



Enhancement of Power Harvesting by Tuning Moment of Inertia

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ABSTRACT

This investigation examines the design of an innovative independent mechanism for tuning the resonant frequency and thus generate greater amounts of energy. The proposed concept is based on adjusting the resonant frequency by automatically modifying the mass moment of inertia. A frequency tuning algorithm is used to maintain the system in the case of resonant continuously, and that by change the proof mass position within the tuning range. The electromagnetic energy harvester is used to convert the vibration energy into electrical energy. The proposed design is analyzed using the methodology of approximate single degree of freedom system to study the effect of the mass position on Eigen-frequency and also to determine the response of the system as well as the harvested power. Analytical results were confirmed and verified to high accuracy with laboratory results. It is worth mentioning that this proposed design is suitable for low frequencies and can be considered as a good step towards the production of autonomous energy harvesters.

Keywords: Vibration energy harvester, Tuning, Resonant, Electromagnetic

1. INTRODUCTION

Recently, with the urgent need for renewable and clean energy sources, the emphasis has been on extracting and generating energy from the surrounding environment. Ambient energy can be in the form of light, heat or mechanical kinetic energy. Harvesting mechanical kinetic energy is an attractive field for easy access to energy from multiple sources, whether from the movement of air or liquids or vibration of mechanical elements or structures. The kinetic energy can be transformed out of the mechanical domain to the electrical domain [1, 2]. In general, there are three popular ways to convert mechanical energy into electrical energy: electrostatic[3-6], electromagnetic[7], and piezoelectric harvesters [8, 9].

The idealistic energy harvester from the vibratory system consists of a mechanical system subjected to external excitation, mechanical mechanisms which are used to transport and amplify the vibratory motion, the power adapter that transforms vibration motion into electrical energy, energy storage device, and control systems for automated harvesting of energy [10].

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The resonant systems prove their worth in the energy harvesting field, where the maximum output power is achieved when the ambient frequency is similar to the natural frequency of the mechanism. It is noticeable that moving away from the resonance zone leads to a significant reduction in the output power. Therefore, there are two ways to overcome this problem: widening the bandwidth of the operating frequency [11-15] and tuning the resonant frequency.

The resonance frequency can be modified using mechanical or electrical methods. Electrical tuning methods usually depend on modifying the electrical loads of the harvester for tuning the resonance frequency. Most of the researches in electrical tuning used piezoelectric generators[16-19], and this due to easy modification of capacitive loads unlike load inductance which is difficult to change and resistive loads which cause low efficiency of electrical energy transferred.

The mechanical tuning technique involves modifying the dimensions of the structure [20, 21], the center of gravity of proof mass[22-24], and spring stiffness continuously or intermittently.

Wu et al [24] has adjusted the natural frequency of piezoelectric energy harvester by using design which consists of a piezoelectric cantilever beam and an effective mass which contains two parts, the first one is fixed and manufactured from a small density material (eg. aluminum) whilst the other is a movable steel stud which is manually moved to change the centroid mechanism position. The resonant frequency of the proposed mechanism was from 130 to 180 Hz with tunability 32.26%. The drawback of this approach is that it ignored the influence of the shape and dimensions of

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the proof mass during analyzing the proposed model, and these parameters have been considered ineffective. Schaufuss et al.[23] developed this design by using a lever mechanism instead of the traditional mass and spring system. It has been used two prototypes of energy harvesters to verify this proposed, one of them was an electrostatic generator and the other was a piezoelectric generator. The position of the proof mass was modified manually by a screw. The mechanism natural frequency can be tuned in the range from 42 to 55 Hz with the tunability of about 26.8% and sensitivity 0.26 Hz/mm.

The variety of researches was interested in tuning the natural frequency tuning by changing the spring stiffness. The basic concept in most of these studies adopted the application of negative spring and this by applying a tuning force. A lot of various methodologies have been used to obtain this tuning force which included electrostatic[25-27], piezoelectric[16-18], and electromagnetic methods[19]. Several studies have also addressed mechanical [28-30]and thermal methods[31, 32].

It was found from the review that several researchers, in the field of tuning the natural frequency, were interested in changing the mechanism stiffness.

This article is therefore concerned with researching an innovative technique for adjusting the resonance frequency of the system by studying the influence of changing the mass moment of inertia. The study also focused on designing a control system to ensure continuity in the resonance state permanently, which leads to harvest the largest amount of energy through electromagnetic harvester.

