Journal of Al-Azhar University Engineering Sector

Vol.15, No. 55, April, 2020, 500-512

STRENGTHENING OF SHEAR AND MOMENTS IN CONTINUOUS BEAMS

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ABSTRACT
Existing structures are subjected to increasing loads. They were often designed using Allowable Stress Design method (ASD). In old industrial buildings, beam dimensions were selected compact by using either rolled steel sections, or built up profiles. A direct design method is presented to strengthen such continuous beam panels above their plastic limit at different locations. Shear yield strain is crucial in the analysis, which is considered by using a numerical solution, and then simplified formulae are presented. The strengthening is based on attaching steel plates at the suitable locations. The position of each plate and its reasonable extension is determined. Material consumption and welding energy are kept as low as possible. Results are compared with previous tests and numerical values. Excessive strengthening is avoided. The proposed method could be applied on beams with other dimensions and/or conditions. The required precautions and restrictions are given.

KEYWORDS: Shear, Strengthening, Continuous Beams, FEM, Plastic Limit.

التحليل لتقوية القص والعزم في الكمرات المستمرة

فرح الـــديب
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الملخص:
تم تقديم طريقة تحليلية مباشرة ومدعومة بحسابات عددية لتقوية الكمرات المستمرة فوق قدرتها القصوى. ومن أهم تلك الحالات هي تلك الكمرات التي تعاني السيولة في مواضع عدة عند الحمل الأقصى وبالأخص ذات العصب الضعيف الذي لا تصل مقاومته للقص ذلك المقدار الذي تتطلبه العزوم القصوى. إن البحث تأسس على استخدام الحد الأدنى من الألواح المجمعة وإلحاقها بأماكن سبق تحديدها بطرق مبسطة مع حساب الحد الأدنى لانتشارها وفرا للمواد والطاقة. وقد تم التحقق من صلاحية طريقة المقاربة بالمقارنة مع الأبحاث السابقة ومع الحسابات العددية. ويقدم البحث للمصمم طريقة مباشرة لتحديد كمية التقوية المطلوبة للوصول لمستوى التقوية المطلوب وتحاشى البحث التقوية المفرطة. ويمكن تطبيق الطريقة على كمرات مختلفة الأبعاد والشروط مع تقديم الاحتياطات الواجب اتخاذها.

الكلمات المفتاحية : القص ، التقوية ، الكمرات المستمرة ، العناصر المحدودة ، حد اللدونة.

1. INTRODUCTION
Strengthening of steel buildings, follow due to increased loads, and change of function and/or rehabilitation of old buildings. It should be done with the minimum amount of material and applying the least possible welding heat energy to the existing steel elements.

Nagaraja and Lambert [1,2] welded steel elements on steel beams and columns while under load. Lambert [2] observed that the welding generated heating had a significant effect on the residual stress redistribution. It rather "corrects" the effect of the residual stresses and could be used for reinforcing purposes. When strengthening columns by using cover plates to the flange and bead weld to the tip of the flange, it was noticed that the column strength improved as the residual stresses redistributed favorably.

Yuan-qing, *et al.*, [3] investigated the effect of the welding energy generated heat on the existing beam numerically as well as experimentally. Upper und lower cover plates are clamped to the flanges, and then the beam is pre-loaded. They applied a systematic welding process and the strengthened beam is loaded until failure.

Lui and Gannon [4] used cover plates to strengthen beams under different levels of pre-loading. A horizontal plate was welded along the whole beam length, and then the beam is tested until failure. The attached another type, as two vertical plates to the tips of both flanges. The later has increased the strength remarkably and prevented lateral buckling under ultimate load. For the cases tested experimentally numerical studies were carried out.

Gendy and El Dib [5] presented the strengthening of slender I-Beams that required the attachment of two vertical plates welded at the tips while under load. Under the ultimate load the beam has failed by web local buckling. Several parameters of the beam were studied. The ultimate moment capacity of the strengthened beam has increased when the strengthening is applied at less loaded beam. Also it was found that the increase in the beam strength is insignificant if the plate length is more than 2/3 of the beam span. A simplified mathematical model was proposed to estimate the ultimate moment capacity of the strengthened beams that are uniformly loaded at the top flange.

2. ASSUMPTIONS

This study investigates the behavior of strengthened continuous beams, which under ultimate load; indicate no local or lateral buckling. The following assumptions are made:

1- The continuous beam, which under ultimate load may create plastic moments at different locations, has a constant section and carries a distributed load.

2- The beam fails, under ultimate load due to excessive yield spread only, triggered by normal and/or shear stresses. No local or lateral failure is expected.

3- To achieve the minimum martial and welding requirements, the best location of the strengthening plates and their extension should be determined.

4- Strengthening follows at a maximum pre-load of 0.8 the ultimate load.

5- The section is I-shaped and its elements are compact and allow for excessive yield spread and can develop plastic shear or moment. The load increases over the beam length simultaneously.

6- In old buildings, the material of the beam is usually ordinary steel (F_y=240 MPa), the strengthening plates can be of higher steel grade (F_y=350 MPa), and E=210 GPa.

