Numerical Simulation of Natural Convection Heat Transfer using Solid and Hollow/Perforated Pin Fin Heat Sinks

محاكاة عددية لانتقال الحرارة بالحمل الحر باستخدام مشتت حراري ذو زعانف صلبة وآخر ذو زعانف مفرغة ومثقبة

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الملخص العربي:- تم اجراء دراسة عددية لانتقال الحرارة بالحمل الطبيعي من مصدر حراري ذو زعانف تبريد دائرية قصيرة مصمتة ومجوفة مثقبة بأشكال مختلفة باستخدام حزم البرامج (Fluent Ansys 16.0). تمت مناقشة اعتمادية انتقال الحرارة على شكل زعانف التبريد الدائريةالمصمتة القصيرة والمجوفة والمثقبة المرصوصة بشكل متعارض على مساحة ثابتة للوح الساخن.تم تطويع الكود سمبل(SIMPLE)لتحليل حجم معين للدراسة (control volume)لحل معادلات (السريان – الحركة – الطاقة) باستخدام التقريب Upwind technique ذو الرتبة الثانية.وقد بينت النتائج التى تم الحصول عليها لمدى رقم رايلى أكبر من9.3×106وأقل من 1.63×107 أن أداء المصادر الحرارية ذو الزعانف الدائرية القصيرة المجوفة والمثقبة أعلى من تلك المصاحبة للمصادر الحرارية ذات الزعانف الدائرية المصمتة. تم إستنتاج أن مصفوفة الزعانف الدائرية القصيرة المجوفة المثقبة ذات نسبة القطر الداخلى الى القطر الخارج 0.5 وقطر ثقب 6 مم تعطى أفضل أداء وأقل فى الوزن من نظيرتها ذو الزعانف الدائرية القصيرة المصمتة

Abstract—Free convection heat transfer from circular solid and hollow/perforated pin fin heat sinks with different geometry and heat flux is numericallystudied using FLUENT ANSYS 16.0. The geometric dependence of heat transfers from heat sinks of solid and hollow/perforated fins with staggered pattern, attached on a fixed area heated base plate is discussed. A SIMPLE code is developed using control volume approach to solve governing equations (mass, momentum and energy), and the second order upwind technique is used. The results were carried out for a range of Rayleigh number, $9.3\times10^6 < Ra < 1.63\times10^7$. It showed that the performance of the hollow/perforated fin heat sink is higher than that for the solid fin heat sink. It is also investigated that hollow/perforated fin array with inner-to-outer diameter ratio ($D_i/D_o=1/2$) and perforation diameter ($d_P=6$ mm) is the best sample giving the maximum performance and less amount of weight compared to the corresponding solid ones.

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I. INTRODUCTION

FREE convection is widely used because it doesn`t require a power supply for a fan and high reliability. For enhancing heat transfer from system components in industry to prevent the destructive effects due to overheating, extended surfaces are used. Pin fins are used in various applications such as cooling electronic and thermoelectric devices, heat exchangers and solar energy applications, etc. The optimization of fins size and modification of their shapes play an important role to achieve minimum cost, weight and size and also increase the rate of heat dissipation. A lot of research work has been conducted to study the effect of geometry and operating parameters on the heat transfer mechanism. Al Essa et al. [1]compared the heat dissipation enhancement from a perforated horizontal rectangular fin under free convection.

They realized that the perforated horizontal rectangular fin gives more heat transfer and a reduced amount of weight compared to corresponding solid fins. Elshafei[2] experimentally studied free convection heat transfer from the circular pin fin heat sink and compared it with perforated. He observed that in the case of perforated fin heat sink, Nusselt number of upward arrangement was lower than sideward arrangement. Bahadure et al.[3] experimentally studied the thermal performance of a pin fin heat sink made from copper, aluminum and mild steel with a number of perforations on the pin fin. They observed that the average heat transfer coefficient is higher for three perforations compared to solid, single, and two perforations respectively. Murtadha et al.[4]studied natural convection heat transfer through solid and perforated square pin fin heat sinks. They observed that the perforated pin fin gives better heat transfer than solid pin fin. Awasarmol et al. [5] investigated the heat transfer characteristics of perforated pin fin array in free convection by changing the perforation diameter from 4-12 mm and inclination angles from 0-90°. They observed that fins with 12 mm diameter of perforation present greater heat transfer at an angle of45°. Al-Widyan et al. [6] studied numerically the enhancement of heat dissipation for a perforated fin in free convection.. They found that the heat transfer increased as the spacing between holes decreased. Senol, etal.[7]carried out a parametric study to investigate the effect of fin spacing, fin length, fin height and concluded that overall heat transfer is superior with the decrease in fin length and the increase in fin height, hence increases with H/L. Al-Essa et al. [8]investigated the free convection heat transfer augmentation from triangular perforated horizontal rectangular fin. They found that temperature drop alongside the solid fin length is lower than that on the equivalent perforated fin. Hanamantet al.[9] numerically and experimentally evaluated natural convection heat transfer and fluid flow characteristics of fin array with lateral circular perforation, equipped on the horizontal flat surface. They concluded that heat dissipate increases with the increase in the size of perforation. Welling et al. [10] discussed the effect of change in the fin height-to-fin spacing ratio (H_f/S_r) on the variation of temperature then proposed the optimum value equation for (H_f/S_r). Their results showed that the vertical orientation of fin array is the most efficient including all other types. Chaddock[11] varied the fin height and its spacing in the large vertical fin arrays and deduced that about 20% of the total heat transfer is contributed by the radiation heat transfer. Naik et al. [12] experimentally studied the free convection heat transfer from heat sink with a vertical rectangular fin with a horizontal shroud situated adjacent to and above the fin tips. They found that the increase in the ratio of the shroud clearance to fin height from (0 to 1) resulted in a higher heat transfer rate from the heat sink. Leung[13] studied the heat transfer from polished duralumin vertical rectangular fins and found that the optimum thickness

