



# Numerical Analysis of Mode I Fracture Toughness for Plain Concrete Beams

## (التحليل العددي لمتانة الكسر في نمط الإنهيار الأول للكرات الخرسانية العادية)

Seleem S. E. Ahmad and Ghada M.A. Gamiaa

### KEYWORDS:

*Fracture mechanics, stress intensity factor, thickness effect, ANSYS, crack depth ratio, strength of concrete, finite element method*

*المخلص العربي:* تعد الخرسانة من أهم المواد المستخدمة في مجال الإنشاءات لذلك يجب دراسة خواصها الميكانيكية بشكل دقيق وكذلك مقاومتها لنمو الشروخ وخاصة أثناء التحميل لمعرفة سلوك هذه العناصر الإنشائية. لقد توالت الأبحاث لدراسة الأسباب التي تؤثر على سلامة المنشآت، وفي نطاق هذا البحث سيتم دراسة سلوك الكسر في الكمرات الخرسانية التي يتواجد بها شروخ ودراسة تأثير كل من: سمك العينة، إجهاد الخرسانة المستخدمة، وطول الشرخ على تحديد قيمة معامل شدة الإجهاد وذلك تحت تأثير الأحمال الاستاتيكية باستخدام برنامج التحليل (ANSYS 16). ودراسة تلك العوامل تبين تأثير كل منها على قيمة معامل شدة الإجهاد حيث تبين النقص في قيمة معامل شدة الإجهاد مع زيادة سمك العينة وتبين زيادة قيمة معامل شدة الإجهاد مع زيادة نسبة طول الشرخ ومقاومة الخرسانة.

**Abstract**— Fracture mechanics is based on the assumption that all engineering materials contain cracks from which failure starts. Structures have sizeable existing cracks that may or may not grow, depending on the load level. When a material has an existing crack, this leads to high stresses near the crack tip. Stress field type in the region of the crack tip can be defined as mode I, where two fracture surfaces are displaced perpendicular to each other in the opposite direction where crack growth takes place. The crack tip stress and displacement field can be uniquely characterized by the stress intensity factor, SIF; it is neither the magnitude of stress nor strain, but it is a unique parameter that describes the effect of loading at the crack tip region and the resistance of the material. The stress intensity factor is valid for a small region around the crack tip. It depends on both the values of stress and crack size. In the present work, numerical models were designed to study the effect of specimen thickness, crack-depth ratio and concrete compressive strength on the SIF in concrete beams. Nine beams were idealized with dimensions of 1500 mm length, 400 mm depth and different thicknesses 120, 250 and 400 mm. The crack-depth ratios were 0.1, 0.2 and 0.3 and the concrete compressive strengths were 25, 40 and 55 MPa. The results proved that there is a decrease in the SIF with increasing the specimen thickness. On the other

hand; there is an increase in the SIF value when increasing either the concrete compressive strength or the crack-depth ratio.

### I. INTRODUCTION

**F**AILURES of structure buildings have occurred due to many reasons such as defects in materials, design, and construction or maintenance. Hence, these causes should be taken into consideration. Although a lot of efforts have been lately devoted to studying fracture damage behavior of concrete, linear elastic fracture mechanics still seems to be useful for studying the concrete failure due to cracks [1]. The cracking strength of concrete is determined by using fracture mechanics concepts. In case of loading a cracked body, it is usually accompanied by inelastic deformation and other nonlinear effects near the crack tip, except for ideally brittle materials. There are, however, situations where the extent of inelastic deformation and the nonlinear effects are very limited compared to the crack size and the length of the body; in such cases the linear theory is used to characterize the stress distribution in the cracked body [2, 3].

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Seleem S.E. Ahmad is a Professor and Head of Engineering Material department, faculty of Engineering Zagazig University, Zagazig, Egypt, (e-mail: seleemahmad62@yahoo.com)

Ghada M.A. Gamiaa is a MSc student, Materials Engineering Dept., faculty of Engineering Zagazig University, Zagazig, Egypt, (e-mail: engineerg49@yahoo.com)

Numerical models of fracture can be classified by the observation scales of fracture, which vary from the atomistic level to the micro level to the macro level. For the modeling of macroscopic cracks, the finite element method (fem) is the best method to represent it. In this method all the complexities of the problems, such as varying shape, boundary conditions and loads are maintained as they are but the solutions obtained are approximate.

