THE EFFECT OF THERMAL DISPERSION ON UNSTEADY MHD CONVECTIVE HEAT TRANSFER THROUGH VERTICAL POROUS

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Abstract

The influence of thermal dispersion on unsteady two-dimensional laminar flow is presented. A viscous incompressible conducting fluid in the vicinity of a semi infinite vertical porous through a moving plate in the presence of a magnetic fluid is studied. A cod (FORTRAN) was constructed for numerical computations for the velocity and temperature for various values of the affected parameters are carried out.

Keywords: MagnetoHydroDynamics, General fluid mechanics, convection in porous media.

Introduction

The study of flow and heat transfer for an electrically conducting fluid through a porous moving plate under the influence of a magnetic field has attracted the interest of many investigators. The applications have been presented in many scientific problems such as magneto-hydro-dynamic (MHD), plasma studies, nuclear reactors, and the field of aerodynamics. Gribben [1] has considered the MHD boundary layer flow over a semi-infinite plate with an aligned magnetic field in the presence of a pressure gradient. He has obtained solutions for large and small magnetic Prandtle numbers using the method of matched asymptotic expansion. Takhar and Ram [2] has studied the MHD free porous convection heat transfer of water at 4°C through a porous medium. Soundalegkar [3] has obtained approximate solutions for two-dimensional flow of an incompressible, viscous fluid past with constant suction velocity normal to the plate. The difference between the temperature of the plate and the free steam is moderately large causing the free convection current. Raptis [4] has studied mathematically the case of time-varying of two-dimensional natural convective heat transfer of an infinite vertical porous plate. The study of Darcian porous MHD is much complex. It is necessary to consider in detail the distribution of velocity and temperature in addition to the surface skin friction across the boundary layer. All these studies assume that thermal diffusivity is constant, However, under the conditions at which the internal effects are prevalent, the thermal dispersion effect become significant as observed in Plumb[5], Hong and Tien [6], Nield and Bejan [7]. Tomer [8] has studied the flow and heat transfer characteristics for natural convection along an inclined plate in a saturated porous medium with an applied magnetic field. Ramana Reddy [9] has analyzed the influence of first order homogenous chemical reaction and thermal radiation on

hydro-magnetic free convection heat and mass transfer of a viscous fluid past a semi infinite vertical moving porous plate embedded in a porous medium.

In the present work, the investigation of the effects of a magnetic field, porous medium, thermal dispersion and the exponential index on unsteady MHD convective heat transfer through a semi-infinite vertical porous moving plate will be carried out. Numerical computations for the velocity and temperature for various values of the parameters will be obtained.

Mathematical analysis

Suppose that two-dimensional unsteady flow of a laminar, incompressible fluid past a semi-infinite vertical porous moving plate embedded in a porous medium and subjected to a transverse magnetic field. Moreover, there is no applied voltage which implies the absence of an electric field. The induced magnetic field is negligible, viscous and Darcy's resistance terms are taken into account with constant permeability of the porous medium.

The MHD term is derived from an order of magnitude analysis of the full Navir-Stokes equations. Under these conditions, the governing equations, which mean the mass, momentum and energy conservation equation expressed in a Cartesian coordinates as:

$$\frac{\partial v^*}{\partial y^*} = 0 \tag{1}$$

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + v\frac{\partial^2 u^*}{\partial y^*{}^2} + g\,\beta\,(T - T_\infty) - v\frac{u^*}{k^*} - \frac{\sigma}{\rho}\beta_0^2 u^* \tag{2}$$

$$\frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{\partial}{\partial y^*}(\alpha_y^* \frac{\partial T}{\partial y^*}) \tag{3}$$

Equation (1) is the continuity equation, Equation (2) is the momentum equation, and Equation (3) is the energy equation. Where x^* and y^* are the dimensional distances in the directions perpendicular to plate plan respectively. u^* , v^* are the components of dimensional velocities along x^* and y^* directions respectively, ρ is the density of the medium, v is the kinematic viscosity and α_y^* is a variable quantity which is the sum of molecular thermal diffusivity α and dispersion thermal diffusivity α_d . The expansion for dispersion thermal diffusivity will be $\alpha_d = \gamma\,d\,u$ Plumb [5]. Where γ is mechanical dispersion coefficient whose value depends on the experiment method, d is the pore diameter, k^* is permeability of the

porous medium, T is temperature, and α_y^* is the component of thermal diffusivity in y^* direction.

