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ALTERNATIVE ALGORITHMS FOR SOLVING CLASSICAL TRANSPORTATION PROBLEMS

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ABSTRACT

The transportation problem has a special nature of linear programming problems. The objective of this problem is to minimize the total cost of distributing products from a number of sources to a number of destinations. In this paper, there are alternative algorithms which are used to solve the classical transportation problem. The solution algorithms for these methods are reformatted in an easy way. Some of these methods give solutions close to the optimal solution while others provide the optimal solutions directly. An illustrative example is used here to explain each method as well as a comparison between the different methods.

1. INTRODUCTION

Transportation economics plays a great role in both automotive, transportation and economic fields. Transportation problem (TP) is one of the real world applications of transportation economics. TP deals with finding the optimal distribution of a certain product from different sources to different destinations. The optimal distribution aims at minimizing the total transportation costs.

Patel et al [1], Taylor [2], and Ellaimony [3] stated that the transportation problem was introduced first by F.L. Hitchcock in 1941. Hitchcock presented a study entitled “The Distribution of a Product from Several sources to numerous Localities”. This presentation is considered to be the first important contribution to the solution of the transportation problems. In 1947 T.C. Koopmans presented in independent study, not related to Hitchcock’s, and called “Optimum Utilization of the Transportation System”. Both Hitchcock and Koopmans presentations helped in the development of transportation methods which involve a number of shipping sources and a number of destinations. The transportation problem, received this name because many of its applications involve determining how to optimally transport goods.

The transportation problem is a kind of linear programming problems that can be solved with the aid of using simplex technique. It consists of primary application in solving issues regarding several product resources and numerous destinations of merchandise, this type of problem is regularly called the Transportation hassle. The two common targets of such issues are: minimize the cost of Transportation m units to n destinations, and maximize the earnings of delivery m units to n destinations.

There are many different methods that can be used for solving the classical transportation problem. The studied methods in this paper are: north-west corner method (NWCM), row minima method (RMM), column minima method (CMM), least cost method (LCM), Vogel’s approximation method (VAM), Russell's approximation method (RAM), solving using excel solver (ESM), solving using lingo software (LSM),

inverse coefficient of variation method (ICVM), and allocation table method (ATM). The stepping stone and the modification distribution method (MODI) are also introduced to illustrate how to test and reach the optimal solution.
The solution algorithms for each of these methods as well as illustrative examples are included. A comparison table for the results of these methods is also introduced.

Solution of the Transportation problems generally includes of the following steps:

- Finding an initial feasible basic solution from sources to destinations.
- Test of optimality for the initial feasible basic solution (examine if the transportation cost is minimum or not).
- Moving towards optimality.

2. The General Structure of the Transportation Problem

For a plant wants to move a number of units of homogenous product from multiple warehouses (sources) to number of retail outlets (destinations). Each store (j) requires a certain number of product units (b_j), while each warehouse (i) can provide a certain amount of product units (a_i). The cost of moving one unit from source (i) to destination (j) is (c_{ij}), and is recognized for all combinations (i, j) to minimize the total Transportation cost (Z).

The quantity shipped from source i to the destination j is (x_{ij}). The total amount that shipped out of i is $a_i \geq 0$, and the sum received by destination j is $b_j \geq 0$. We temporarily impose restrictions on the total quantity shipped equals the total quantity received, i.e, $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$. The cost of shipping x_{ij} units is $c_{ij}\ x_{ij}$. Since the negative shipment has no valid explanation for the problem, we limit each x_{ij} to non-negative. Therefore, the Transportation problem can be mathematically formulated as follows:

$$\text{Min.} Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

Subject to:

$$\sum_{j=1}^{n} x_{ij} = a_i, i = 1, 2, \dots, m.$$

$$\sum_{i=1}^{nm} x_{ij} = b_j, j = 1, 2, \dots, n.$$

$$x_{ij} \geq o\ \forall\ i\ and\ j.$$

3. METHODS OF SOLVING CLASSICAL TRANSPORTATION PROBLEM

3.1 North-West Corner Method (NWCM)

To find an initial basic feasible solution to the transportation problem, the northwest corner method [4, 5, 6, 7 and 8] suggests that the quantities transferred from factories (sources) to warehouses (destinations) should start in the upper left corner (northwest). When using this entire path, factory requirements or full storage are used, depending on the lower number, the rollback is either set by the factory or warehouse capacity to the new row or column until it is fully used. Using this procedure, the table is filled from the upper left cell to the bottom right cell, using the entire warehouse requirements, then factory capabilities, etc.

