Effect of tilting angle on natural convection heat transfer from a cylinder suspended in stagnant water

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ABSTRACT

Natural convection heat transfer from a cylinder suspended in water at different tilting angles has been analyzed numerically by varying the Rayleigh number (Ra) in laminar regime ($10^4 \leq Ra \leq 10^8$). The numerical analysis is carried out at different tilting angles in the range from 0 to 90 from vertical position using Ansys 15.0- Fluent computer code. The temperature and velocity contours as well as the average Nusselt number (Nu) and surface temperature have been obtained at different Ra for each tilting angle. It has been found that the average Nusselt number (Nu) increases while the average surface temperature decreases with the increase of Ra for all of the cases. By increasing the tilting angle from the vertical position, the average Nu increases whereas the average surface temperature decreases until a tilting angle of 45º. Further increase in the tilting angle results in decrease of the average Nu, whereas average surface temperature changes slightly. A single correlation relating Nu with Ra for natural convection from cylinder inclined at any arbitrary angle was developed.

Nomenclature:

At	total area of convective surface, m^2
D	diameter of the cylinder, m
Gr	Grashof number, $g\beta \, \Delta T L^3/\nu^2$
g	gravitational acceleration, m/s^2
h	average convective heat transfer coefficient, $W/m^2 \, k$
k	thermal conductivity of fluid, W/mK
L	length of the cylinder, m
m·	mass flow rate, (kg/sec)
Nu	average surface Nusselt number
Nu*	Nusselt number based on L_c (section 5.3)
Pr	Prandtl number
P	pressure, N/m^2
Q	total heat transfer rate, W
Ra	Rayleigh number
R	radius of the cylinder, m
Ra*	Rayleigh number based on L_c
r	radial coordinate
T	fluid temperature, K
Tw	cylinder surface temperature, K
Tb	bulk temperature, K
t	time, sec
V_X, V_Y, V_Z	velocity components, m/s

Greek Symbols:

B	thermal expansion coefficient, 1/K
α	thermal diffusivity, m^2/s
ν	kinematic viscosity, m^2/s
ρ	density, kg/m^3
μ	dynamic viscosity, kg/ms

1. INTRODUCTION

Heat transfer by natural convection is involved in many practical situations as it has many advantages. Some of these advantages are less noise, less coast, and it does not need maintenance as it does not involve moving parts. Among the practical situations in which natural convection is involved, is natural convection from heated cylinders. Heat transfer by natural convection from cylinders is found in many practical situations such as using hollow cylinders in medical operations where it is placed around light sources. In these cases, the cylinder is heated up and then cooled by natural convection in ambient air. Natural convection from cylinders is found also in steel industries where vertical cylinders are sometimes suspended in air for immediate cooling. One of the most important practical situations in which natural convection from cylinders are encountered is in research reactors where the process of radioisotope production is performed by placing the targets to be irradiated in a cylindrical can, and heat is generated from these targets due to irradiation. Cooling of this can is usually achieved by forced convection of water. Safety analysis of these irradiation processes requires that the calculation is done in a conservative way, assuming heat removal by natural convection in stagnant water. Many researches on heat transfer by natural convection from cylinders have been conducted. Swastik Acharya et.al [1] studied numerically heat transfer by natural convection from a vertical hollow cylinder suspended in ambient air by varying the Rayleigh number (Ra) in the laminar region ($10^4 \leq Ra \leq 10^8$). They studied the effect of the ratio of length to pipe diameter (L/D) by varying it in the range $0.05 \leq L/D \leq 20$. They demonstrated that in all the cases of L/D, the average Nu for both the outer and the inner surface of the cylinder increases as the Ra increases. The Nusselt number (Nu) of the outer surface of a hollow cylinder exactly matches with that of the solid cylinder for all L/D when Ra is less than 10^6 but for a Ra beyond 10^6 the Nu of the outer surface of the hollow cylinder is marginally higher than that of the solid cylinder. Churchill and Chu [2] and Churchill [3], experimentally

studied the natural convection from a horizontal cylinder and vertical flat plate respectively. They reported a correlation for average Nu as a function of Prandtl number (Pr) and Ra. The effect of curvature and the criteria for a vertical cylinder to be treated as a vertical flat plate have been studied by some researchers as given by Gebhart et al.[4] and explained in Ozisik [5] and Holman [6]. LeFevre and Ede [7] proposed a solution that accounts for the effect of wall curvature in the laminar range. Fujji and Uehara [8] compared the heat transfer rate for the case of laminar natural convection from a vertical cylinder with that from a vertical flat plate. Acharya and Dash [9,10] studied numerically the natural convection heat transfer from an isothermal horizontal hollow cylinder with and without perforation. They showed the flow and thermal contours around the cylinder and proposed a correlation for the value of Nu. In addition to numerical studies, also many experimental studies have been performed during the last three decades and interesting results have been presented. L. Davidson et.al [11] studied the natural convection phenomenon in vertical shell and tube. They studied the effect of different inlet conditions and geometrical dimensions on the developed thermal and velocity boundary layers. They showed that the larger the inlet velocity, the larger the Nu especially near the transition region where the variation in Nu is large. Whereas, this variation in Nu gradually vanishes in the fully turbulent region. Crane [12] discussed how, high Pr affect free convection through vertical cylinder. C.O.Popiel [13] studied the effect of curvature of the cylinder boundary layer thickness when it is comparable with the diameter of the cylinder. Modified integral method is used for boundary layer calculations. Shiri et.al [14] studied experimentally the natural convection in the near wall region of vertical cylinder. They measured the mean and turbulence quantities in the near wall region, where there is considerable variation of thermal properties due to high temperature gradient there. A new set of boundary layer equations are established to represent the variable properties of the flow in this region. In this paper natural convection heat transfer from a cylinder suspended in

water at different tilting angles, from the vertical position, has been analyzed numerically by varying the Ra in laminar regime ($10^4 \leq$ Ra $\leq 10^8$). The cylinder under study is a solid cylinder of 2.5 cm diameter and 8 cm length with a constant heat generation of 250000 W. This cylinder simulates the can used for isotope production in ETRR-2, whereas different targets are placed inside this can during irradiation.