Benarbia and benguediab [4] studied the cracks propagation in concrete structures, they reached that cracks propagation is a phenomenon which can be described by linear-elastic fracture mechanics concept and perfect description of cracks propagation starts from the criterion of the critical stress intensity factor. This method makes it possible to quantify the effects of the presence of a crack and its influence on the behavior with the fracture of the structure. Hence, studying fracture parameters will help to prevent catastrophic failures of structures and will be one of the important aids in material engineering.

Prasanth [5] studied the fracture parameter critical stress intensity factor for the concrete grade 30, and reported that the critical stress intensity factor depends on depth ratio. He reported that the critical stress at which failure occurs can be predicted for any structure subjected to crack. Simetal [6] studied the size of specimen that affects the compressive strength of lightweight concrete. He used specimens with dimensions varying from 50 to 150 mm. This study revealed that the strength decreases when the specimen size increases.

Fladr and bily [7] determined the relation between the specimen size and the mechanical properties of high-strength fiber-reinforced concrete (hsfrc). The compressive strengths ranged from 100 to 175 MPa. They used four sizes of cubic specimens 40, 100, 150 and 200 mm. The compressive strengths were measured for cubes specimens under loading rate 0.5MPa/s. They reached that whenever the specimen size increases the strength decreases to a certain value and afterwards the curve closes to straight line.

Subbarao and venkateswararoe [8] studied three beams, having sizes of 150 x 250 x3200 mm, 150 x 250 x 3100 mm and 150 x 250 x 3000 mm length. All the beams were tested for a simply supported span condition under bending. ANSYS was employed for predicting the load displacement response of the beams with cracks and without cracks. They reached that the presence of a crack reduced the strength of the beam when compared with the normal beam. The results obtained from a normal beam using the finite element analysis were in good correlation with the theoretical values.

The determination of stress intensity factor is a critical task in fracture mechanics. Numerical methods such as the finite element and boundary element methods [9] are used to determine the stress intensity factor. These methods are applicable to the description of not only different size specimens but also specimens with varying geometry and strength [10, 11].

In this work we aimed firstly to understand the behavior of cracked concrete beams under vertical load in order to improve their design. Using the finite element analysis program

(ANSYS) to develop a model to study the parameters that affect the behavior of cracked concrete beams and also to determine the stress intensity factor. Several parameters of concrete were checked using a general three dimensional finite element model idealization. The behavior of concrete beam with edge cracks with various geometries have been generated and loaded by vertical monotonic concentrated loading conditions. The effect of crack-depth ratio, specimen thickness and compressive strength on SIF have been investigated.

## II. METHODOLOGY

In the development of the 3D finite element model using ANSYS, concrete was modeled using 3D solid element. The model was meshed as tetragonal mapped elements [12]. The support conditions provided are simple supports with an effective span of 1300 mm. The model was assumed as linear elastic and homogeneous and was analyzed under static loading condition. The solid element used was SOLID186. The element is defined by 20 nodes having three translation degrees of freedom at each node in the x, y and z directions. In the present numerical analysis, an idealization was performed for 9 beams of 1500 mm length, L, and 400 mm, depth, d, and various beam widths, b, were used ,120, 250 and 400 mm. The crack-depth ratios (a/d), in all beams were maintained as 0.1, 0.2 and 0.3, and three levels of concrete strengths were used 25, 40 and 55MPa. The concrete properties and elements of the numerical program used in this study are shown in table (1). The 3D model of the beam is shown in Figs. 1 to 3.