NWCM Solution Algorithm

<u>Step 1</u>:

1. The first assignment is made within the cell occupying the upper left-hand (north-west) corner of the matrix; cell (1,1).
2. The most feasible quantity is allocated right here i.e. X_{11} = min (a_1, b_1). This value of x_{11} is then entered in the cell (1,1) of the Transportation table.

<u>Step 2:</u>

1. If $b_1 > a_1$, move vertically down to the second row and make the second allocation of quantity x_{21} = min (a_2, $b_1 - x_{11}$) in the cell (2, 1).
2. If $b_1 < a_1$, move horizontally right side to the second column and make the second allocation of quantity x_{12} = min ($a_1 - x_{11}$, b_2) inside the cell (1, 2).

3.If $b_1 = a_1$, there may be a tie for the second allocation. You will make a second allocation of value $x_{12} = \min(a_1 - a_1, b_2)$ in the cell(1, 2) or $x_{21} = \min(a_2, b_1 - b_1)$ in the cell (2, 1).

Step 3:
Begin from the brand new north-west corner of the transportation matrix and repeat steps 1 and 2 until all the requirements are satisfied.

Step 4:
The generated solution may not be the optimal solution. So we need to do a test of optimality and if needed a moving towards optimality. This can be done using the modified distribution (MODI) method which is illustrated in section 7 of this paper.

3.2 Row Minima Method (RMM)

The row minima method [8, and 9] is used to solve the transportation problem using the least element in the row, although its simplicity, it gives considerable better results than the north-west corner method.

RMM Solution Algorithm

1. Start by choosing the lower cost cell at the first row. A customization of this cell is made within the availability of row and column requirements. The row or column that is exhausted or satisfied should not be considered from further estimates.
2. If the row is exhausted we move on to the second row and continue with the same procedure at the second row. If the column is satisfied we choose the next lowest cost element in the first row and make an allocation that meets the grade availability (balance after the first assignment) and the column requirement.
3. The procedure is repeated until the first row is guaranteed.
4. Then go to the second row and repeat the procedure.
5. This procedure is repeated until all sources are exhausted and all destinations are satisfaction.
6. When the minimum cost element in a row is not unique randomly select between the minimum.
7. If you meet the availability of the row and the column requirement at one time we override the column only. Then look for the least expensive item in the row and assign zero units to that cell. Then cross the row and move on to the next row.
8. The generated solution may not be the optimal solution. So we need to do a test of optimality and if needed a moving towards optimality. This can be done using the modified distribution (MODI) method which is illustrated in section 7 of this paper.

3.3 Column Minima Method (CMM)

The column minima method [8, and 9] use the same steps as the RMM, but we start with columns instead of rows. CMM usually gives better solutions than the NWCM.

CMM Solution Algorithm

1. Start by choosing the first column instead of the first row as in RMM. The minimum cost element in this column is then determined for customization under the row availability constraints and column requirements.
2. The procedure here is the same as the procedure that was performed in the Minimum Row Method, except moving from one column to another rather than moving from one row to another. This way the procedure continues until the last column requirements are satisfied.
3. The generated solution may not be the optimal solution. So we need to do a test of optimality and if needed a moving towards optimality. This can be done using the modified distribution (MODI) method which is illustrated in section 7 of this paper.