2. Problem description

Fig. (1.a) shows isometric view of the studied geometry where half of the solid cylinder surrounded by stagnant water is simulated because of symmetry of the problem, while Fig (1.b) shows 2D view of the computational domain with boundary conditions to be used in the numerical computations. The cylinder under study is of 2.5 cm in diameter and 8 cm in length. Ansys Fluent release 15 computer code is used for carrying out the numerical simulation of the problem. The dimensions shown in Fig (1.b) are derived by carrying out domain independent tests which show that the most effective dimension is the distance between the top of the cylinder and top end of the computational domain as reported by Swastik Acharya et.al [1]. Whereas the distance between the bottom of the cylinder and the bottom of the domain and the distance from the side surfaces of the cylinder to the side surfaces of the domain do not influence the result at all.
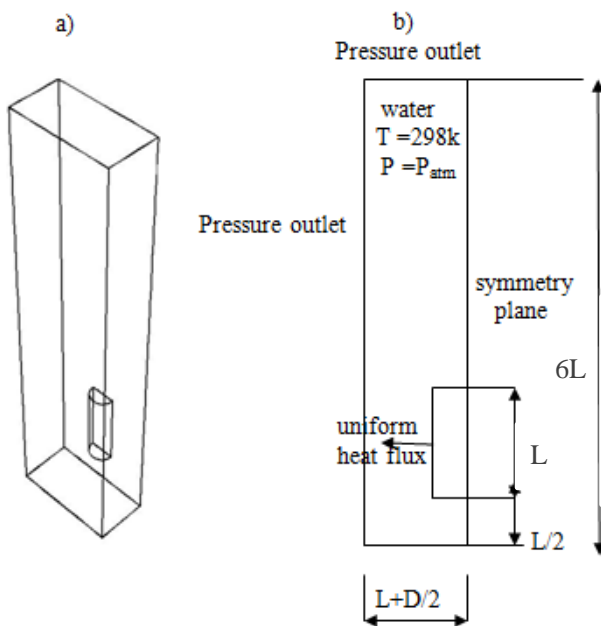

Fig. (1). a) 3D view of cylinder suspended in stagnant water

b) Dimensions of the computational domain

3. METHODOLOGY

3D steady laminar flow around cylinder in stagnant water is simulated at different tilting angles from the vertical position. The mathematical model for this simulation is shown below:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad \text{(continuity)}$$

$$w\frac{\partial u}{\partial z} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

(x momentum)

$$w\frac{\partial w}{\partial z} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{\mu}{\rho}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) + g\beta(T - T_o)$$

(y momentum)

$$w\frac{\partial v}{\partial z} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

(z momentum)

$$\rho C_p\left(w\frac{\partial T}{\partial z} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + Q$$

(energy)

Boussinesq approximation is included in the Y momentum equation to account for the natural convection. In the term of buoyancy, the reference temperature corresponds to the bulk fluid temperature and is fixed at 298 K. Hence temperature difference between the wall and bulk fluid is relatively small throughout all of the runs and the Boussinesq approximation remains valid. With respect to the boundary conditions, constant heat generation of 250000 W/m^3 is applied inside the can throughout all of the runs. Constant value for the pressure corresponds to atmospheric pressure value is applied at all of the water boundary except the right plane shown in Fig.1 which is the symmetry plane. The set of differential equations are discretized using the finite volume technique to obtain a set of algebraic equations. The resulting equations are solved numerically using Ansys-Fluent version 15. The number of elements used for numerical analysis is 126600. Greater number of elements does not affect the solution. For each value of Ra, and tilting angle, the temperature and velocity contours as well as the average value of Nu are deduced.

Nu = hL/k

Q= hA(T$_w$-T$_b$)

A = 2Πdl

T$_w$ is calculated by Ansys Fluent computer code.

4. VALIDATION

Figure 2 shows the variation of the average Nu with Ra for the case of vertical cylinder. The results that were obtained from the present study are compared with the correlation of Churchil [3] which is:

$$Nu = F[0.825 + 0.387(GrPr)^{1/6}/\{1+(0.492/Pr)^{9/16}\}^{8/27}]^2$$

In this correlation F is a correction factor accounts for the curvature effect since the correlation of Churchil is for flat plat and can be used for the case of cylinder in provided that D/L $\geq 35/(Gr^{1/4})$. The correction factor F was proposed by Minkowycz and