TABLE 1  
MECHANICAL PROPERTIES OF CONCRETE AND NUMERICAL PROGRAM

Compressive strength	f <sub>cu</sub> = 55MPa			f <sub>cu</sub> = 40MPa			f <sub>cu</sub> = 25MPa		
Bending tensile strength	4.95MPa			3.52MPa			2.1MPa		
Modulus of elasticity , MPa	54120			39360			24600		
Passion's ratio	0.18			0.18			0.2		
	P ( N )			P ( N )			P ( N )		
a/d									
Thickness, mm	0.1	0.2	0.3	0.1	0.2	0.3	0.1	0.2	0.3
T=120mm	48730			34650			20670		
T=250mm	101530			72200			43070		
T=400mm	162460			115520			68923		

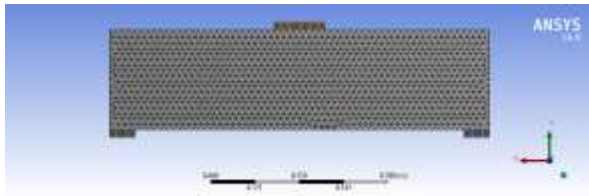


Fig. 1. 3-D meshed beam model.

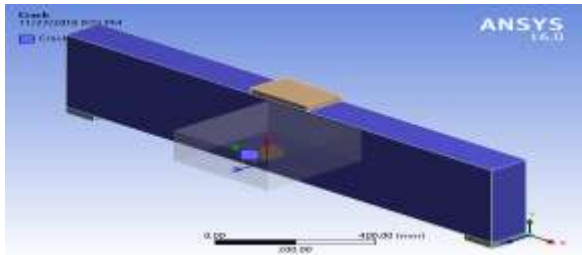


Fig. 1. Idealization of crack geometry in beam

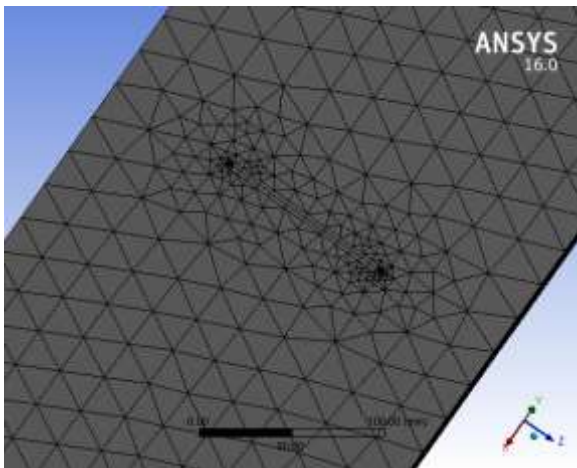


Fig. 3. Crack idealization

**III. RESULTS AND DISCUSSION**

The values of stress intensity factors obtained from ANSYS are shown in table (2). The effect of thickness *b*, crack depth ratio *a/d* and compressive strength *f<sub>cu</sub>* on the stress intensity factor are illustrated in Figs. 4 to 6

TABLE 2  
THE VALUES OF STRESS INTENSITY FACTOR OBTAINED FROM ANSYS

Compressive strength	<i>f<sub>cu</sub></i> = 55 MPa			<i>f<sub>cu</sub></i> = 40 MPa			<i>f<sub>cu</sub></i> = 25 MPa			
	<i>a/d</i>	0.1	0.2	0.3	0.1	0.2	0.3	0.1	0.2	0.3
Thickness, mm										
T=120mm		18.721	26.428	29.349	13.312	18.792	20.869	7.9411	11.21	12.449
T=250mm		16.84	21.409	22.848	11.975	15.224	16.248	7.1435	9.0819	9.692
T=400mm		16.81	21.005	22.468	11.958	14.936	15.976	7.1312	8.9107	9.531

Tables 3 and 4 show the values of stress intensity factor KIC obtained from the experimental result (M. Karamloo, M. Mazloom and G. Payganeh, 2016) and (H.Chen , R. K L Su and A. Kwan,2010)

TABLE 3  
THE VALUES OF STRESS INTENSITY FACTOR OBTAINED FROM EXPERIMENTAL RESULT

Compressive strength, <i>f<sub>cu</sub></i> MPa	48.83	40.45	27.02
Stress intensity factor, KIC, MPa√mm	30.18	27.04	18.04