3.4 Least Cost Method (LCM)

The least cost method [3, 4, 6, 7, and 8] gives better solutions than the previous three methods because it takes into account the entire transportation network and not a row or column. What is used in this method is to determine the available quantity of the least cost variable per unit and exclude the column or row that meets the needs. Then adjust the supply and demand for all non-excluded. Then repeat the process by determining the available quantity of the least expensive variable for each unit that is not excluded and continue to solve until the distribution on the entire transportation network is done.

LCM Solution Algorithm

1. Start by choosing the cell which has the lowest unit cost in the whole matrix. The amount that should be added to this cell must be the minimum of its row availability and its column requirements. The row or column that allocates its entire amount is subtracted from further estimates.
2. If the row or column is exhausted we move on to the next least cost cell and continue with the same procedure as in step 1.
3. This procedure is repeated until all sources and destinations are satisfied.
4. When the minimum cost element in the matrix is not unique, we randomly select between the minimum.
5. The generated solution may not be the optimal solution. So we need to do a test of optimality and if needed a moving towards optimality. This can be done using the modified distribution (MODI) method which is illustrated in section 7 of this paper.

3.5 Vogel Approximation Method (VAM)

Vogel approximation method [3, 4, 5, 6, 7, and 8] is an improved method from the previous methods as it often gives better results that are optimal or very close to optimal. The steps to solve the transportation problem can be summarized with Vogel's method as follows:

VAM Solution Algorithm

1. For every row of the table assign the smallest and the second less value. Calculate the difference between them for each row. Those are called penalties. Type the result of the difference between the values outside the table next to the values. Repeat this step for the columns as well.
2. Identify the row or column with the biggest penalty. If a tie happens then use an arbitrary choice. Let the largest penalty exists at the i^{th} row and have its lower unit cost c_{ij}. Allocate the biggest feasible quantity x_{ij} = min (a_i, b_j) within the cell (i, j).
3. Once more compute the row and column penalties for the decreased table and then go to step2. Repeat the previous method until all requirements are satisfied.
4. The generated solution may not be the optimal solution. So we need to do a test of optimality and if needed a moving towards optimality. This can be done using the modified distribution (MODI) method which is illustrated in section 7 of this paper.

3.6 Russell's Approximation Method (RAM)

Although Russell's method [5] is one of the methods to find the initial feasible basic solution to the transportation problem, it often gives results that are close to the optimal solution and in many cases give the optimal solution directly.

RAM Solution Algorithm

1.At each row (i) let $\overline{u_i}$ = the highest unit cost c_{ij} at this row. At each column (j) let $\overline{v_j}$= the highest unit cost c_{ij} at this column.
2.Prepare new matrix withΔ_{ij}=C_{ij} - $\overline{u_i}$ -$\overline{v_j}$
3. Determine the cell (i, j); in the new matrix; with the highest negative Δ_{ij} and allocate to it the largest possible amount of products such that x_{ij}= min(a_i,b_j).
4. Delete the row or column that is exhausted or satisfied and prepare a new matrix as illustrated in step 2.
5. Repeat step 3 till all sources and destinations are satisfied.
6. The generated solution may not be the optimal solution. So we need to do a test of optimality and if needed a moving towards optimality. This can be done using the modified distribution (MODI) method which is illustrated in section 7 of this paper.

3.7 Inverse Coefficient of Variation Method (ICVM)

The inverse coefficient of variation method (ICVM) [11] is based on statistical methods which uses arithmetic average and standard deviation to find the initial basic feasible solution for the transportation problem. It is possible to take advantage of computer programs such as "Microsoft Excel" to solve the statistical methods used in (ICVM). It may save the user time and effort to solve the problem.