The third term at the RHS of the momentum equation (2) denotes buoyancy effects. The fourth term is the bulk matrix linear resistance i.e. Darcy term. The fifth is the MHD term. It is assumed that porous plate moves with constant velocity in the direction of fluid flow, and the free stream velocity follows the exponentially increasing small perturbation law. Moreover, the plate temperature velocity is exponentially varying with time.

Under these assumptions, taken into account $u^* = u_p^*$, the appropriate boundary conditions for the velocity and temperature are

$$T = T_w + \varepsilon \, (T_w + T_\infty) \, e^{n^* t^*} \qquad at \qquad y^* = 0 \qquad (4)$$

$$u^* \to u_\infty^* = u_0 \, (1 + \varepsilon \, e^{n^* t^*}), \quad T \to T_\infty \quad as \quad y^* \to \infty \qquad (5)$$

From the continuity equation (1), it is clear that the suction velocity normal to the plate is a function of time only which yields to

$$v^* = - v_0 \, (1 + \varepsilon \, A \, e^{n^* t^*}) \qquad (6)$$

Where A is a real positive constant, ε is a small parameter, so that εA still smaller than unity, and v_0 is a scale of suction velocity which has non-zero positive constants. Outside the boundary layer the momentum equation (2) gives

$$-\frac{1}{\rho} \frac{dp^*}{dx^*} = \frac{du_\infty^*}{dt^*} - \frac{v}{k^*} u_\infty^* + \frac{v}{k^*} u_\infty^* + \frac{\sigma}{\rho} B_0^2 \, u_\infty^2 \qquad (7)$$

The dimensionless variables will be introduced as

$$u = \frac{u^*}{u_0} \qquad v = \frac{v^*}{v_0} \qquad y = \frac{v_0}{v} \, y^*$$

$$u_\infty = \frac{u_\infty^*}{u_0} \qquad u_p = \frac{u_p^*}{u_0} \qquad t = \frac{t^* v_0^2}{v} \qquad (8)$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} \qquad n = \frac{n^* v}{v_0^2} \qquad k = \frac{k^* v_0^2}{v^2}$$

Then

$$P_r = \frac{v \, \rho \, c_p}{k} = \frac{v}{\alpha} \qquad\qquad M = \frac{\sigma \, B_0^2 \, v}{\rho \, v_0^2}$$

$$(9)$$

$$G = \frac{v \, B \, g \, (T_w - T_\infty)}{u_0 \, v_0^2}$$

In view of equations (6)-(9) the governing equations (2) and (3) are reduced to the non-dimensional form

$$\frac{\partial u}{\partial t} - (1 + \varepsilon \, A \, e^{nt}) \frac{\partial u}{\partial y} = \frac{dU_\infty}{dt} + \frac{\partial^2 u}{\partial y^2} + G \, \theta + N \, (U_\infty - u)$$

$$(10)$$

$$\frac{\partial \theta}{\partial t} - (1 + \varepsilon \, A \, e^{n*t*}) \frac{\partial \theta}{\partial y} = \frac{1}{P_r} \frac{\partial^2 \theta}{\partial y^2} + \frac{\gamma \, d \, u_0}{v} \frac{\partial u}{\partial y} \frac{\partial \theta}{\partial y}$$

$$(11)$$

Where $\quad N = (M + \frac{1}{k})$

The boundary conditions (4) and (5) are then given in terms of dimensionless form

$$u = u_p \qquad\qquad \theta = 1 + \varepsilon \, e^{nt} \qquad at \qquad y = 0$$

$$(12)$$

$$u \to u_\infty \qquad\qquad \theta \to 0 \qquad as \qquad y \to 0$$

Solution of the problem

In order to reduce the above system of partial differential equations in dimensionless form, the velocity and temperature may be represented as

$$u = f_0(y) + \varepsilon \, e^{nt} f_1(y) + o(\varepsilon^2) + ...$$

$$(13)$$

$$\theta = g_0(y) + \varepsilon \, e^{nt} g_1(y) + o(\varepsilon^2) + ...$$

$$(14)$$

Substituting from Equations (13) and (14) into Equations (10) and (11), then equating the non-harmonic terms neglecting the coefficients of $o(\varepsilon^2)$, then

$$f_0'' = N f_0' - N - G g_0 - f_0'$$

$$(15)$$

$$g_0'' = \frac{1}{1 + Bg_0}(Bg_0'^2 + P_r g_0')$$

$$(16)$$

$$f_1'' = (N + n)(f_1 - 1) - Af_0' - Gg_1 - f_1'$$

$$(17)$$

$$g_1'' = \frac{1}{1 + g_0 B}[Bg_1 - 2Bg_0' g_1' - P_r g_1' + nP_r g_1] + Bg_1 \frac{Bg_0'^2 - P_r g_1'}{(1 + Bg_0)^2} \qquad (18)$$