M. Karamloo, M. Mazloom and G. Payganeh [14] studied the fracture parameters of self-compacting concrete, they concluded that as the compressive strength decreased, the fracture toughness decreased for the same crack /depth ratio

TABLE 4  
THE VALUES OF STRESS INTENSITY FACTOR OBTAINED FROM EXPERIMENTAL RESULT

Crack depth ratio , <i>a/d</i>	.382	.401	.407	Crack depth ratio , <i>a/d</i>
Stress intensity factor, KIC, MPa√mm	39.84	40.48	42.37	Stress intensity factor, KIC, MPa√mm

Chen, R. K L Su and A. Kwan [15] studied the fracture parameters of plain concrete using three-point bending tests. They concluded that as the crack /depth ratio increased, the fracture toughness increased.

*Effect of Specimen Thickness*

Fig. 4A, B and C show the variation of the stress intensity factor, K1C, against specimen thickness, *b*, for crack-depth ratios, *a/d*, = 0.1, 0.2 and 0.3 respectively. The data for each *a/d* are plotted at different compressive strength, *f<sub>cu</sub>*, 55, 40 and 25MPa. As shown in the figures, the results indicate a general trend, where the fracture toughness value of concrete beams, K1C, decreases with increasing the specimen thickness. These findings are typical for all crack-depth ratios. Similar trend was reported in [13] for conditions of plane stress or plane strain state.

As seen, also, from these figures, the rate of decrease in the values of K1C is mainly affected by the specimen's thickness

and crack-depth ratios. For example, from Fig. 3A, for  $f_{cu} = 55\text{MPa}$  and  $a/d = 0.1$ , as the thickness of specimen increases from 120 mm to 250 mm the rate of decrease in K1C value is about 10.0475%. After that, the K1C decreases with a steady state manner when the thickness increases from 250 mm to 400 mm, by about 0.1781%.

For  $f_{cu} 25\text{MPa}$  and  $a/d = 0.1$ , as the thickness of specimen increases from 120 mm to 250 mm the rate of decrease in K1C is about 10.0427%. After that, K1C decreases with steady state manner when the thickness increases from 250 mm to 400 mm, by about 0.1736%. We can notice that the rate of decrease in K1C values when the thickness of specimen increases from 120 mm to 250 mm and also when the thickness of specimen increases from 250 mm to 400 mm exhibit the same response for  $f_{cu} 40\text{MPa}$ .

From Fig. 3B, when  $a/d = 0.2$ , K1C shows a greater value as compared with that obtained from  $a/d = 0.1$ . Moreover, the rate of change in K1C values are higher than that obtained from case  $a/d = 0.1$ . For example when  $f_{cu} = 55\text{MPa}$  and  $a/d = 0.2$ , as the thickness of specimen increases from 120 mm to 250 mm the rate of decrease in K1C values is about 18.9912%. After that, K1C decreases with a value of about 1.8871% when the thickness increases from 250 mm to 400 mm.

For  $f_{cu} 25\text{MPa}$  and  $a/d = 0.2$ , as the thickness of specimen increases from 120 mm to 250 mm the rate of decrease in K1C is about 18.9839%. After that, K1C decreases with steady state manner when the thickness increases from 250 mm to 400 mm, by about 1.8851%. We can notice that the rate of decrease in K1C values when the thickness of specimen increases from 120 mm to 250 mm and also when the thickness of specimen increases from 250 mm to 400 mm exhibit the same response for  $f_{cu} 40\text{MPa}$ .

From Fig. 3C, when  $a/d = 0.3$ , K1C have the greatest values compared with that obtained for  $a/d = 0.1$  and  $a/d = 0.2$ . For example when  $f_{cu} = 55\text{MPa}$  and  $a/d = 0.3$ , as the thickness of specimen increases from 120 mm to 250 mm the rate of decrease in K1C values is about 22.1507%. After that K1C decreases with a value of about 1.6632% when the thickness increases from 250 mm to 400 mm.