ICVM Solution Algorithm

1. Make sure that the transportation tableau is balanced. If not change it into a balanced one.
2. Calculate the mean $(\overline{c_i})$;$\overline{c_i} = \frac{\sum_{i=1}^{n} c_{ij}}{n}$, (j= 1, 2, …, n) and the standard deviation (S_i); $S_i = \sqrt{\frac{\sum_{i=1}^{n}(c_{ij}-\overline{c_i})^2}{n-1}}$, (j= 1, 2, …, n), for each row (i; i=1,2,…, m), and then calculate the respective inverse of coefficient of variation $(CV)^{-1}$, $CV^{-1} = \frac{\overline{c_i}}{Si}$ by dividing the arithmetic mean ($\overline{c_i}$) on the standard deviation (S_i).
3. Calculate the mean $(\overline{c_j})$;$\overline{c_i} = \frac{\sum_{i=1}^{n} c_{ij}}{n}$, (i= 1, 2, …, m) and the standard deviation (S_j); $S_j = \sqrt{\frac{\sum_{i=1}^{n}(c_{ij}-\overline{c_j})^2}{n-1}}$, (i= 1, 2, …, m), for each column (j; j=1,2,…, n), and then calculate the respective inverse of coefficient of variation $(CV)^{-1}$, $CV^{-1} = \frac{\overline{c_j}}{Sj}$ by dividing the arithmetic mean ($\overline{c_j}$) on the standard deviation (S_j).
4. Specify the lowest value for the respective inverse of coefficient of variation $(CV)^{-1}$and specify the lowest transfer cost in the row or column corresponding to the least value for the parameter (min c_{ij}) and start filling the selected cell as much as possible.
5. Delete the row or column that was satisfied and then repeat the previous steps(Steps 2 to 4) till all sources and destinations are satisfied. Then calculate the total transportation cost.
6. The generated solution may not be the optimal solution. So we need to do a test of optimality and if needed a moving towards optimality. This can be done using the modified distribution (MODI) method which is illustrated in section 7 of this paper.

3.8Allocation Table Method (ATM)

The allocation table method (ATM) [12] is another one of the advanced methods of finding the initial basic feasible solution to the transportation problem.

ATM Solution Algorithm

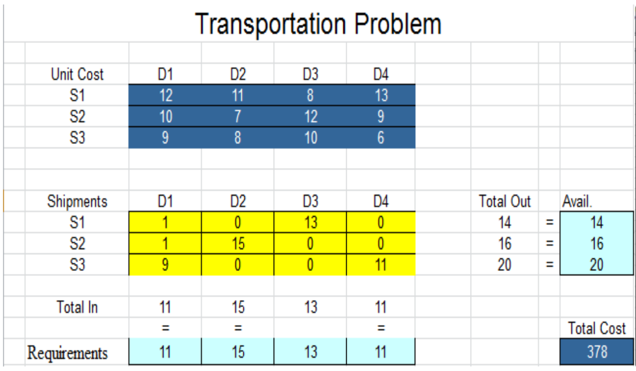
1. Make sure that the supply and demand are balanced in the matrix, if not around it to a balanced.
2. Specify the lowest unit cost cell with an odd number in the transportation network (c_{ij}). If there is no odd unit cost is available, divide all transportation unit costs by 2 and select the lowest unit cost in the new network.
3. Then prepare a new table where its unit costs can be calculated by subtracting the lower transportation unit cost value which is specified in step (2) from all the odd transportation unit costs (only) in the original network. Don't change the even unit costs.
4. Start by filling products for the lowest odd cell that have been identified in step (2).Then headed for the less even value cell in the transportation network (all values will be an even basically) and start allocating values, in accordance with supply and demand. In case of there is more than one less even value, select the cell that will need a smaller amounts of products than others and start with them and then go to other cells afterwards.
5. Repeat the previous step until all supply and demand are satisfied and calculate the final transportation cost using the original unit costs.
6. The generated solution may not be the optimal solution. So we need to do a test of optimality and if needed a moving towards optimality. This can be done using the modified distribution (MODI) method which is illustrated in section 7 of this paper.

3.9 Solving Using Excel Solver

The use of M.S. Excel solver [2, 4, 5, and 7] in the analysis of operations research, linear programming and Transportation problem has become a great advantage for users. Software has become solving problems that have been time consuming in a few minutes saving time and effort. The advantages of Excel solver is, it considered cheap and easy to purchase if compared to other programs. To solve the transportation problem using Excel solver we must do the following: Identify the decision variables, Identify the constraints and Program the objective function. This method gives the optimal solution directly. Figures (1 and 2) illustrate a typical illustrative example solved by M.S. Excel solver.