Sparrow [15] for gases with Prandtl number Pr=0.7 , where

$$F = 1.3 \left[\frac{\frac{L}{D}}{Gr_D}\right]^{0.25} +1$$

Figure 2 shows that the results obtained from the present study are in a good agreement with the correlation of Churchil. From this comparison for the case of vertical cylinder, we can proceed to simulate numerically the case of natural convection at different angles with respect to the vertical position of the cylinder and with the same grid size and domain dimensions.
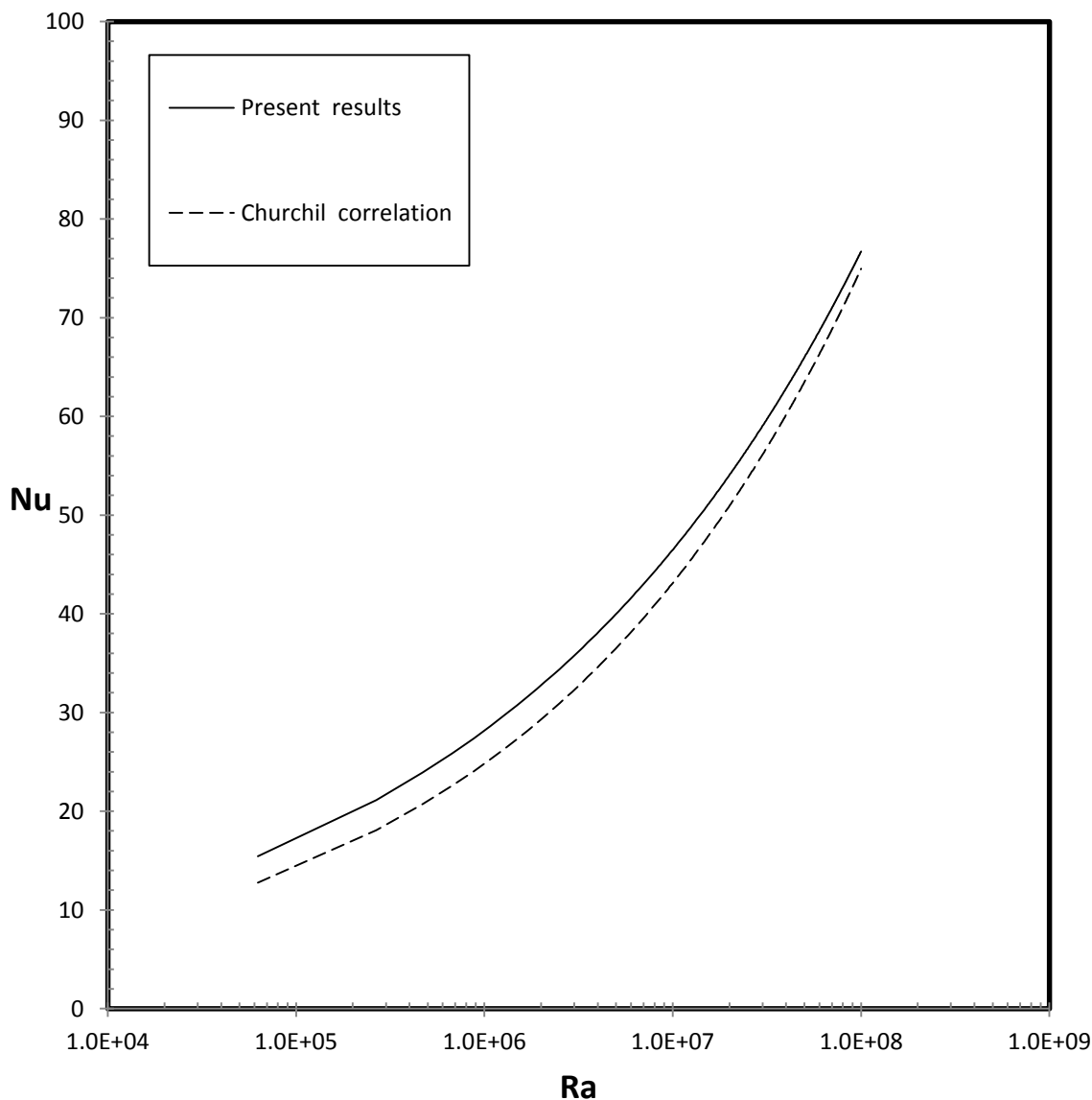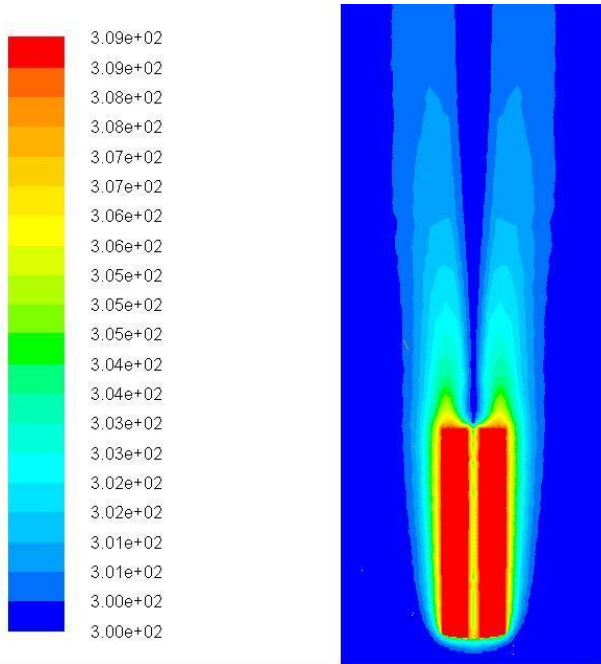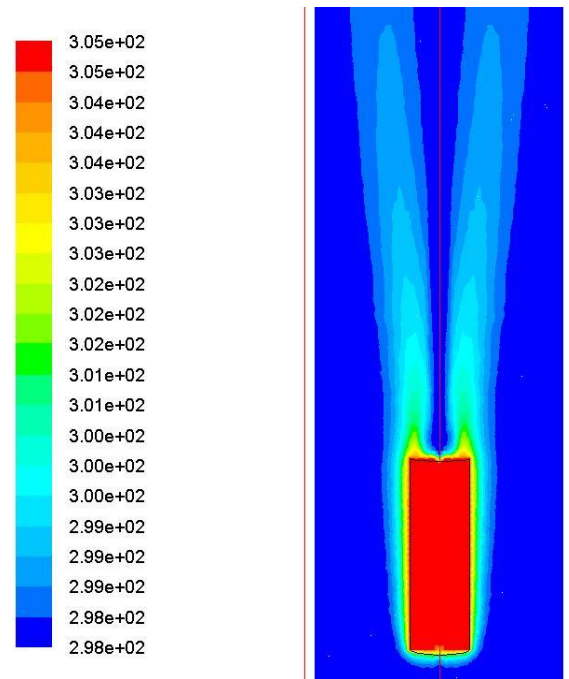