For  $f_{cu} 25\text{MPa}$  and  $a/d = 0.3$ , as the thickness of specimen increases from 120 mm to 250 mm the rate of decrease in K1C is about 22.1431%. After that K1C decreases with steady state manner when the thickness increases from 250 mm to 400 mm by about 1.6611%. We can notice that the rate of decrease in K1C values when the thickness of specimen increases from 120 mm to 250mm and also when the thickness of specimen increases from 250mm to 400mm exhibit the same response for  $f_{cu} 40\text{MPa}$ . From the previous results, we can notice that the K1C values are mainly controlled by the crack depth ratio and concrete compressive strength

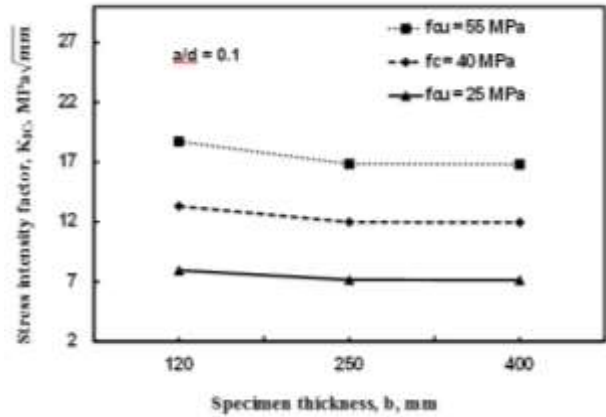


Fig. 4. Stress intensity factor, K1C, against specimen thickness, b, for crack-depth ratios,  $a/d, = 0.1$ , at different compressive strength,  $f_{cu}$ , 55, 40 and 25MPa

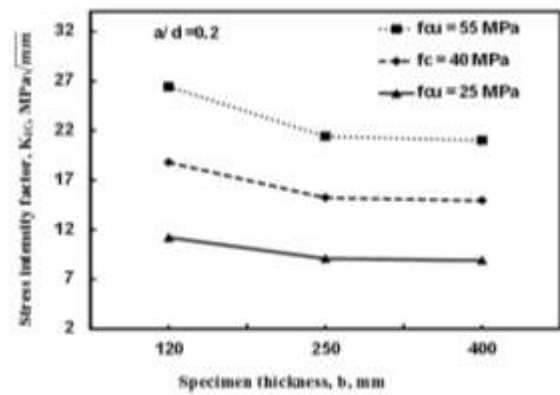


Fig. 4.B Stress intensity factor, K1C, against specimen thickness, b, for crack-depth ratios,  $a/d = 0.2$ , at different compressive strength,  $f_{cu}$ , 55, 40 and 25MPa.

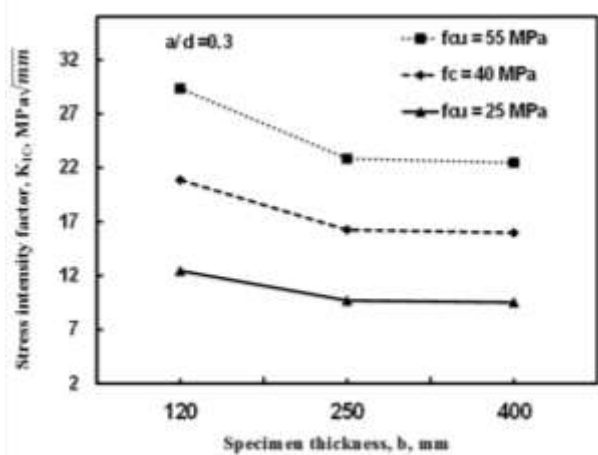


Fig. 4.C, shows behavior of stress intensity factor, K1C, against specimen thickness, b, for crack-depth ratios,  $a/d, = 0.3$ , at different compressive strength,  $f_{cu}$ , 55, 40 and 25MPa.

*Effect of Crack-Depth Ratio*

The variations of stress intensity factor, K<sub>1C</sub>, against crack-depth ratios, a/d, for specimen thickness, b = 120 mm, 250 mm and 400 mm are shown in Figs. 5A, B and C, respectively. The data for each beam thickness are given for different compressive strength, f<sub>cu</sub>. From figure 4, we can notice that, K<sub>1C</sub> increases as the crack-depth ratio increases. These results are typical for all specimen thicknesses. Similar results were presented in [13, 15] and were attributed to the size of process zone ahead of the crack tip.