The primes referred to differentiation with respect to y. The corresponding boundary conditions can be written as

$$f_0 = u_p \quad f_1 = 0 \quad g_0 = 1 \quad g_1 = 1 \quad at \quad y = 0 \qquad (19.1)$$

$$f_0 = 1 \quad f_1 = 1 \quad g_0 \to 0 \quad g_1 \to 0 \ at \quad y = \infty \qquad (19.2)$$

Results and Discussion

The flow of fluid over boundaries of porous materials has many applications such as boundary layer control. The study of unsteady boundary layer owes its importance to the fact that all boundary layers that occur in real life are, in a sense, unsteady. Consequently, the solution of many practical fluid mechanics problems hinges on the understanding of the behaviour of the unsteady boundary layer. The investigation of the effects of a magnetic field, porous medium, thermal dispersion and the exponential index on unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate has been carried out. This enables the numerically computations for the velocity and temperature for various values of these parameters to be carry out. A cod (FORTRAN) was constructed to apply these assumptions numerically.

Conclusion

Numerical computations for the velocity and temperature for various values of the affected parameters such as, thermal dispersion, exponential index, permeability, and magnetic field, were carried out. Hence the following are concluded.

i-The dimensionless velocity u is proportional to the thermal dispersion parameter β, the exponential index n, the dimensionless porous medium parameter k, and inversely to the magnetic field parameter M.

ii-The dimensionless temperature θ is proportional to the thermal dispersion parameter β, and the exponential index n,

iii-Figures (1) and (2) show that the dimensionless velocity u and the dimensionless temperature θ are increasing as the thermal dispersion parameter β is increasing respectively.

iv- Figures (3) and (4) show that the dimensionless velocity u and the dimensionless temperature θ are increasing as the exponential index n is increasing respectively.

v-Figure (5) shows that the dimensionless velocity u is increasing with increasing the dimensionless porous medium parameter k. Physically, this result can be achieved when the holes of the porous medium are very large so that the resistance of the medium could be neglected.

vi-Figure (6) represents the relation between the magnetic field parameter M and the velocity profiles from which it is clear that the existence of the magnetic field decreases the velocity.
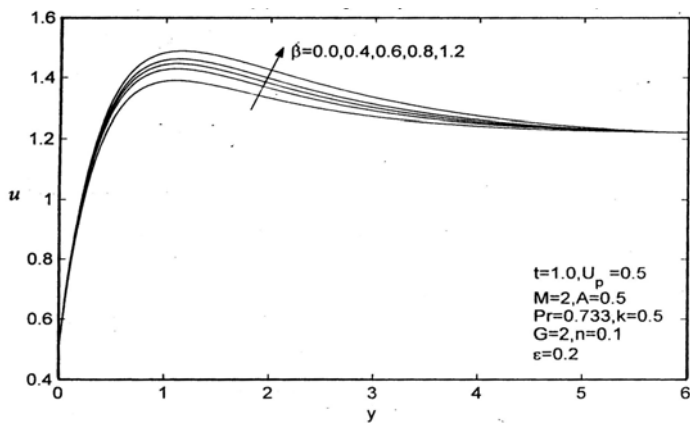
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Fig.(1):velocity profiles against y for different values of thermal dispersion β.
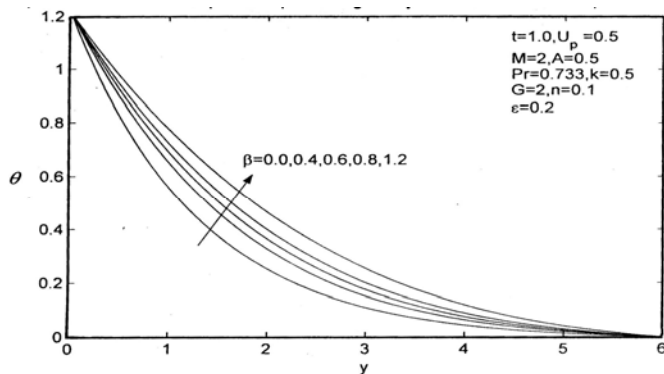

