac{2}{\gamma}\int_{p_0}^{p_1}\frac{dp}{P} = \frac{-2g}{P_0V_O}\int_{p_0}^{p_1} Pdp \tag{12A}$$

Noting that $\frac{(x_f - x_0)}{\Delta x} = z$, and $\frac{2}{\gamma} \approx 1$ yields:

$$\dot{m} = A\sqrt{(P_0/V_0)\left(\left[1-(P_f/p_0)^2\right]/\left[Z + \ln\left(\frac{P_0}{P_f}\right)\right]\right)} \tag{13A}$$

Then, an empirical flow coefficient (α) and carry-over coefficient (ε) are added to correct the equation for the non-ideal nature of the flow and also to account for carry-over, resulting in:

$$\dot{m} = \alpha\varepsilon A\sqrt{(P_0/V_0)\left(\left[1-(P_f/p_0)^2\right]/\left[Z + \ln\left(\frac{P_0}{P_f}\right)\right]\right)} \tag{14A}$$