Fig. (2): variation of the average Nu with Ra for the case of vertical cylinder

5. RESULTS AND DISCUSSION

5.1 Effect of Ra

Ra = 6x10^4
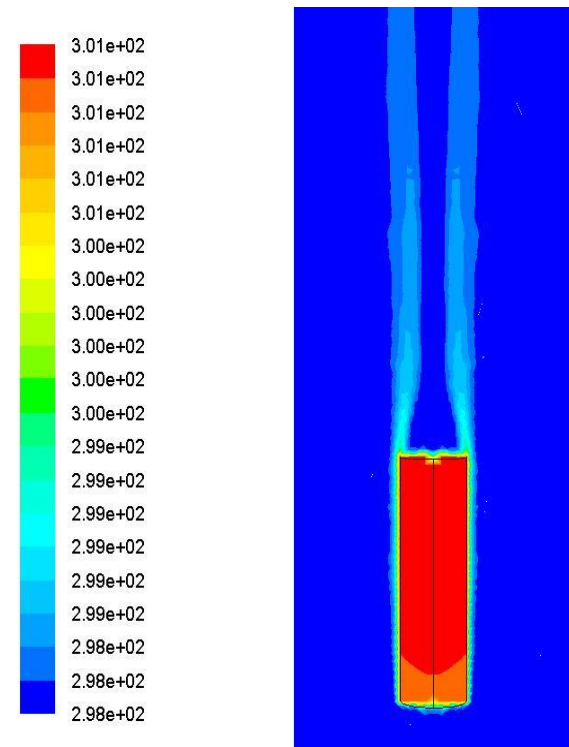

Ra = 5x10^5

Ra = 4x10^6
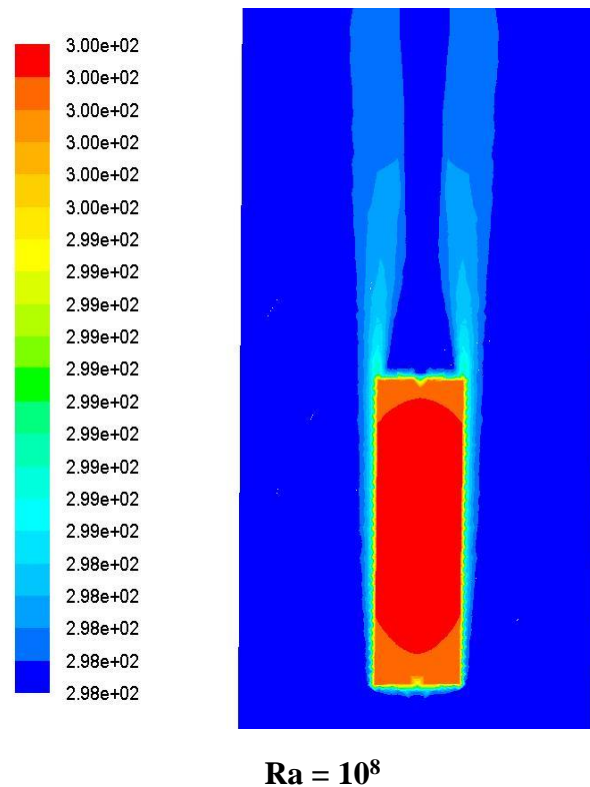

Ra = 3x10^7

$$Ra = 10^8$$
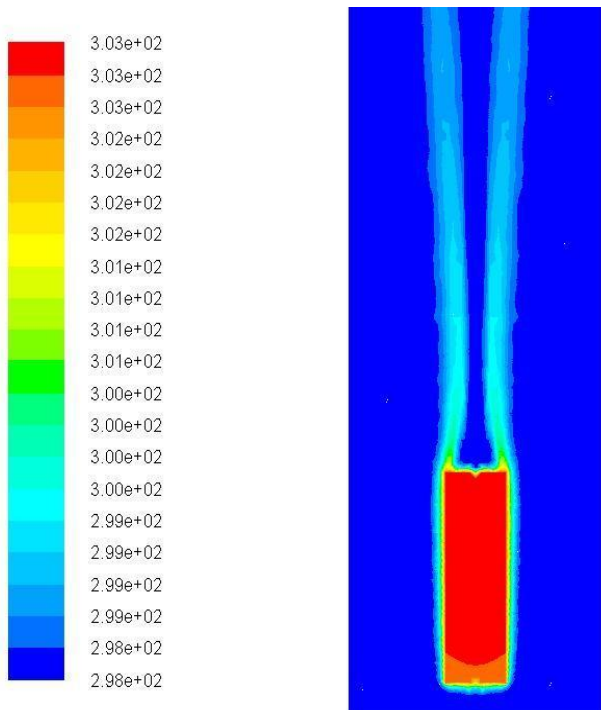
Fig. (3): Temperature contours at different Ra for the case of vertical position

Figure 3 shows the temperature contours at various Ra for the case of the cylinder being at vertical position. From this figure it can be seen that by increasing Ra the thermal plume (region of high temperature) around the cylinder becomes thinner. This can be attributed to the fact that at Ra becomes lower the thermal boundary layer becomes thicker, and hence the thermal plume around the cylinder becomes thicker. As Ra becomes higher the flow around the cylinder becomes higher and the thermal plume becomes thinner. Fig.4 shows the flow field around the vertical cylinder at various Ra ranging from $6x10^4$ to 10^8. As seen from this figure, the fluid nearby the side of the cylinder rises towards upward direction. There are two wake regions: the first is directly around the bottom of the cylinder, where the flow coming from the colder fluid region and travelling downward to replace the hotter fluid adjacent to the cylinder at its lower part, changing direction from downward to upward as it becomes hotter. Whereas the second wake region is directly at the top of the cylinder, as the rising fluid leaving the side of the cylinder at its top end becomes closer and the velocity increases as the flow area decreases, hence a wake region is generated directly above the center region of the top of the cylinder. The diameter of the plume at maximum velocity region is high for low Ra and becomes smaller as Ra decreases. The value of the maximum velocity at Ra = 10^8 is about 24 times the value at Ra = $6x10^4$. Figure 5 shows the effect of Ra on average Nu for various values of tilting angle. From this figure it can be shown that average Nu increases with the increase in Ra, and this increase is more pronounced at higher values of Ra for all of the cases of tilting angle. By varying the tilting angle from vertical position, the average Nu increases until the tilting angle is 45° (half the way between vertical and horizontal position, then by increasing the tilting angle furthermore the average Nu starts to decrease.
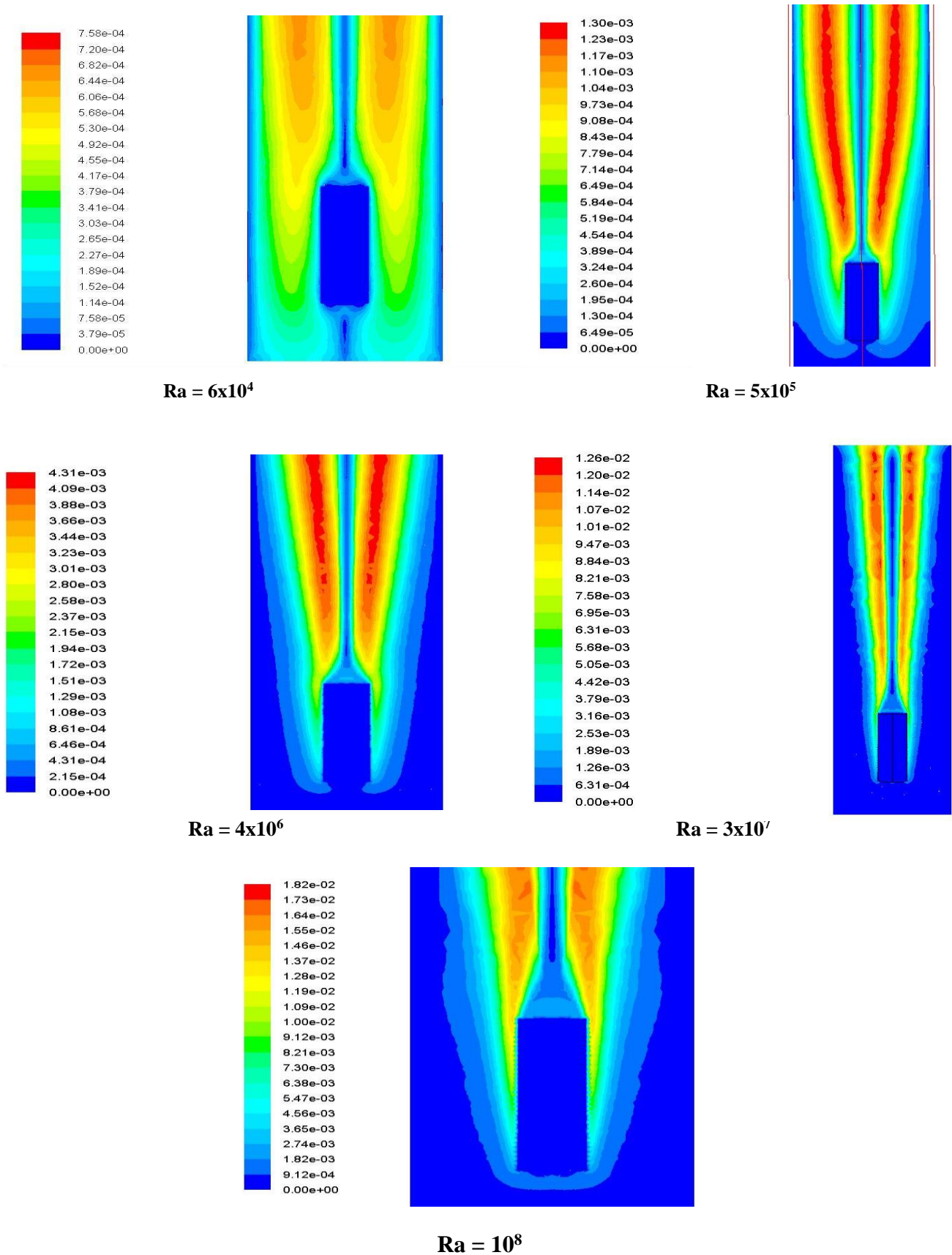
Ra = 6x10⁴