2. DESCRIPTION OF THE SYSTEM

The proposed energy harvester is based on a vibrating cantilever beam with a tuning mechanism that is mounted on its free end to reach the resonance state as illustrated in Figure 1. The concept of the proposed design depends on tuning the resonance frequency by adjusting the moment of inertia of the tuning mechanism by changing the position of the proof mass automatically and thus, the natural frequency of the tuning mechanism matches with the ambient frequency whatever the change in frequency.



tuning mechanism

The tuning mechanism consists of a movable tuning mass on a lead screw which derives its motion from a DC motor. Two guide shafts fixed on the acrylic frame are used to convert a rotational motion of the motor into a linear motion for the proof mass. When the position of the proof mass is changed, the natural frequency of the system will change accordingly. The resonance state can be maintained by changing the position of the mass so that the natural frequency of the system is matched with the ambient frequency. To keep the system continuously in the resonance zone, the automatic tuning technique is used in which the external frequency is measured periodically and the position of the mass is adjusted accordingly so that the natural frequency of the system matches with the ambient frequency whatever change. When maintaining the resonance state, the highest response is achieved and then mechanical energy can be converted into electrical energy.

In the proposed design, electromagnetic energy harvester is used by attaching a permanent magnet (NdFeB N42) at the free end of the mechanism and by placing a coil on a separate holder in front of this magnet. The coil is connected to a decade resistance box and oscilloscope (Tektronix TDS2024C) which were utilized to measure the output voltage from this energy harvester.

3. DYNAMIC MODEL

The proposed design which illustrated in the previous section is analyzed using the methodology of approximate single degree of freedom system, and it is solved with MATLAB program. The proposed mechanism can be reduced to an equivalent mass (M_e) related to the vibrator base by spring (K) and damper (C) [33]. This mass moves with a displacement x(t) while the vibrating base moves with a displacement y(t) as shown in Figure 2.



Figure 2: Equivalent system of the proposed energy harvester

The equation of motion of this system can be represented by the following equation:

$$\mathbf{M}_{\mathbf{e}}\ddot{\mathbf{x}} + \mathbf{C}(\dot{\mathbf{x}} - \dot{\mathbf{y}}) + \mathbf{k}(\mathbf{x} - \mathbf{y}) = \mathbf{F}_{\mathbf{m}}$$
(1)

The cantilever effective stiffness (k) can be determined as:

$$K = \frac{3EI}{L^3} \tag{2}$$

Where (E) is the cantilever modulus of elasticity, (I) is the cantilever second moment of area, and (L) is the length of the cantilever. The effective mass of the system (M_e) can also express as the aggregate of the total mass of the tuning mechanism (M_t) attached in the tip of the cantilever beam, the effective cantilever beam mass (m_c), and the mass moment of inertia of the tuning mechanism (J) as illustrated in the following equation.

$$M_{e} = M_{t} + 0.24 m_{c} + \frac{3J}{L^{2}}$$
(3)

$$M_t = m + 2m_g + m_s + m_{mot} + m_a$$
 (4)

$$J = J_{c} + J_{mot} + J_{m} + 2J_{g} + J_{s} + J_{a}$$
(5)

Where (m) is the movable tuning mass, and $(m_g, m_s, m_{mot}, and m_a)$ represent the mass of guide shafts, screw, DC motor, and acrylic frame respectively. Also each of $(J_c, J_{mot}, J_m, J_g, J_s, and J_a)$ are the mass moment of inertia of the cantilever beam, DC motor, movable tuning mass, guide shafts, screw, and acrylic frame respectively.

As the base motion is varied harmonically with the ambient excitation frequency (Ω) and amplitude (y), the effective mass response is also changing harmonically as expressed in equations (5) and (6) where $(x \text{ and } \phi)$ denote the phase angle and the maximum amplitude of the effective mass.

$$Y(t) = y \sin \Omega t \tag{5}$$

$$X(t) = x \sin(\Omega t + \phi) \tag{6}$$

The equation of motion of this system is solved by substituting Equations (5) and (6) into Equation (1) to obtain the general solution of the maximum amplitude of the system response in the following form:

$$\frac{x}{y} = \sqrt{\frac{k^2 + (C \,\Omega)^2}{(\Omega C_t)^2 + (k - M_e \Omega^2)^2}} \tag{7}$$

The relative oscillations cause a variation in the magnetic flux and this creates a voltage or an electromotive force (V_{EMF}) in the coil and this voltage is proportional to the time-varying magnetic flux according to Faraday's law of induction [2, 7, 34-36].