3. FINITE ELEMENT MODEL

The continuous beam with two spans that carries a vertical uniform load is simulated using a finite element model. Two vertical strengthening plates, in addition to a horizontal one are used to adequately strengthen the continuous beam panel. To prevent shear failure at supports, additional vertical plates designated as (V2) may be attached at the right support. Since the continuous beam is loaded on the top flange, the horizontal plate can only be attached under the bottom flange. The negative moment at the intermediate support is assumed to increase up to the plastic moment the cross section can take. The two vertical plates are therefore attached around the intermediate support, and then extend as needed in both sides. It should be noted that the plate lengths and location A, B and C displayed in Figure 1, should be accurately determined from case to case and according to need.

As shown in Figure 1, the uniform distributed load is simulated by applying a vertical concentrated load at each element node at the junction of the web and the top flange along the whole beam length. The magnitude of the plastic moments in both spans is the same and the ultimate load becomes smaller in the longer panels. The boundary conditions are modeled as pin or roller supports located at the bottom flange. The translations in all three directions x, y and z are prevented at the intermediate support, and the translations in y and z directions are prevented at the outer ones. An initial imperfection of L/1000 at the maximum location is assumed to account for manufacturing pre-deformations, where L1 and L2 are the span lengths from support to support, and the imperfections have the shape of the first buckling mode.

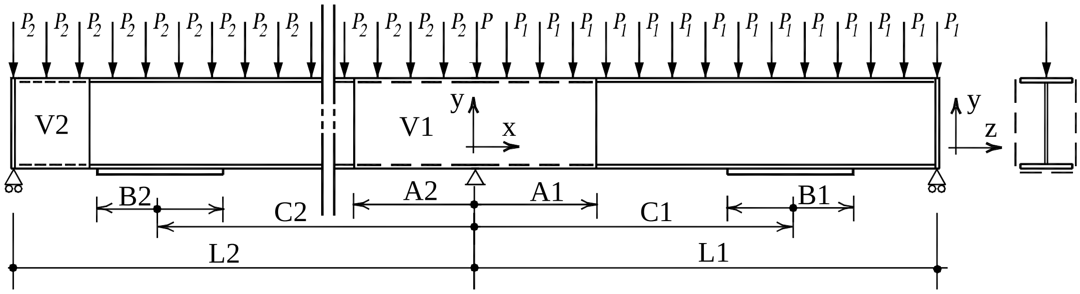


Figure 1: System, loads and boundary conditions.

A plastic hinge at intermediate support behaves as a hinge. This property makes it possible to investigate each panel separately. The beam elements and the welded plates are represented by using a 4-node shell element “SHELL 181” provided by ANSYS [6]. It is suitable for the analysis of thin to moderately thick plates. The material is assumed perfectly elastic-plastic. A sample model built with an I-section HEB 800, panel length 10ms, and five cover plates attached for strengthening is displayed in Figure 2 for demonstration. The deformations, for a fixed left support that simulates the continuity, are demonstrated.

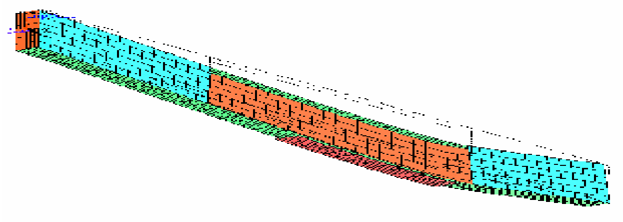


Figure 2: Panel System, Finite Element Model.

4. NUMERICAL ANALYSIS PROCEDURE

The procedure is summarized in two major steps as follows: In the first step the ultimate load of the un-strengthened system is determined; it is denoted as “ULT1”. In the second step the un-strengthened beam is usually loaded up to a value of “0.8 ULT1”, at this load level the strengthening plates are attached and the loading is then increased until failure. This step is denoted as “ULT2”. In both steps the proposed geometrical imperfections, as well as the material non-linearity is accurately included as related to the first Eigen shape of the structural system.

5. VERIFICATION OF THE FINITE ELEMENT MODEL

The strengthening methods used in the comparison samples are different: Horizontal plates are attached on top and bottom flanges of a beam that is loaded with one concentrated load at mid-span, as given by Yuan-qing, *et al.* [3]. Other two vertical plates are welded to the tips of both flanges, and then the beams are loaded with two concentrated loads at 1/3 and 2 /3 of the span length. The given beams have failed due to local- or lateral buckling as given by Lui and Gannon [4], and Gendy and El Dib [5]. The results comparison is presented in Table 1.

Table 1: Comparison between the proposed model and previous results.