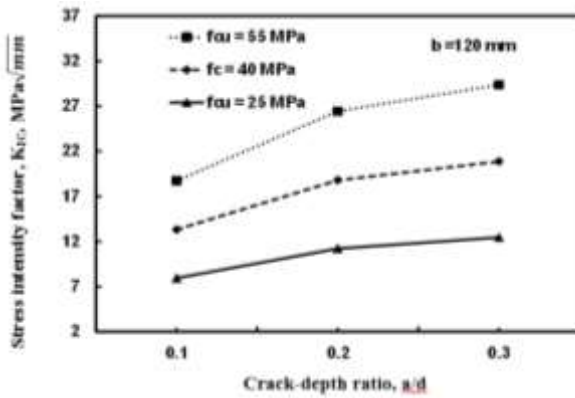


Fig. 5A The stress intensity factor, K<sub>1C</sub>, against specimen thickness, b, for crack-depth ratios for specimen thickness, b = 120 mm.

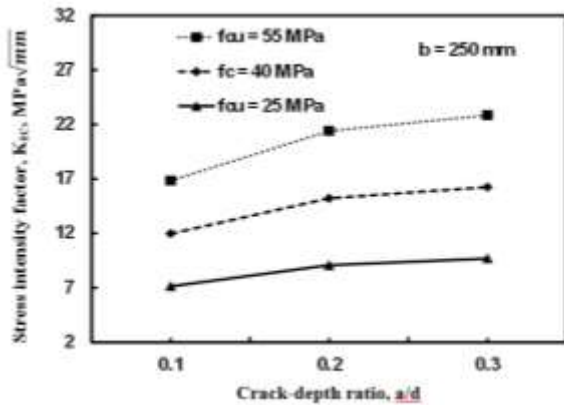


Fig. 5B The stress intensity factor, K<sub>1C</sub>, against specimen thickness, b, for crack-depth ratios for specimen thickness, b = 250 mm.

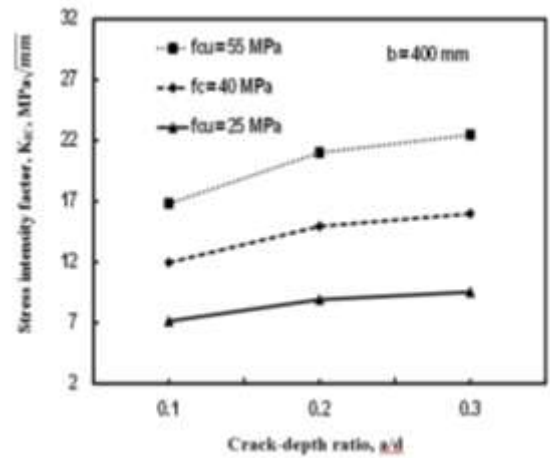


Fig. 5C The stress intensity factor, K<sub>1C</sub>, against specimen thickness, b, for crack-depth ratios for specimen thickness, b = 400 mm.

*Effect of Concrete Compressive Strength*

Figs. 6A, B and C show the variation of stress intensity factor, K<sub>1C</sub>, against concrete compressive strength for specimen thickness, b= 120, 250 and 400 mm, respectively. The data for each beam thickness are plotted at different crack-depth ratios, a/d. As shown in Fig. 6, the K<sub>1C</sub> increases as the compressive strength increase.

For example, for b = 120 mm and a/d = 0.1, Fig. 6A, as the compressive strength increases from 25MPa to 40MPa, the value of K<sub>1C</sub> increases from 7.9411 to 13.312MPa√mm. After that K<sub>1C</sub> increases from 13.312 to 18.721MPa√mm when the compressive strength increases from 40 to 55MPa by about 40.63%. Similar trend was found in case of a/d= 0.2 and 0.3.