Ra = 5x10⁵

Ra = 4x10⁶

Ra = 3x10⁷

Ra = 10⁸

Fig. (4): Velocity contours at different Ra for the case of vertical position
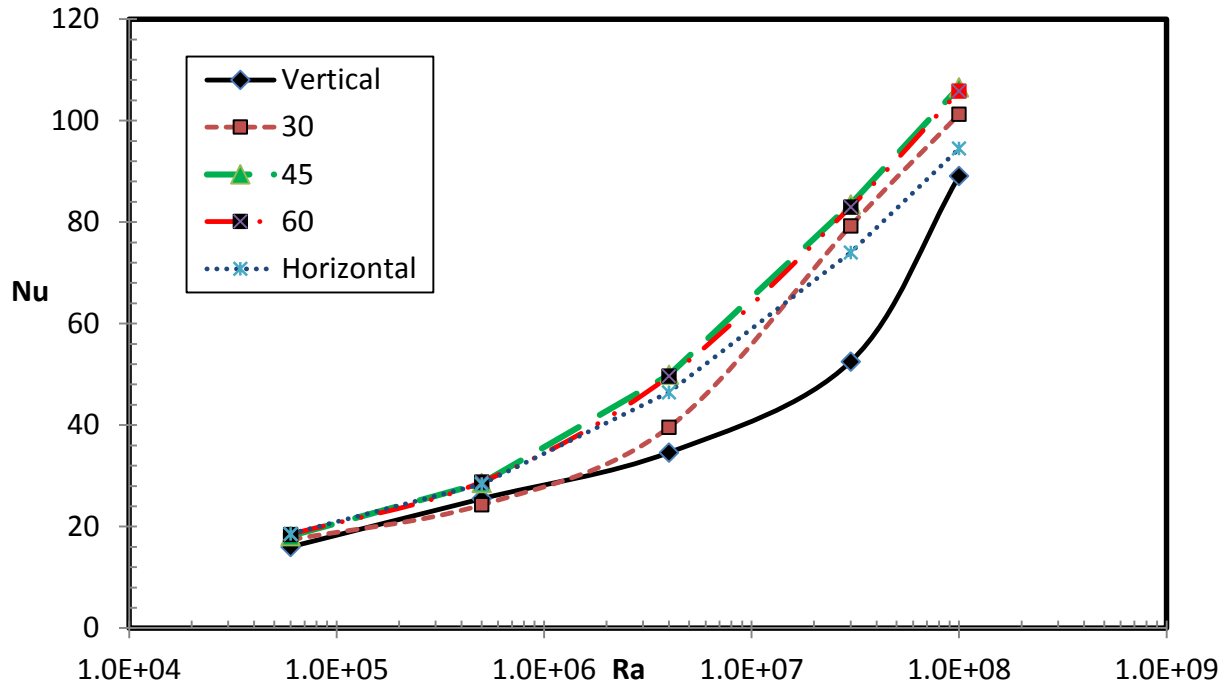
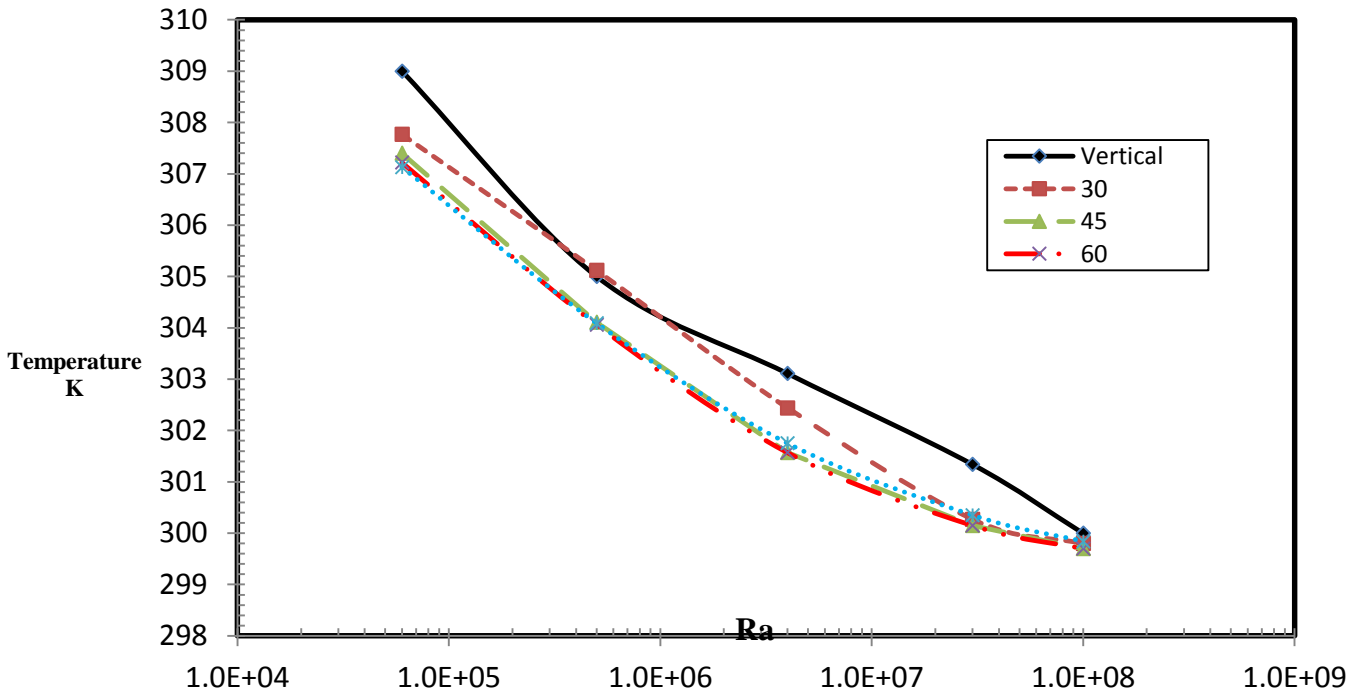**Fig. (5): Variation of average Nu with Ra at different tilting angles**