3.10 Solving Using Lingo Codes

LINGO [7 and 10] is a software tool designed to efficiently work on optimization models and linear and nonlinear models and solve them effectively.



Transportation Problem

Unit Cost	D1	D2	D3	D4			
S1	12	11	8	13			
S2	10	7	12	9			
S3	9	8	10	6			
Shipments	D1	D2	D3	D4	Total Out		Avail.
S1	1	0	13	0	14	=	14
S2	1	15	0	0	16	=	16
S3	9	0	0	11	20	=	20
Total In	11	15	13	11			
	=	=	=	=			Total Cost
Requirements	11	15	13	11			378

Fig.1. T.P. Example by Excel Solver.

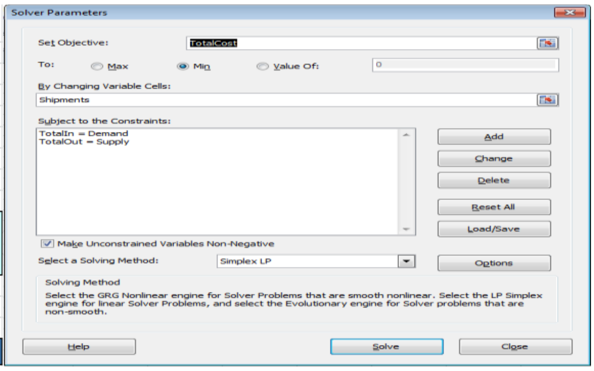


Fig.2. Excel Solver Parameter

It is relatively difficult to deal with the reverse program and its price is high compared to others, but it has the ability to solve many problems. In transportation problems, the program has the ability to solve problems where transportation costs are in real numbers and not only integer numbers, which may be complicated in different programs. Tables (1 and 2) illustrate the Lingo code & Lingo output for an illustrative example which is subsequently solved.

Table 1. A typical Lingo code for an T.P. example.

```
SETS:
SOURCES/S1,S2,S3/:Availabilities;
DESTINATIONS/D1,D2,D3,D4/:Requirements;
LINKS(SOURCES,DESTINATIONS):COST,SHIP;
ENDSETS
MIN=@SUM(LINKS:COST*SHIP);
@FOR(DESTINATIONS(J):
@SUM(SOURCES(I):SHIP(I,J))>Requirements(J));
@FOR(SOURCES(I):
@SUM(DESTINATIONS(J):SHIP(I,J))<Availabilities(I));
DATA:
Availabilities=14,16,20;
Requirements=11,15,13,11;
COST=12,11,8,13,
10,7,12,9,
9,8,10,6;
ENDDATA
END
```

Table 2. Lingo code output for the T.P studied example.