Ref	Result Type	Span (mm)	Pre-load %	P_{ULT}-Ref (kN)	P_{ULT} (FE) (kN)	Error %
Yuan-qing [3]	Experiment	3000	74	575.86	584.50	+1.50
Lui [4]	Experiment	2400	0	188	191	+1.60
Lui [4]	Experiment	2400	69	788.5	785.5	- 0.38
Lui [4]	Numerical	3000	0	103	104.4	+1.36
Lui [4]	Numerical	3000	31	797.3	795.95	- 0.17
Lui [4]	Numerical	3000	62	796.4	795.75	- 0.08
Gendy [5]	Numerical	3000	93	792.1	795.09	+0.38

6. THE PLASTIC MOMENTS AT ULT1

It is necessary to determine the relationships between the post strengthening beam plastic moments. They represent the upper bound of the bending moments created under ultimate loads: Consider that the beam panel shown in Figure 3 has reached the ultimate load. The theoretical plastic moments M_p at the intermediate support and at the field are equal. By taking the parameter $M_o = wL^2/8$ as an indication of the load, then $8M_o/L^2$ represents the vertical load w, which is the load per unit length on the top of the beam. The term $4M_o/L$, thus, represents the shear in a simply supported panel wL/2 …etc. To establish dimensionless relationships, the ultimate load is expressed in terms of M_o, which in turn is related to the cross section plastic moment M_P, or to its yield moment M_y.

By using the principles of the plastic theory, the panel left reaction at mid support is:

$$R = \frac{wL}{2} - \frac{M_{PL}}{L} = \frac{1}{L}(4M_0 - M_{PL}) \quad (1)$$

$$(1-C) = \frac{R}{w} = \frac{1}{wL}(4M_0 - M_{PL}) = L\left[\frac{1}{2} - \frac{1}{8(M_0/M_{PL})}\right]. \quad (2)$$

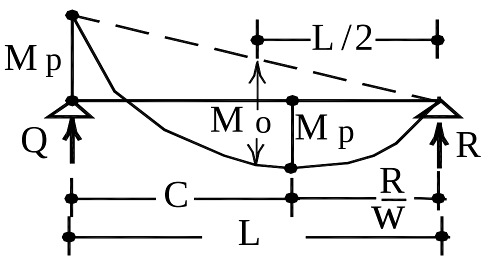


Figure 3: Ultimate moment diagram of the proposed beams.

The maximum positive moment is:

$$M_{PL} = \frac{1}{16M_0}(16M_0^2 + M_{PL}^2 - 8M_0M_{PL}) \; . \quad (3)$$

By taking $m = \frac{M_0}{M_{PL}}$, then:

$$\left(\frac{M_0}{M_{PL}}\right)^2 - \frac{3}{2}\frac{M_0}{M_{PL}} + \frac{1}{16} = 0 \quad . \qquad (4)$$

$$16\,m^2 - 24\,m + 1 = 0$$

And the distance is $C = L\left[\frac{1}{2} + \frac{1}{8m}\right]$. (5)

The solution of Equation (4) is $m = \frac{3}{4} \pm \frac{1}{\sqrt{2}} = \frac{1.4571}{0.0429}$, (6)

and, the corresponding distance is $C = \frac{0.5858}{3.4137}L$. (7)

In Equations 6 and 7, the correct μ and C values are the upper ones. If these two values are true, the system can develop the plastic hinges, shear, or moment. In addition, it indicates that the system can resist the shear triggered by the ultimate moments. Only then, the analytical direct formulae, presented later, are valid and can be applicable.

7. THE PLASTIC SHEAR AT ULT1

The assumption, that all cross section elements are compact and can sustain excessive yield strain, means that the web is strong enough as to prevent shear failure. In case, the web cannot take the shear force triggered by the ultimate load, the whole cross section at this location fails, in bending as well as in shear. Then the full plastic moment cannot develop, but only a reduced moment related to the maximum shear capacity of this section.
The web fails, when the pure shear strain value exceeds its yield limit. It is thus necessary to estimate the maximum shear force created by the ultimate moments that are determined in the previous section: Knowing that μ_{ULT1} = 1.4571, from Equation 6, □ □ then Equation 1 becomes:

$$Q_{ULT1} = \frac{wL}{2} + \frac{M_{PL}}{L} = \frac{1}{L}(4M_0 + M_{PL}) = \frac{M_{PL}}{L}.(4m+1) = 6.83\frac{M_{PL}}{L} \qquad (8)$$

The plastic moment distribution in Figure 3, when applied on the system it creates the maximum shearing force at the left support given by Equation 8. This shear failure check is crucial in short span panels. The shear stress due to Q_{ULT1} is thus:

$$\frac{Q_{ULT1}}{A_W} \le \frac{F_y}{\sqrt{3}} \qquad (9)$$

Additional strengthening elements are necessary in short span panels that cannot take the shear the plastic moments create. Strengthening the shear resistance requires attaching additional elements. The following case of a shear failure demonstrates Figure 4: Considered is a two-span continuous beam, each span is 6.0 ms long. The beam cross section is HEB 800 (h/ b/ s/ t: 800, 300, 17.5 and 33 mm). When increasing the carried distributed load until “ULT1”, the beam fails due to shear failure at the mid support. Note that the beam could develop no plastic support moment at failure.

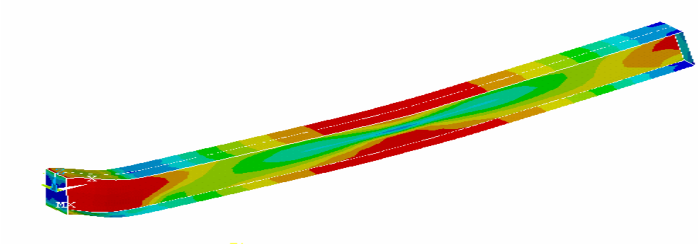


Figure 4: Example shear failure.