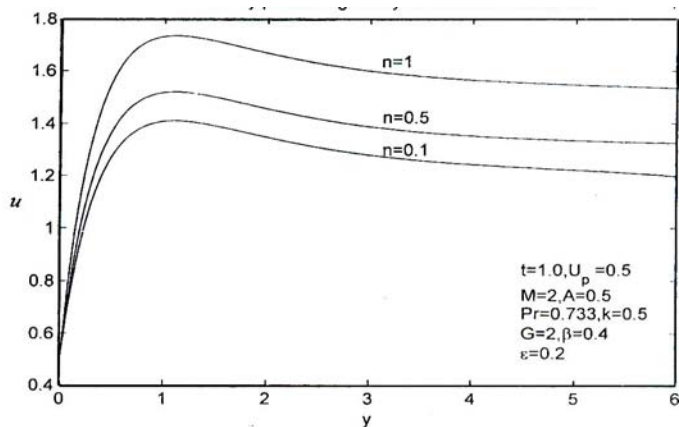Fig.(2): Temperature profiles against y for different values of thermal dispersion β.

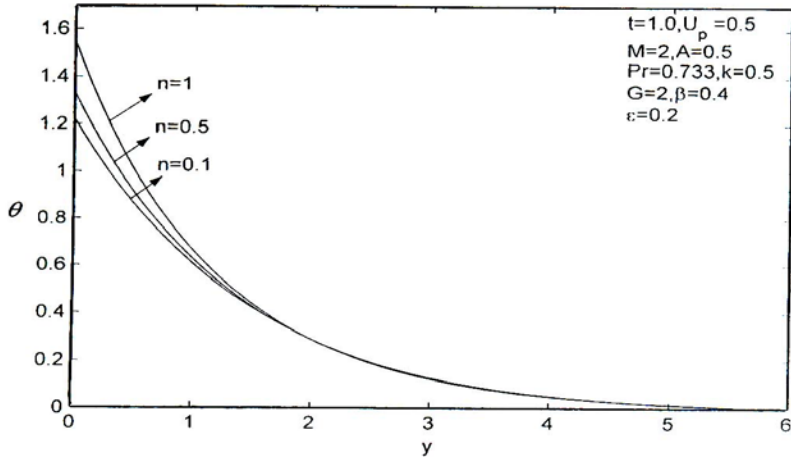Fig.(3): Velocity profiles against y for different values of exponential index n.

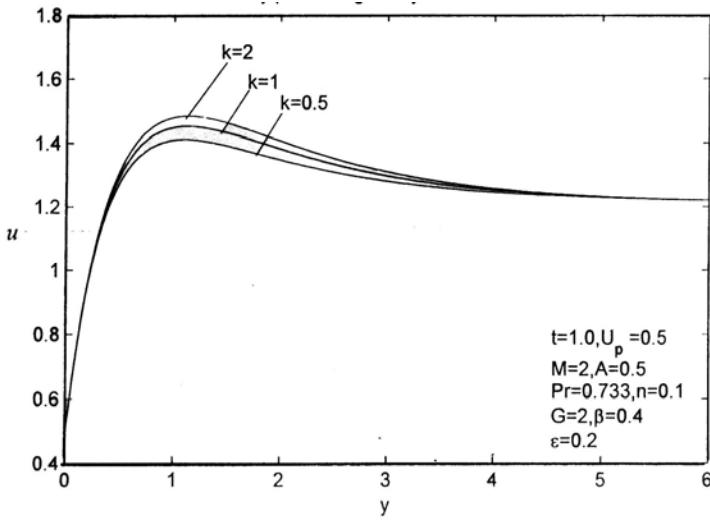**Fig.(4): Temperature profiles against y for different values of exponential index n.**

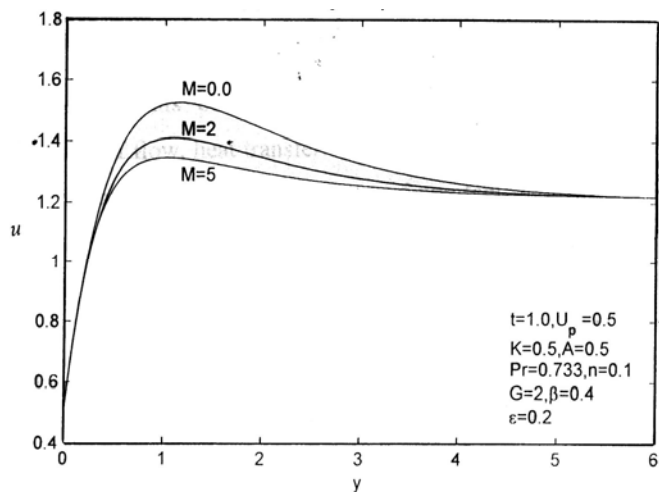**Fig.(5): velocity profiles against y for different values of permeability k.**

Fig.(6): velocity profiles against y for different values of magnetic parameter M.

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