Lingo Output	COST(S2, D2) 7.000000 0.000000
Global optimal solution found.	COST(S2, D3) 12.00000 0.000000
Objective value:	COST(S2, D4) 9.000000 0.000000
378.0000	COST(S3, D1) 9.000000 0.000000
Infeasibilities: 0.000000	COST(S3, D2) 8.000000 0.000000
Total solver iterations: 6	COST(S3, D3) 10.00000 0.000000
Elapsed runtime seconds:	COST(S3, D4) 6.000000 0.000000
0.11	SHIP(S1, D1) 1.000000 0.000000
Model Class: LP	SHIP(S1, D2) 0.000000 2.000000
Total variables: 12	SHIP(S1, D3) 13.00000 0.000000
Nonlinear variables: 0	SHIP(S1, D4) 0.000000 4.000000
Integer variables: 0	SHIP(S2, D1) 1.000000 0.000000
Total constraints: 8	SHIP(S2, D2) 15.00000 0.000000
Nonlinear constraints: 0	SHIP(S2, D3) 0.000000 6.000000
Total nonzeros: 36	SHIP(S2, D4) 0.000000 2.000000
Nonlinear nonzeros: 0	SHIP(S3, D1) 9.000000 0.000000
	SHIP(S3, D2) 0.000000 2.000000
Variable Value Reduced Cost	SHIP(S3, D3) 0.000000 5.000000
AVAILABILITIES(S1) 14.00000	SHIP(S3, D4) 11.00000 0.000000
0.000000	Row Slack or Surplus Dual Price
AVAILABILITIES(S2) 16.00000	1 378.0000 -1.000000
0.000000	2 0.000000 -12.00000
AVAILABILITIES(S3) 20.00000	3 0.000000 -9.000000
0.000000	4 0.000000 -8.000000
REQUIREMENTS(D1) 11.00000	5 0.000000 -9.000000
0.000000	6 0.000000 0.000000
REQUIREMENTS(D2) 15.00000	7 0.000000 2.000000
0.000000	8 0.000000 3.000000
REQUIREMENTS(D3) 13.00000	
0.000000	
REQUIREMENTS(D4) 11.00000	
0.000000	
COST(S1, D1) 12.00000	
0.000000	
COST(S1, D2) 11.00000	
0.000000	
COST(S1, D3) 8.000000	
0.000000	
COST(S1, D4) 13.00000	
0.000000	
COST(S2, D1) 10.00000	
0.000000	

4. Comparison between the Results of the Studied Methods by Using an Illustrative Example

Table (3) illustrates a transportation problem example including the All availabilities, requirements by tons, and the transportation cost by dollars per ton.

The above example is solved by the 10 illustrated methods previously to find the initial basic feasible solution by each one. Table (4) below gives a comparison between the results of the 10 method. Both the optimal solution and total transportation cost of each method are included in this Table.

Table3. T.P. Illustrative Example.

	D_1	D_2	D_3	D_4	Avail.
S_1	12	11	8	13	14
S_2	10	7	12	9	16
S_3	9	8	10	6	20
Req.	11	15	13	11	50

Table4. Comparison between the studied T.P. solution methods.

No.	Method	Optimal Solution	Total Cost	No.	Method	Optimal Solution	Total Cost
1	North-West Corner Method (NWCM)	x_{11}= 11, x_{12}=3, x_{22}=12, x_{23}=4, x_{33}=9, x_{34}=11	453	6	Russell's Approximation Method (RAM)	x_{11}= 1, x_{13}=13, x_{21}=1, x_{22}=15, x_{31}=9, x_{34}=11	378
2	Row Minima Method (RMM)	x_{12}= 1, x_{13}=13, x_{22}=14, x_{24}=2, x_{31}=11, x_{34}=9	384	7	Inverse Coefficient of Variation Method (ICVM)	x_{11}= 1, x_{13}=13, x_{21}=1, x_{22}=15, x_{31}=9, x_{34}=11	378
3	Column Minima Method (CMM)	x_{13}= 13, x_{14}=1, x_{22}=15, x_{24}=1, x_{31}=11, x_{34}=9	384	8	Allocation Table Method (ATM)	x_{13}= 13, x_{14}=1, x_{22}=15, x_{24}=1, x_{31}=11, x_{34}=9	384
4	Least Cost Method (LCM)	x_{11}= 1, x_{13}=13, x_{21}=1, x_{22}=15, x_{31}=9, x_{34}=11	378	9	Solving Using Excel Solver	x_{11}= 1, x_{13}=13, x_{21}=1, x_{22}=15, x_{31}=9, x_{34}=11	378
5	Vogel Approximation Method (VAM)	x_{11}= 1, x_{13}=13, x_{21}=1, x_{22}=15, x_{31}=9, x_{34}=11	378	10	Solving Using Lingo Codes	x_{11}= 1, x_{13}=13, x_{21}=1, x_{22}=15, x_{31}=9, x_{34}=11	378