8. THE PLASTIC MOMENTS AT ULT2

Increasing the load on the strengthened continuous beam until failure usually creates two plastic hinges in the panel. The created plastic hinges depend mainly on the strengthening technique. They are usually not equal as shown in Figure 5. Similar to the procedure, given in Section 6, Equation 3 considers a reduced plastic moment at field equals z $M_{\mathbf{p}}$ set on the left hand side as follows:

$$z\ M_{PL} = \frac{1}{16M_0}(16M_0^2 + M_{PL}^2 - 8M_0M_{PL}) \qquad (10)$$

$$\left(\frac{M_0}{M_{PL}}\right)^2 - (z + \frac{1}{2})\frac{M_0}{M_{PL}} + \frac{1}{16} = 0$$

Equation 4 becomes, *Taking* $m = \dfrac{M_0}{M_{PL}}$: , (11)

$$16\,m^2 - (16z + 8)m + 1 = 0$$

As for the distance C, Equation 5 is still valid.

The value of z is usually < 1.0; this is the case of a section, strengthened by attaching only one horizontal plate under the lower flange. Equation 11 is still valid for z > 1.0, in such a case, the value of C gets shorter, but never less than 0.5 L as plotted in Figure 6.

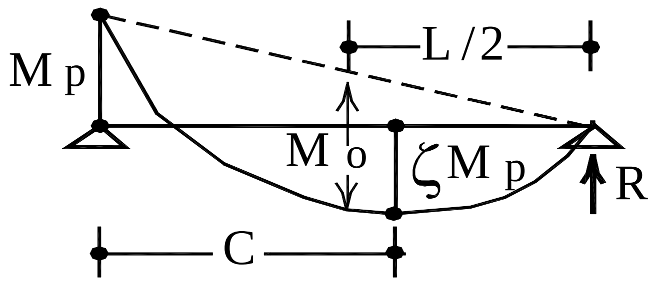


Figure 5: Plastic moments at ULT2.

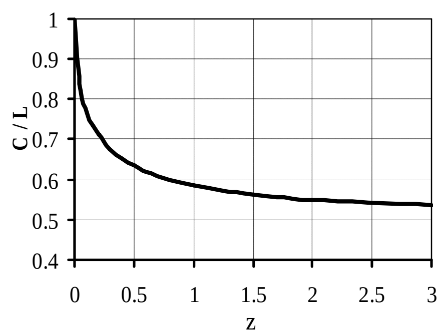


Figure 6: Variation of C/L vs. z

9. THE PLASTIC SHEAR AT ULT2

The ultimate load carried by the strengthened beam panel (ULT2) may create a high shear at the left support. In case this shear exceeds the ultimate capacity of the web, the ultimate load reduces its maximum value due to the reduced plastic moment at intermediate support x $M_{\mathbf{p}}$.

The actual ultimate web shear Q determines the value of x. The equilibrium equations of forces solve this problem to estimate the maximum value of Q that the web can take.
In the following, the same rules used in Sections 6 and 7 are applied, and then the equilibrium at the right reaction support gives the relationship between Q and x as follows:

$$R = wL - Q; \quad RL = 8M_0 - QL = 4M_0 - x\,M_{PL} \tag{12}$$

The right distance to max. +ve M is:

$$(1-C) = \frac{R}{w} = L\left[\frac{1}{2} - \frac{x}{8(M_0/M_{PL})}\right], \tag{13}$$

And max $M_p$is:

$$\begin{aligned} M_{PL} &= \frac{R(1-C)}{2} = RL\left[\frac{1}{4} - \frac{x}{16(M_0/M_{PL})}\right] \\ &= (8M_0 - QL)\left[\frac{1}{4} - \frac{x}{16(M_0/M_{PL})}\right] \end{aligned} \tag{14}$$

$$\begin{aligned} &Taking\ k = \frac{QL}{M_{PL}}; \qquad and\ x = \frac{QL}{M_{PL}} - \frac{4M_0}{M_{PL}} = k - 4m, \\ &and\ substituting\ in: \\ &1 = \frac{2M_0}{M_{PL}} - \frac{x}{2} - \frac{k}{4} + \frac{k}{16}\frac{x}{(M_0/M_{PL})}, \\ &Taking\quad m = \frac{M_0}{M_{PL}}, then \\ &64\,m^2 - 16(1+k)m + k^2 = 0 \end{aligned} \tag{15}$$

$$C = L\left[\frac{1}{2} + \frac{x}{8m}\right], \quad 0.5L < C < 0.625L \tag{16}$$