Fig. (6): Variation of average surface temperature with Ra at different tilting angles

Figure 6 shows the variation of average surface temperature of the cylinder with Ra at different tilting angles. As shown in this figure, the average surface temperature decreases with the increase in Ra for all of the cases of tilting angle. As the tilting angle varies from the vertical position, the average surface temperature shows reverse trend w.r.t the trend of Nu which has been

previously discussed, i.e the temperature decreases until the tilting angle is 45°. Further increase of the tilting angle after 45° seems to have no significant effect on the average surface temperature, but the decrease in Nu which occurs at greater tilting angle than 45° results from the fact that the surface heat flux decreases for tilting angles greater than 45°.

5.2 Effect of tilting angle

Vertical

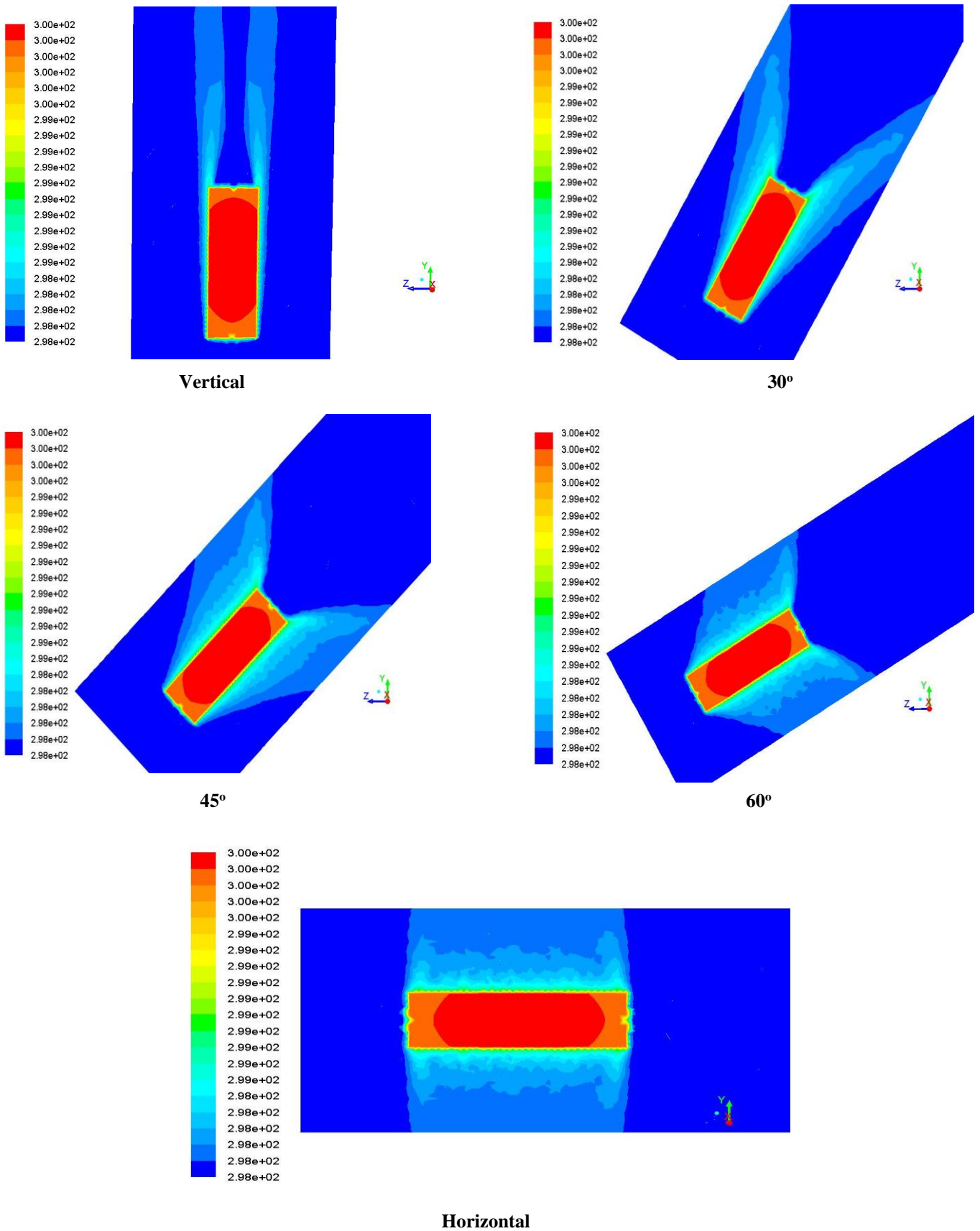**30º**

45º

60º

Horizontal

Fig. (7): temperature contours at different tilting angles for the case of Ra = 10^8