5. Stepping Stone and The Modified Distribution Methods

The different methods that are used to find an initial basic feasible solution to the transportation problem which are reviewed in this paper provide basic solutions that may be close or far from the optimal solution. Some of them provide the optimal solution from the first iteration. But only two methods; solving using excel solver and solving using lingo code; provide the optimal solution directly for all transportation problems.
It could be said that the transportation problem solution is optimal if and only if its total cost (Z) is the least expensive amount, and we never find any other solution to the same problem that gives less value than the optimal one.
In general, the basic feasible initial solution is optimal if and only if ($c_{ij} - u_i - v_j \geq 0$) for every (i, j) such that x_{ij} is non-basic [5], where u_i and v_j can be calculated as in the solution algorithm below.
In order to determine whether the solution that was found in these methods is an optimal solution or not, we are doing the optimality test using one of the following methods: Stepping Stone Method and Modified Distribution Method (MODI).

Stepping stone method [3, and 6] derives its name from the fact that a closed loop of occupied cells is used to evaluate each empty cell (non-basic variables). These occupied cells are thought of as stepping stones in a pond-the pond being the entire tableau. The modified distribution method (MODI) is illustrated in [3, 4, 5, 6, 7, and 8]. MODI solution algorithm and flow chart is illustrated below.

The Modified Distribution Method Solution Algorithm

1. Solve the transportation problem by any of the initial basic feasible solution methods.
2. Ensure the requirement of the optimization test which states that the number of occupied (filled) cells in the transportation network from step (1) equals m+n-1.
3.For the occupied cells only determine (u_i; i=1,2, ..., m) for each row (i), and (v_j; j= 1,2,…, n) for each columns (j) according to the equation ($u_i + v_j = c_{ij}$). Start by assuming one of the values of u_i or v_j = 0. Then calculate the other values of u_i and v_j. After calculating the values of the other u_i and v_j for all rows and columns, calculate ($\Delta_{ij} = c_{ij} - u_i - v_j$) for all non-occupied cells.

4. Prepare a new matrix for the calculated values of (Δ_{ij}) as indicated in step 3.

- If all Δ_{ij} non-occupied cells is greater than zero, the initial basic solution would be an optimal solution. Stop.
- If one or more of Δ_{ij} at the non-occupied cells equal zero, and Δ_{ij} at all other non-occupied cells is greater than zero, this means that the current solution is optimal, but there is one more other optimal solution as well. We can stop here, or we can find the other optimal solution(s) as in step 5 below.
- If there is one or more of Δ_{ij} less than zero, the solution is not optimal and from here determine the cell which includes the value of the largest negative number. This cell must be filled by as much products as we can to minimize the total transportation cost. Go to step 5 below.

5. If the initial basic feasible solution is not optimal, start with the cell which includes the highest negative (Δ_{ij}) and draw a square or a rectangular closed path with 4 cells. Give this cell a (+) sign which means that this cell must receive products. The next cell in the closed path corner should include a (-) sign which means this cell must give products. This cell must have products (from the initial basic feasible solution). The next cell in the closed path corner should include a (+) sign, and the last one should include a (-) sign. Remember that the two cells with the (-) signs must have products to give.

6. Now transfer as much products from the two (-) cells to the two (+) cells such that the transferred amounts should not exceed the amounts exists in any of the giver cells. By this distribution the total transportation cost should be reduced.

7. After making the previous improvements as in step (6) go back to Step (3) to test the optimization and do the consecutive steps till we reach the optimal solution.