of fins was from 20mm to 60mm. Ko et al. [14] experimentally studied the heat transfer from vertical rectangular fin array made from stainless-steel under free convective conditions. They concluded that the optimum fin spacing relating to changes in temperature excess from 20 to 40°C.Natural convection and radiation heat transfer from a heat sink with five staggered cylindrical pin fins were experimentally studied by Sparrow and Vemuri [15]. They found that the radiation was a dominant factor and contributing 25-40% of the total heat transfer. Alessio et al. [16]experimentally investigated the free convection and radiation heat transfer from a heat sink of inclined staggered fins with inclination angles of 30° and 60°down from the horizontal. They reported that the rate of heat transfer decreased by 10% lower than the horizontal array. Zografos and Sunderland [17] studied natural convection and radiation from pin fin arrays which were arranged inline and staggered. It was presented that the inline arrangements presented higher heat dissipation rate than the staggered ones. Huang et al. [18] experimentally investigated the free convection heat dissipation from pin fin heat sinks subjected to the effect of orientation. They found that the downward facing orientation gave a lower heat transfer coefficient than sideward and upward orientation. Bahadur and Cohen [19] carried out an optimization of free convection fin arrays formed of polyphenylenesulphide polymer. They found that the geometrical parameters were the dominant factor affecting the heat dissipation rate.

From the previous review, many researchers deal with the heat transfered from heat sink but a very little of them deal with the hollow perforated pin fins relaying on natural convection mode. So, the present work deals with the free convection heat transfer performance of solid and hollow perforated pin fin heat sinks and also studied the heat sink performance of pin fin diameter ratio (D_i/D_o)and perforation diameter (d_p) effect for horizontal base plate orientation.

II. NUMERICAL METHOD

A. Physical Model

In this study, a perspective view of heat sinks are composed of an array of eight (a) solid pin fins (SPF), (b) hollow/perforated pin fins (HPPF)and (c) top view of heat sink reshown in Fig.1. The base plate of the heat sink is 150mm wide, 150mm long and 10mm thickness. For the heat sinks, each fin is 50mm long, 12mm outer diameter, inner-to-outer diameter(D_i/D_o=1/3,1/2,2/3)and perforation diameter (d_p= 3.5,4,5,6mm). The heat sink was made of aluminum (k= 237W/mK). Pin fins were arranged in a staggered manner with inter-fin spacing ratios of (S_h/D_o=3.5, S_v/D_o=5). The perforated hole was at 5mm from the base plate of the heat sink. Detailed dimensions of investigated heat sink samples are listed in Table 1.
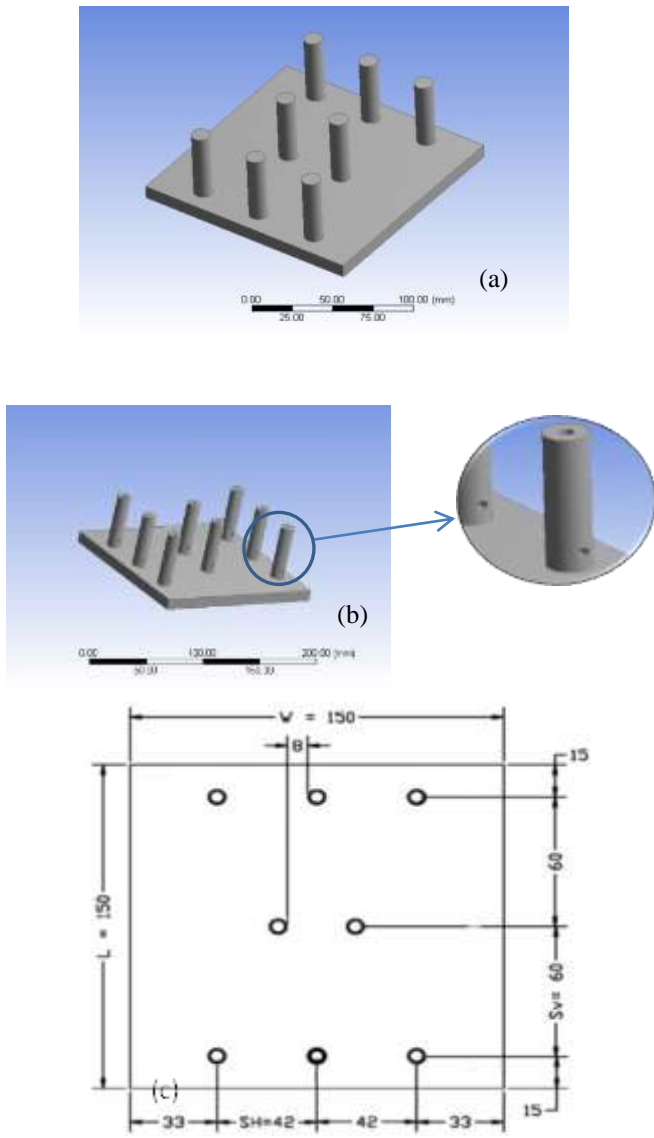
(a)