Figure 7 shows the temperature contours at different tilting angles for the case of Ra = 10^8. From this figure it can be shown that in all cases the maximum temperature region is at the center of the cylinder surface and the temperature becomes lower towards the two ends of the cylinder. In the water region, the temperature contour has two branches starting from the two sides of the cylinder. These two branches are closed to each other for the case of vertical position forming a V shape and the divergence of this V shape increases as the tilting angle increases and finally the two branches become completely separate for the case of horizontal position. The temperature values differ slightly with the change in the tilting angle for the same Ra and heat generated, whereas the temperature strongly depends on Ra as discussed before.
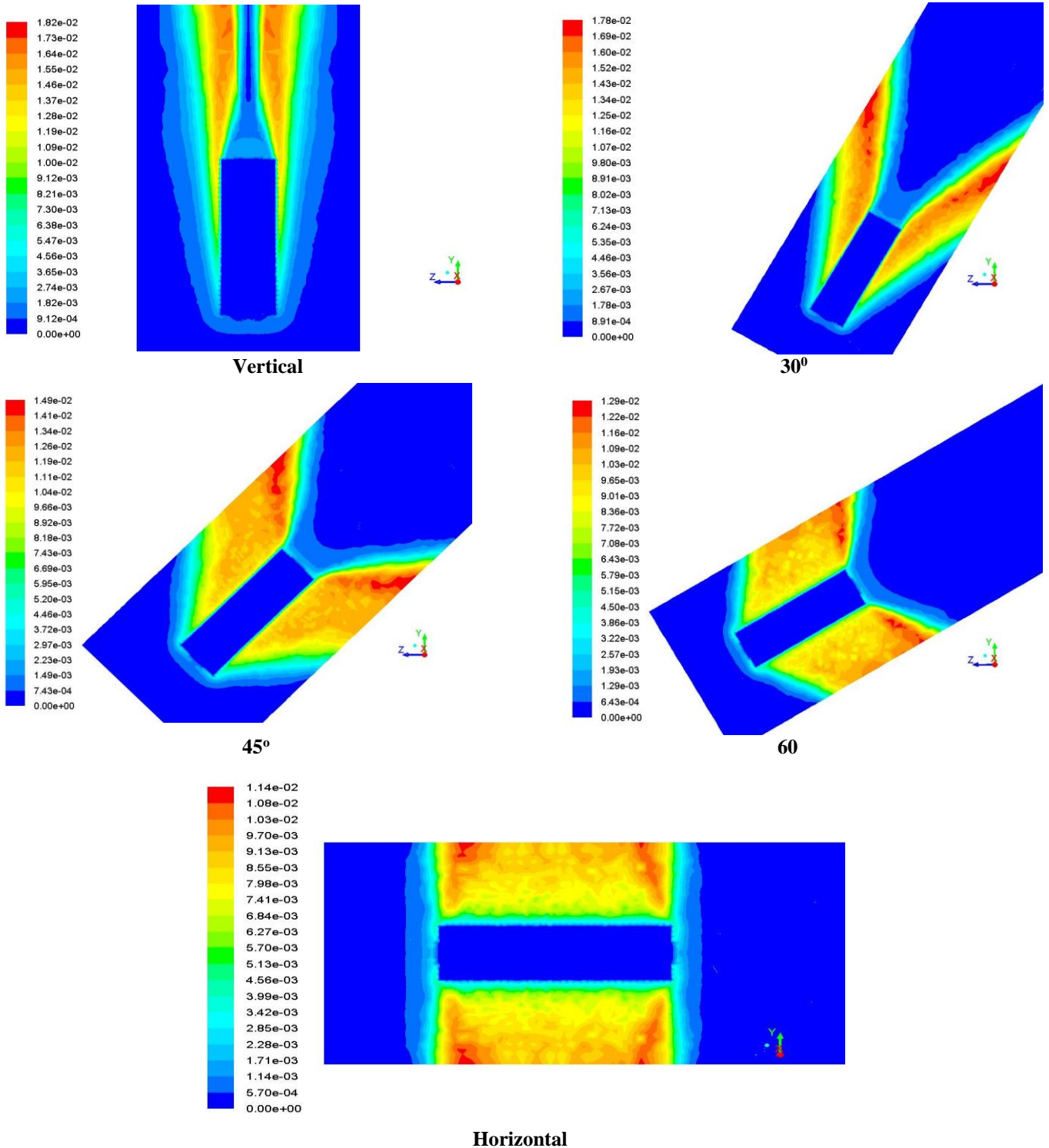

Vertical

30⁰

45⁰

60

Horizontal

Fig. (8): Velocity contours at different tilting angles for the case of Ra = 10^8

Figure 8 shows the flow field (velocity contours) around the cylinder at different tilting angle ranging from 0° (which corresponds to vertical position) to 90° (which corresponds to horizontal position) for the case of Ra = 10^8. From this figure it can be shown that for all cases flow separation occurs due to the stagnation which occurs at the lower part of the cylinder body. As for the two wake regions which are previously discussed in Fig.4, the one which is directly around the bottom of the cylinder still exists. After leaving the cylinder at its top end, the two branches of the fluid leaving the two sides of the cylinder comes closer for the case of vertical position. Whereas by tilting the cylinder from vertical position, these two branches diverges, and this divergence increase with the increase in tilting angle from the vertical position, and the maximum value of the flow velocity decreases as well.

Figure 9 shows the variation of Nu with tilting angle at different values of Ra. As shown from the figure, Nu increase with Ra at the same tilting angle as mentioned before. For a certain value of Ra, the Nu increases with the increase in tilting angle until a value of 45° and then starts to decrease. But this trend is more pronounced for higher values of Ra. Fig. 10 shows the variation of average surface temperature with tilting angle at different values of Ra. Nearly a reverse trend to the trend of Nu is shown in this figure. I.e the average surface temperature decreases with the increase in Ra at the same tilting angle. For the same Ra, the temperature decreases by increasing the tilting angle until reaching 45° and then it starts to increase by further increase in the tilting angle. The increase in the average surface temperature for tilting angles greater than 45° is more pronounced for higher Ra. For Ra values of E4, E5, the temperature is nearly constant for tilting angles greater than 45°.
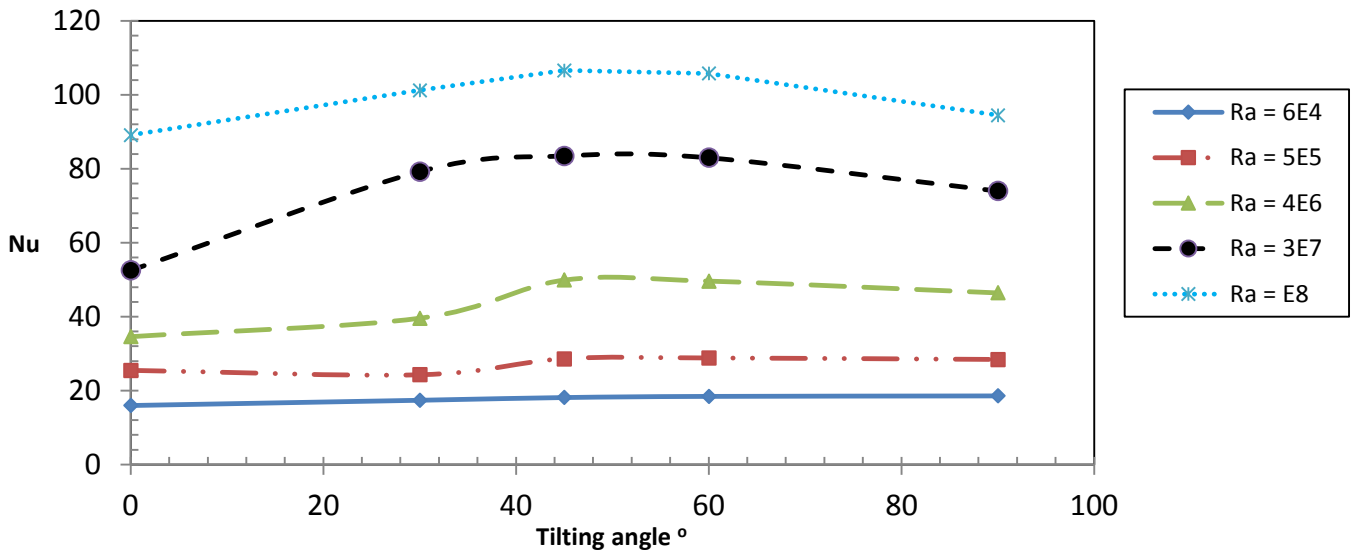