$$V_{EMF} = -BL_w \dot{x} \tag{8}$$

Where (B) is the magnetic field strength, (L_w) is the total length of the coil wire. This magnetic field will

generate a magnetic force (F_m) which has a reverse direction to the magnet movement direction.

$$F_m = BL_w i(t) \tag{9}$$

This electromagnetic force is proportional to the magnet velocity and it can be expressed also as the product of velocity with the electromagnetic damping (C_m)

$$F_m = C_m \dot{x} \tag{10}$$

The induced current passing through the coil circuit can be written as:

$$i(t) = \frac{V_{EMF}}{R_1 + R_c + j\Omega L_c}$$
(11)

Where (R_1) is a load resistance, (R_c) is the coil resistance, and (L_c) is the inductance of the coil. At the low frequencies (less than 1 kHz) [2], the coil inductance term can be neglected and the coil resistance dominates its impedance. From equations (9), (10), and (11) the electromagnetic damping can be given from the following equation:

$$C_m = \frac{(BL_w)^2}{R_l + R_c} \tag{12}$$

and the harvested power can be calculated as

$$P = R_l i^2(t) = \frac{[BL_w \dot{x}]^2 R_l}{(R_l + R_c)^2}$$
(13)

The main parameters and material properties of the cantilever beam, tuning mechanism, and Electromagnetic Generator which are used in the proposed design are listed in Table 1. A MATLAB program is used to solve this analytical model to find the natural frequency at the different mass position and also to calculate the response of the system and the harvested power.

 Table 1: Main parameters of the proposed design

Parameter	Value
The density of cantilever (ρ)	7850 Kg/m ³
Cantilever modulus of elasticity (E)	213 GPa
Cantilever length (L)	0.06 m
Movable tuning mass (m)	0.118 Kg
Mass of guide shafts (mg)	0.049 Kg
Mass of screw (m _s)	0.056 Kg
DC motor mass (m _m)	0.06 Kg
Width of movable mass (b)	0.0095 m
Screw length (L _s)	0.32 m
Guide shafts length (Lg)	0.3 m
Magnetic field strength (B)	1.3 Tesla
The inner diameter of a coil	40 mm
The outer diameter of a coil	50 mm
Height of a coil	15 mm

4. EXPERIMENTAL PROCEDURES

As a first step, it is necessary to find the natural frequency value of the energy harvester at the different positions of the tuning mass. The proof mass position (L_m) is measured from the free end of the cantilever beam every 10 mm from L_m = 40 mm to L_m = 280 mm. The natural frequency is measured at each position using the OptoNCDT1402 laser sensor by applying a simple motion to the free end of the cantilever beam allowing it to move with its natural frequency and measuring it. Whereas the laser sensor depends on measuring a period in the time-domain and the signal is in the form of a sinusoidal excitation, the natural frequency can be calculated by finding the inverse of the time between two consecutive cycles. Figure 3 demonstrates the flow diagram of the sequences which is used to determine the natural frequency at each mass distance. After that its necessary to plot the relationship between the mass distance and the natural frequency at each distance from the free end of the cantilever, and find the trend line equation.



Figure 3: The flow diagram of the sequences used to determine the natural frequency versus tuning mass position

Figure 4 shows the experimental set-up and the components of the electromagnetic energy harvester with the proposed tuning mechanism. A cantilever beam with length 60 mm, width 28 mm, and thickness 1 mm beam is mounted on a shaker (Brüel & Kjær type 3386-062) to obtain and control the ambient excitation frequency whilst the tuning mechanism is attached with the other free end. The shaker and the cantilever beam are placed so that the vibration movement is in the horizontal plane to avoid the weight effect of the cantilever beam and the tuning mechanism components during the vibration motion.



Figure 4: Experimental set-up of the autonomous electromagnetic energy harvester

To automatically adjust the frequency, a slotted strip is fixed to the cantilever beam end and a photoelectric infrared radiation count encoder sensor is placed underneath it. The sensor is connected to the Arduino UNO board as input to count the total number of complete cycles per a specified periodic time to calculate the external excitation frequency. The DC motor is also connected to the Arduino UNO board as output and is initially calibrated to control it. A limit switch is added to determine the zero position of the tuning mass. The Arduino board is connected to the computer and supplied with a 6-volt using an adapter.