When the ultimate shear of the web is $Q_w = A_w\,.0.58\,F_y$, where A_w is the web area, then μ can be determined by using κ from Equations 15 in Equation 16. The plastic moment distribution thus relates to the reduced plastic moment at the intermediate support. Note that, at ultimate load in a simple beam: $Q = wL/2 = 4M_o/L$, $M_o = M_p$. The corresponding κ = 4, and thus Equations 15 and 16 give the values of x = 0 and μ = 1, while in an elastic two span continuous beam panel, the value of the shear $Q = 5\,wL/8 = 5\,M_o/L$.
In case of two similar plastic hinges under ultimate load, and from Equation 8, the ultimate shear $Q_{ULT} = 6.83\,M_o / L$, and κ = 6.83. Thus, in such a case, the range of κ is between 4 and 6.83. In this range, the κ − μ relationship is approximately linear and gives the formula:

$$m = 1 + 0.1615(k-4), \quad 1 < m < 1.4571. \tag{17}$$

Substituting in Equation 16, $x = 0.3535(k-4), \quad 0 < x < 1.0.$ (18)

In a continuous beam panel with a reduced plastic moment at the mid-support, the above relationships make it possible to predict directly the ultimate strengthened load related to web shear ultimate failure (Figure 7).

It is also important to check the shear failure at the right support “R” at ULT2. It depends mainly on the amount of relative increase η in ULT1 at ULT2:

h = *(ULT2-ULT1)/ULT1,* and $R_{ULT2} = (1+\eta)\, R_{ULT1}$. (19)

By substituting in Equations 1 and 6, then:

$$R_{ULT2} = (4 \times 1.4571 - 1) \times (1+h)\frac{M_{PL}}{L} = 4.83\,(1+h)\frac{M_{PL}}{L} \quad (20)$$

It should be noted that, at the right support, no such check is necessary when $\eta < 0.41$. The check at the mid-support, using Equation 8 is sufficient. In case “R_{ULT2}” exceeds the ultimate web shear stresses, use two vertical cover plates at the right support “V2”, as shown later.

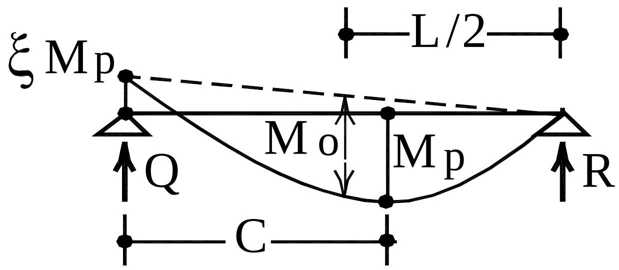


Figure 7: The case of reduced support plastic moment.

10. ANALYTICAL EVALUATION

Two types of cover plate elements strengthen the beam. Vertical plates, welded to the flange tips and extends as needed, and another horizontal plate, with variable length and location strengthens the beam positive moment at the bottom flange. The following example represents the numerical model, and explores the behavior of strengthening in general. In this example, the cross section of the beam is taken as HEB 800 (800, 300, 17.5, 33 mms, F_y 240 MPa, E 210 GPa, and the calculated M_p = 2388.09 kNm). The beam extends over two spans, 16 ms each. The symmetry makes it possible to reduce the stiffness matrix and to analyze one panel, with one fixed left end to simulate the continuity.

Two vertical plates (2x767x15 mms, F_y 360 MPa) attached to the beam tips strengthen the mid-support region, and extend up to 0.75 and 1.0 times the panel length in each panel. The model section is compact and fulfils the assumptions stated in Section 2. After attaching the plates, at 0.95 ULT1 the loading starts increasing up to ULT2. The load deformation relationship related to the vertical deflection at 0.6 L shows all loading phases in Figure 8. By applying the relationships, given in Sections 6 and 7, a direct evaluation comparison between analytical and numerical values is possible.

By inspecting the numerical results in Figure 8, a straight-line relationship starts at the origin and extends up to μ = 0.87834 with the corresponding δ = 33.841 mm. The analytically determined δ in the elastic domain is only 31.747 mm. By considering the elastic shear deformations, which are included in the stiffness elements of the FEM, the final δ = 31.747+1.857=33.604 mm (G=E/2.6) and the Error decreases to 0.7 %.

The small curved part, following the straight line represents the first plastic hinge formation over the mid support. The next part is another straight line, with smaller inclination indicating smaller stiffness, and represents the beam behavior that follows the completed formation of the first plastic hinge and extends between $\mu_1 = 1.2053$ and $\mu_2 = 1.3869$.

Knowing that the elastic positive $\delta_{max} = M_o.L^2/(c.EI_x)$, where c=23.075 for continuous beam and 9.6 for the simple beam, the corresponding increase in δ relative to $(\mu_2 - \mu_1) = 0.1816$ is $\delta_{1,2}$ = (68.928-53.263) =15.666 mm. In this case, the calculated c = 9.670, which almost matches the constant of the simple beam (the calculated $I_x = 3.491.10^{9}$ mm^4).