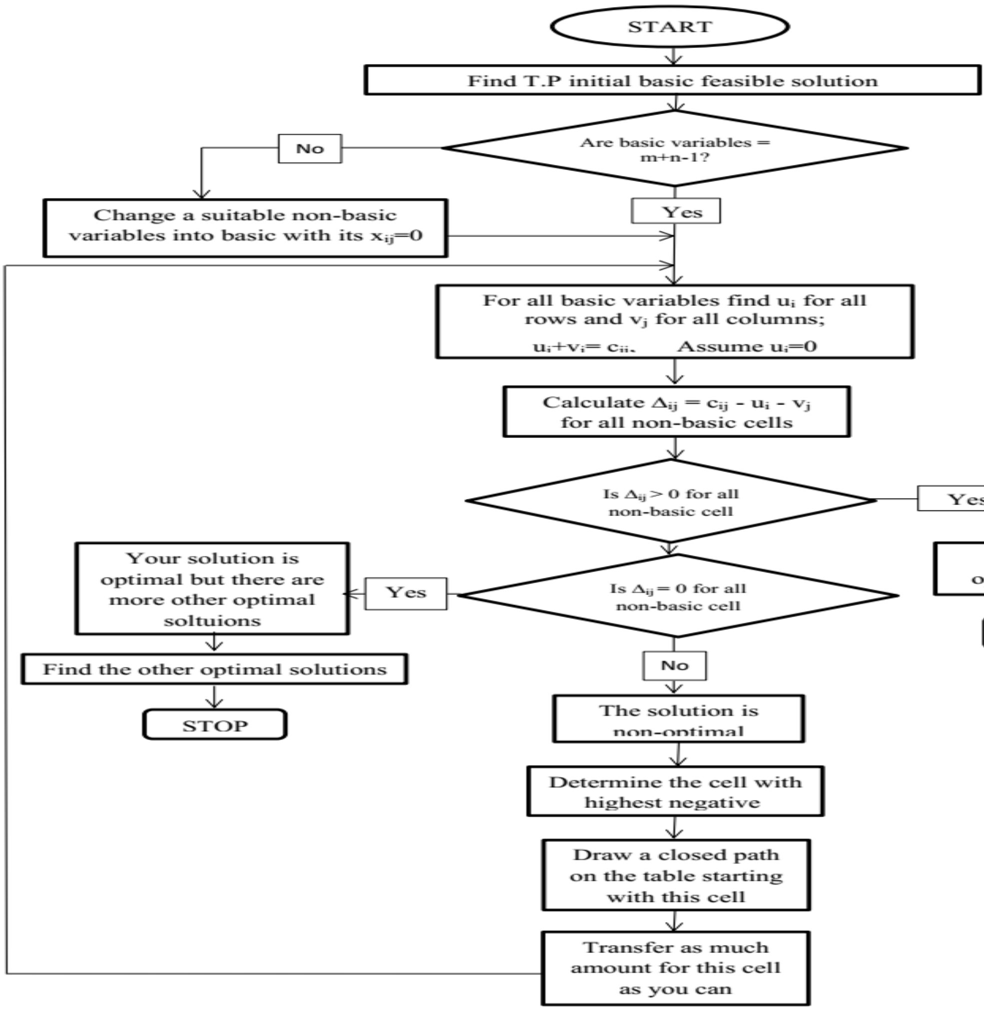


Fig.3. MODI flow chart.

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خوارزميات بديلة لحل مشكلة الفعل الكلاسيكية

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الملخص :

تعتبر مشكلة النقل هي إحدى صور تطبيقات البرمجة الخطية، الهدف من هذا البحث هو تقليل مجموع التكاليف عند نقل المنتجات من مجموعة من المصادر إلى مجموعة من الأهداف، في هذا البحث تم صياغة خوارزميات حل لمجموعة من الطرق التي تعطي الحل المبدأي المتاح لمشكلة النقل الكلاسيكية –ذات الهدف الوحيد– في شكل مبسط، بعض هذه الطرق تعطي حلول مقاربة جداً للحل الأمثل لمشكلة النقل وربما الحل الأمثل نفسه في بعض التطبيقات ، كما تم اقتراح خوارزميات حل لإيجاد الحل الأمثل لمشكلة النقل باستخدام برمجيات مثل معالج الإكسيل وبرنامج لينجو، وتم تقديم مثال توضيحي وإيجاد الحل المبدأي الأمثل لمشكلة النقل بكافة الطرق التي تم عرض خوارزميات حل لها وإظهار التباين في النتائج بين الطرق المختلفة ومقارنته بالحل الأمثل.