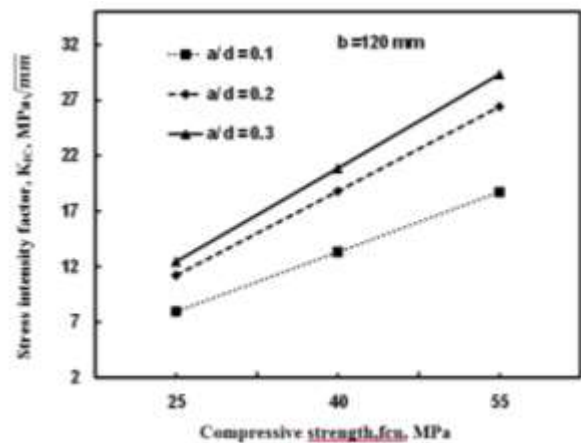


Fig. 6A The stress intensity factor, K<sub>1C</sub>, against concrete compressive strength for specimen thickness, b = 120 mm.



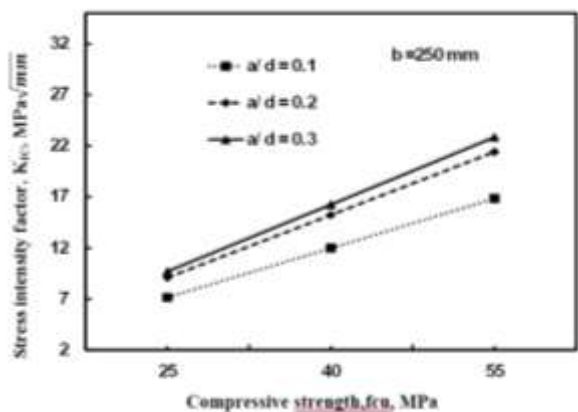


Fig. 6B The stress intensity factor, K<sub>1C</sub>, against concrete compressive strength for specimen thickness, b = 250 mm.

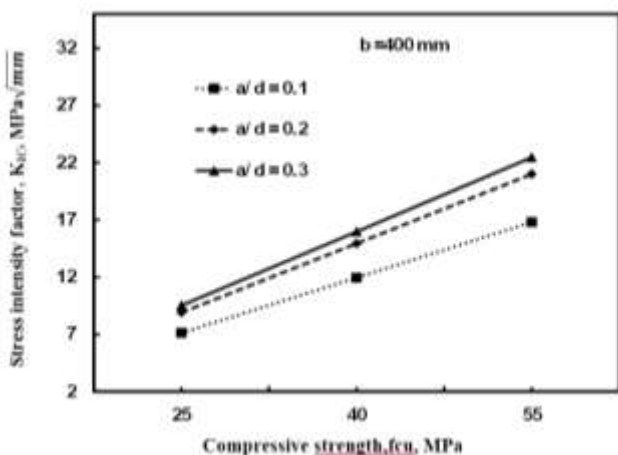


Fig. 6C The stress intensity factor, K<sub>1C</sub>, against concrete compressive strength for specimen thickness, b = 400 mm.

**IV. CONCLUSIONS**

In this work a numerical investigation for the effect of specimen thickness, crack depth ratio and compressive strength on the critical stress intensity factor, K<sub>1C</sub>, was conducted. Concrete beam specimens were tested and idealized with the aid of the finite element program ANSYS. Based on the obtained results, the following conclusions can be drawn.

1. The stress intensity factor, K<sub>1C</sub>, depends on the thickness of specimen and is found to be reducing upon increasing the thickness of specimen.
2. K<sub>1C</sub> changed with high rate when the thickness increased from 120 mm to 250 mm then it changed with very smooth rate when the thickness increased from 250 mm to 400mm.
3. The value of the stress intensity factor depends on the

crack-depth ratio and is found to be increasing upon increasing the crack length. It is noticed that K<sub>1C</sub> has the highest value when crack-depth was equal to 0.3.

4. K<sub>1C</sub> changed with high rate when the crack-depth ratio increased from 0.1 to 0.2, then the rate decrease when the crack-depth ratio increased from 0.2 to 0.3.

5. The stress intensity factor depends on the strength of concrete and is found to be increasing upon increasing the concrete strength; it was found that K<sub>1C</sub> has the highest value when the strength of concrete was 55 MPa.

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