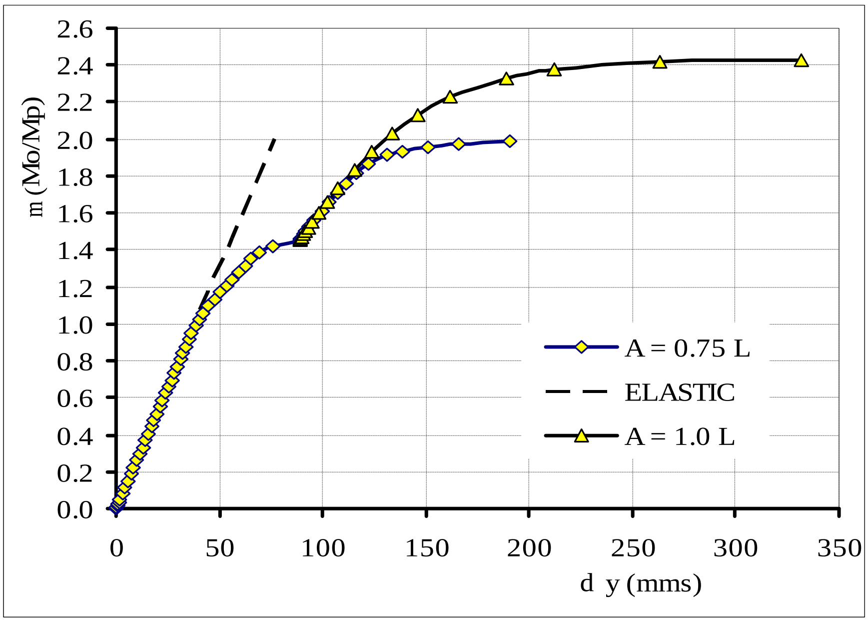


Figure 8: P-d **relationships.**

Therefore, after plastic moment formation, and by increasing the load, the full plastic hinge at mid-support behaves structurally as a hinge. Then the continuous beam deformations start following the rules of a simple beam.

The ultimate load ULT1 is found numerically at $\mu = M_o/M_p = 1.48278$. The corresponding analytical value in Equation 6 is 1.4571. In this example, in case the vertical plates extend over the full panel length (A_1 = 1.0 L), eventually, yield spreads over all section elements, the original elements and the strengthening ones. The total plastic moment of the strengthened section is thus $M_{PL,T}$ = 3976.47 kN.m, and $M_{O,ULT2}$ = 1.4571* 3976.47. The corresponding $\mu_{\square\ ULT2} = M_{O,ULT2} / M_{PL}$. Knowing that M_{PL}, calculated at the original section is = 2388.09 kNm, then $\mu_{\square ULT2}$ = 2.4262. Its numerical mate value is 2.4277 making a small error. The deformations in the final phase up to ULT2 extend up to $\mu_{\square \mathbf{ULT2}}$. Note that a partially strengthened beam panel is a numerical, not an analytical issue (Figure 9).

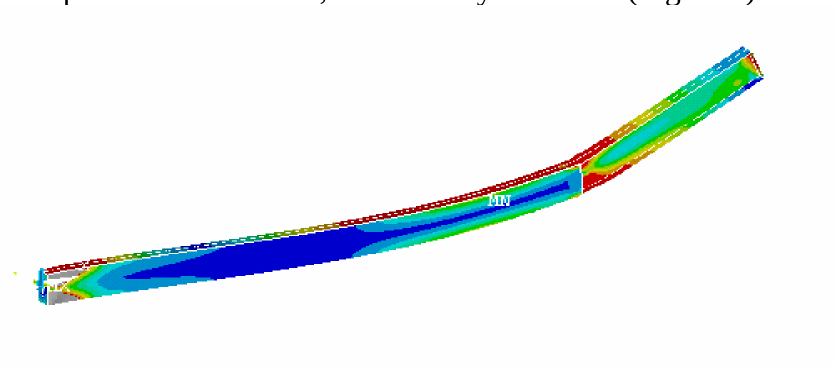


Figure 9: Example partially strengthened beam panel.

11. EFFECTIVE STRENGTHENING SYSTEM

Table 2 presents two different I-section types to study and verify the effective strengthening arrangements. All selected sections, usually used in old industrial buildings are hot rolled and have compact elements. For each section type the plastic moment is given for F_y = 240 MPa, where $\eta_S = \eta_{Section}$ is the relative increase in the cross section plastic moment caused by the attached plates. Note that the increase in system ultimate load is much smaller than η_S, yet these values are important parameters in estimating ULT2. To estimate the location and extension of the added plates, we extend gradually the attached vertical plates, at the

intermediate support, over eight locations until full panel length. The investigated beam lengths L are 4, 6, 8, 10, and 12 ms, the values L/H_{Web} varies between 8 up to 24, and the ratio A_{Web}/A_{Total} are 0.27 to 0.45. The efficiency "I" is defined as the increase in ULT2 related to the increase in total system steel weight. The results gained from all the 56 runs are plotted together on one plot. By inspecting Figure 10, all cases show the same trend, with the exception of few cases, where the system failed due to shear failure. In the following, a direct systematic procedure to determine ULT2 is given, including a special approach for the cases of shear failure.
Strengthening follows by combining both vertical and horizontal cover plates (V, H, V+H or V1+H+V2 followed by plate thickness in mms). Hence the specimens can be designated by their section type (I or II), beam length, pre-load ratio, cover plates (i.e. II, 8, 0.9, V10, and an asterix "*" denotes a variable field etc.).

Table 2: Proposed cross section types.