Figure 5: The procedure used to control the tuning mechanism

The tuning procedure is illustrated in detail in Figure 5. Initially, when the system is turned on the tuning mass will return until the touch the limit switch to specify the mass zero position. After that, the external excitation frequency is calculated by counting the number of full cycles at a specified time using a photoelectric infrared radiation count encoder sensor through the Arduino program code. The program then compares the excitation frequency value to the pre-programmed natural frequency limit. If the external excitation frequency is not within the tuning range, the external frequency is recalculated after a specified time. When the excitation frequency is within the tuning range, the program finds the appropriate position of the movable tuning mass through the equation between the natural frequency and the position of the tuning mass. The motor is switched on until the mass reaches the appropriate position and the external frequency is calculated again after a specified time to maintain the system in the case resonant continuously.

5. RESULTS AND DISCUSSION

While analyzing the effect of changing the mass moment of inertia through the analytical model of the proposed design, it was clear that once the mass moment of inertia changed from 0.0105 to 0.0232 Kg/m2, there was a considerable variation in the system natural frequency from 2.62 to 3.84 Hz. Figure 6 indicates that the natural frequency of the system is inversely proportional to the mass moment of inertia that can be changed by modifying one of the studied parameters, such as the effective mass value, position, shape, or thickness.



Since the current design idea is based on automatic resonance adjustment, the most suitable solution is to change the effective mass position to adjust the natural frequency and then continuously attain the resonance state. Figure 7 illustrates the effect of changing the position of the proof mass from 0.04 to 0.28 m on the natural frequency as a significant parameter. Upon verification of the theoretical findings with those of the experimental work, considerable compatibility has emerged. Where the natural frequency was tuned practically from 2.65 to 3.75 Hz with tunability 38.4%, whilst the tunability was 38.02% theoretically.



The behavior of the system was studied at the mass position 0.04, 0.15, and 0.28 m respectively. The system response was calculated in the frequency range from 2 to 4.5 Hz theoretically and compared it with the experimental measurements as shown in Figure 8.



Figure 8: Influence of ambient frequency on system response for various tuning mass positions

It is known that the response of any system increases upon approaching the resonance zone until hits the maximum value. The experimental results revealed that the mass position affects the maximum system response in the resonance state, where the maximum response reduced by 18.4% when the mass is at the end of the path (L=0.28 m) compared to the beginning at (L= 0.04 m).

The analytical study did not show a noticeable difference in the maximum system response with the change of mass position at the resonance zone, however, there was a difference between the practical and theoretical results when moving away from the resonance zone. This may be because the system has been simplified and reduced by analyzing it using the methodology of approximate single degree of freedom system.

The harvested voltage and power in the resonance state were calculated for the same three mass positions by changing the load resistance (R_L) from 0 to 500 Ω , to obtain the optimum load resistance for each case, and accordingly harvest a large amount of energy as illustrated in Figures 9 and 10.



Figure 9: Influence of load resistance on the output voltage for various tuning mass positions



It can be seen from Fig. 9 that the output voltage increases with the increase of the load resistance until reaching its maximum value, then a constancy state occurs to the output voltage value. It was also observed that the harvested power was increased with increasing the resistance load until reaching its maximum value at a certain resistance load which represented optimum value. After that, the harvested power reduced again by increasing the resistance load. Also, it is noticeable that the tuning mass position has a significant effect on the output voltage and hence the harvested power in the resonance state. When the tuning mass was at positions 0.04, 0.15, and 0.28 m, the harvested powers were 30.6, 26.4, 11.8 MW with optimum resistance load 32, 26, and 18 Ω respectively.

6. CONCLUSIONS

This paper presented a study of the moment of inertia effect on the natural frequency of the system to reach the resonance state and consequently to harvest the maximum amount of energy. A self-tuning technique has been used to ensure the survival of the system in the resonance state continuously. This system has been analyzed using the methodology of approximate single degree of freedom system and verified by comparing it with the experimental prototype. This technique was able to experimentally adjust the natural frequency within a range from 2.65 to 3.75 Hz with tunability 38.4% by automatically modifying the proof mass position when the ambient frequency varies. The proposed design has achieved remarkable success in harvesting energy from low frequencies where it was able to reap about 30.6 MW using an electromagnetic energy harvester.

Credit Authorship Contribution Statement

Shahenda M. Attya: Methodology, Software, Formal analysis, Experimental work, Writing original draft. Ebtisam F. Abd-Gwad: Writing review & editing, Supervision.

Declaration of Competing Interest

The authors declare that there is no conflict of interest regarding the publication of this paper

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