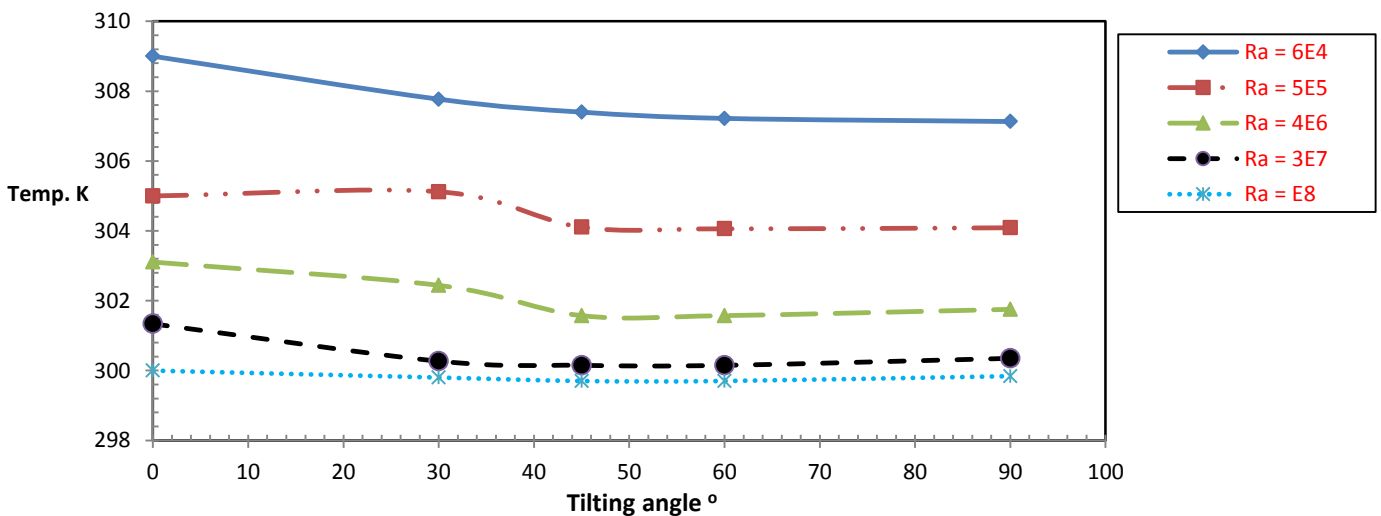**Fig. (9): Variation of Nu with tilting angles at different Ra**

Fig. (10): Variation of average surface temperature with tilting angle at different Ra

5.3 Development of a single correlation for all inclination angles

A single correlation relating Nu with Ra irrespective of the inclination angle can be developed using the characteristic length proposed by Neetu Rani et al. [16]. This characteristic length is:

$$L_c = \left[\frac{LD}{L/D\cos\theta + D/L\sin\theta}\right]^{1/2}$$

Where: θ is the angel from the horizontal position.

All of the data relating Nu with Ra at different inclination angle were fitted as shown in Fig.11 and a correlation in the form Nu = a Ra^b was tested. The best fit equation for the data trend line was:
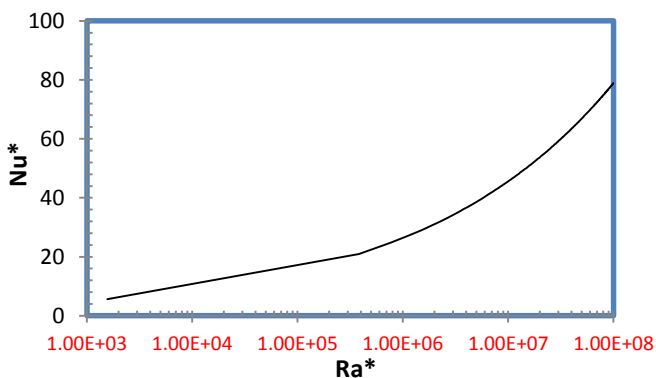
Nu = 0.978 Ra$^{0.238}$

Fig. (11): Best fit curve for Nu* with Ra* at any tilting angle

6. CONCLUSIONS

Analytical analysis using Ansys 15.0- Fluent computer code.of natural convection heat transfer from a cylinder suspended in water at different tilting angle has been analyzed numerically by varying the Ra in laminar regime ($10^4 \leq$ Ra $\leq 10^8$). The numerical analysis is carried out at different tilting angles in the range from 0º to 90º from vertical position. The results obtained can be summarized as follow:

1- By increasing Ra, the average Nu as well as flow velocity increase while the average surface temperature decreases and the thermal and velocity plume become thinner for all cases.

2- By increasing the tilting angle from the vertical position, the average Nu increases whereas the average surface temperature decreases until a tilting angle of 45º. Further increase in the tilting angle results in decrease in the average Nu, whereas average surface temperature increases for high values of Ra, while it nearly remains constant for relatively low values of Ra. Whereas the thermal and velocity plumes have two branches of V shape which diverge by increasing the tilting angle.

3- The data relating Nu with Ra at any inclination angel can be correlated with a single correlation using a previously proposed characteristic length which depends on the inclination angle.

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