Type	ID	M_{PL} Kn m	Q_{ULT} kN	h_S for $t_{VL, Plate}$ 0.8t_w	1.0t_w	1.2t_w
I	IPE500	505.75	661.45	0.618	0.773	0.928
II	HEA500	915.29	738.27	0.362	0.452	0.543

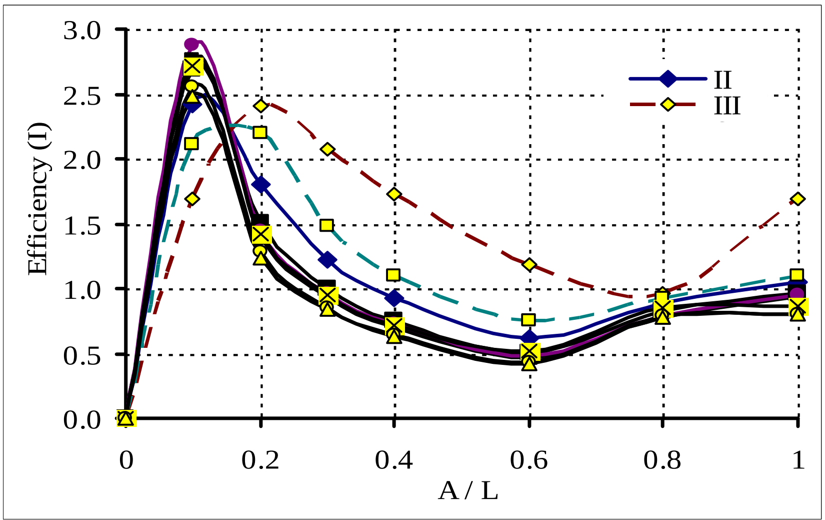


Figure 10: Length efficiency of vertical strengthening plates.
(HEA 500, IPE 500, 2L,L=*,V1:*,0.8ULT1)

Based on the variation of "I" vs. plate length, the extensions from zero to 0.2 L (A1), from 0.4L to 0.75L (B1) and from 0.8L to 1.0L (V2) are economic (Figure 10). Within these distances, "I" increases the efficiently, i.e. indicating positive second derivative. Note that B1 in Figure 1 represents the horizontal plate, which makes similar trend as the vertical one. Vertical plates have variable thicknesses, but the horizontal plate has the same flange breadth and its area is at 0.5 – 0.8 the web area. This strengthening system is efficient in hot rolled I-section continuous beams and applies throughout the following analyses.

12. DIRECT METHOD: ANALYTICAL AND NUMERICAL RESULTS

Under the conditions of ULT1, the shear resistance of the web plate is**:**

$$Q_w = A_w . F_y / \sqrt{3} \,, \tag{21}$$

which determines the efficiency of the strengthening. By using Equation 15, then:

$$k_W = \frac{Q_w L}{M_{PL}}, 4.0 \le k_W \le 6.83, \qquad (22)$$

When the system creates the full plastic moments, then its resistance increase at ULT2 is:

$$h_{Bending} = 0.45 \sqrt[3]{h_S}, \qquad (23)$$

where $h_{Bending}$ is the relative increase in ultimate load of the continuous beam that is based on the theoretical value: $M_o=1.4571.\ M_{PL}$, and on h_S that is the section relative increase in plastic moments due V1.

In case, $k_W \ge 6.83$, □ Equation 23 is valid: apply only V1 + H.

In case $4.0 \le k_W \le 6.83$: $h_{Shear} = h_{Bending} \cdot \frac{k_W}{6.83}$, (24)

and the shear strengthening system becomes: V1 + H + V2, which means adding vertical plates V2 at the outer supports between 0.8-1.0 L (Figure 11). In this case the selected thickness of V1 (0.8-1.2) t_W, must be increased by Dt, to avoid web failure and to substitute the difference between available Q_w and required Q_U that creates full plastic moments:

$$\Delta t = \frac{(Q_U - Q_W)}{2.H.(F_y/\sqrt{3})} \qquad (25)$$

Note that for V1: A1/L=A2/L= 0.25 with t=0.8-1.2t_w, for H: B1/L=B2/L=0.35 and is centered at C/L from Equations 16 - 18, on both panels. For simplification, B/L is taken 0.4L and is centered at approximately 0.6L. V2, if needed, extends at 0.8-1.0 L and has the same thickness as V1. When needed, it is placed in both panels simultaneously. V1+H+V2 are of grade 350 MPa. The original system is grade 240 MPa and has two equal spans L.

In case $\kappa_W < 0.4$, shear failure prevails. No strengthening is recommended. No plastic hinge at intermediate support is formed. The panel then behaves almost as a simple beam. In the following, bench mark examples are collected in Table 3 to explain procedure and to assure validity. At ULT2, the von Mises stress distribution, on system and on strengthening plates, which are detached on Figure 11, is separately depicted for demonstration.

Table 3a: Example Cases, System, Strengthening Positions.

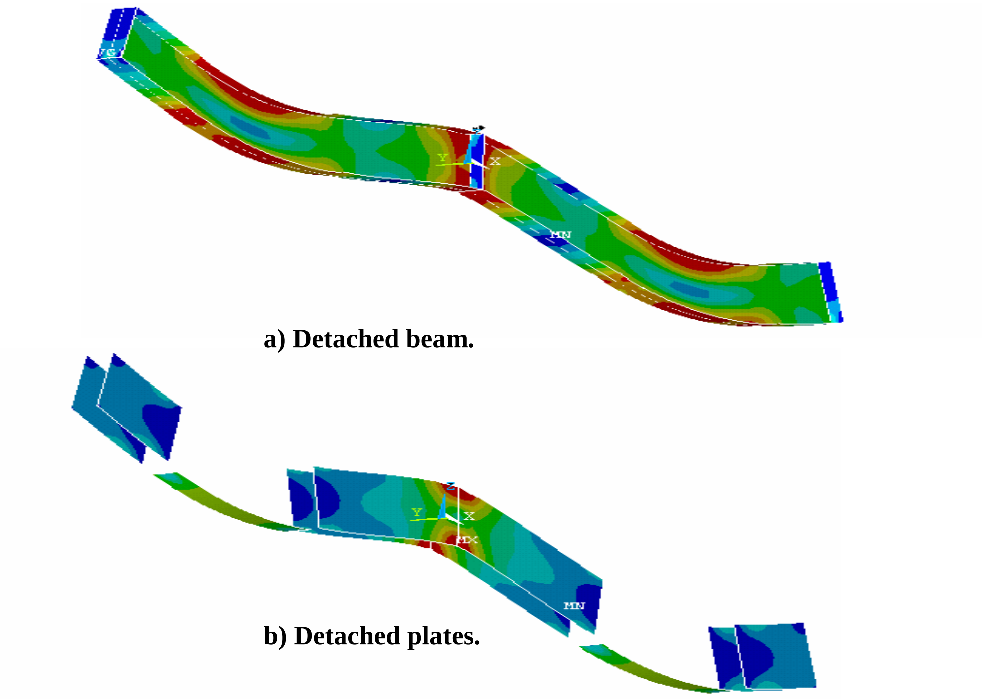
Case #	Sec. ID	Span (m)	V1: t_S / t_w	V1:t_S+Δt (mm)	κ_{avail}	ξ	μ	C/L
Col. #	1	2	3	4	5	6	7	8
1	I	4	0.8	9.2	5.23	0.44	1.1989	0.5454
2	I	8	0.8	8.2	6.83	1	1.4571	0.5858
3	I	12	0.8	8.2	6.83	1	1.4571	0.5858
4	I	4	1.0	11.3	5.23	0.44	1.1989	0.5454
5	I	8	1.0	10.2	6.83	1	1.4571	0.5858
6	I	12	1.0	10.2	6.83	1	1.4571	0.5858
7	I	4	1.2	13.3	5.23	0.44	1.1989	0.5454
8	I	8	1.2	12.2	6.83	1	1.4571	0.5858
9	I	12	1.2	12.2	6.83	1	1.4571	0.5858
10	II	4	0.8	14.2	4.00	0.00	1.0000	0.5858
11	II	8	0.8	9.8	6.45	0.87	1.3961	0.5858
12	II	12	0.8	9.6	6.83	1.00	1.4571	0.5858
13	II	4	1.0	16.6	4.00	0.00	1.0000	0.5858
14	II	8	1.0	12.2	6.45	0.87	1.3961	0.5858
15	II	12	1.0	12.0	6.83	1.00	1.4571	0.5858
16	II	4	1.2	19.0	4.00	0.00	1.0000	0.5858
17	II	8	1.2	14.6	6.45	0.87	1.3961	0.5776
18	II	12	1.2	14.4	6.83	1.00	1.4571	0.5858

Table (3b): Example cases, solved analytically and numerically.

Case #		η_S	η_{Shear}	η_{FE}	Note
Col. #		9	10	11	12
1		0.618	0.294	0.404	V2
2		0.618	0.383	0.375	-
3		0.618	0.383	0.399	-
4		0.773	0.316	0.400	V2
5		0.773	0.413	0.427	-
6		0.773	0.413	0.448	-
7		0.928	0.336	0.390	V2
8		0.928	0.439	0.470	-
9		0.928	0.439	0.497	-
10		0.362	0.188	0.209	V2
11		0.362	0.303	0.313	-
12		0.362	0.321	0.320	-
13		0.452	0.202	0.212	V2
14		0.452	0.326	0.353	-
15		0.452	0.346	0.368	-
16		0.543	0.215	0.212	V2
17		0.543	0.347	0.363	-
18		0.543	0.367	0.352	-

13. DISCUSSION AND RECOMMENDATIONS

Premature web shear failure prevents the continuous beams from developing the plastic moments and reduces their ultimate load. A strengthening system is presented, which substitutes the missing shear resistance and provides the system with additional strength above the plastic moments ultimate load. The given system shows regular behavior and can be computed and designed directly by using the given equations. The method can also consider continuous beams with strong webs that create full plastic moments.



a) Detached beam.

b) Detached plates.

Figure 11: Case #4: Detached strengthening system at ULT2.

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