(b)

Fig. 1. A schematic of pin fin heat sink: (a) solid fins; (b) hollow/perforated fins; (c) top view of heat sink.

TABLE 1
GEOMETRIC DETAILS OF INVESTIGATED HEAT SINK MODULES.

	sample	S_h mm	S_v mm	W,L mm	H_f mm	D_o mm	D_i mm	d_p mm	Ψ (A_t/A_{bp})
Solid	1						0	0	1.74049
Hollow/perforated d_p=Const.	2						4	3.5	1.99665
	3						6	3.5	2.09231
	4	42	60	150	50	12	8	3.5	2.18187
Hollow/perforated D_i,D_o=Const.	5						6	4	2.09577
	6						6	5	2.10268
	7						6	6	2.10959

B. Data reduction

The finning factor Ψ is defined as:

$$\Psi = A_t/A_{bp} \tag{1}$$

$$A_{bp} = WL \tag{2}$$

$$A_{fp} = N\left[\frac{\pi}{4}(D_o^2 - D_i^2)\right] \tag{3}$$

$$A_t = A_{bp} + A_f \tag{4}$$

$$A_f = N\left[\frac{\pi}{4}(D_o^2 - D_i^2) + \pi(D_o + D_i)H_f \right.$$
$$\left. + \pi(D_o - D_i)d_h\right] \tag{5}$$

The terms A_{bp}, A_t, and A_f are defined as the unfinned portion of the base plate, total heat sink and pin fins surface areas. Also, A_{fp} is the pin fins projected area, W and L are the width and length of the base plate, respectively, N is the number of fins, D_i and D_o are the inner and outer diameter of the pin fins, respectively, H_f is the height of the pin fin, and d_p is the perforation diameter.

The average Nusselt number, $\overline{Nu}$ based on the length of the base plate, can be expressed as:

$$\overline{Nu} = \overline{h}L/k \tag{6}$$

The Rayleigh number is defined as:

$$Ra = \frac{g\beta(T_{sm} - T_\infty)L^3}{\nu\alpha} \tag{7}$$

The mean surface temperature is defined as:

$$T_{sm} = (A_{bp}T_{bp} + A_f T_{fm})/A_t \tag{8}$$

Where, T_{bp} and T_{fm} are the base plate and mean fin temperatures and the thermo physical properties of the air in the $\overline{Nu}$ and Ra numbers expressions are evaluated at the film temperature, T_m, which is given by:

$$T_m = (T_\infty + T_{sm})/2 \tag{9}$$

C. Conservation equations

Mass equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{10}$$

Momentum equation:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$
$$= -\frac{1}{\rho}\frac{\partial p}{\partial x}$$
$$+ \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \tag{11}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$
$$= -\frac{1}{\rho}\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$
$$+ g\beta(T - T_\infty) \tag{12}$$

$$u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$
$$= -\frac{1}{\rho}\frac{\partial p}{\partial z}$$
$$+ \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) \qquad (13)$$

Energy equation:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) \qquad (14)$$

D. Computational Modeling and Simulation

The computational fluid dynamics modeling, simulation and solving were performed in FLUENT of ANSYS 16.0. In the CFD modeling to carry out fluid analysis, the geometry by design Modeler of ANSYS is created as shown in Fig. 1. The domain was created using "Enclosure" option around the heat sink of size 350mm×210mm×110 mm. The following assumptions are taken for the 3D numerical simulation:

1) *Natural convection and radiation heat transfer are considered.*
2) *Bothsteady fluid flow and heat transfer in the simulation study are considered.*
3) *The thermal conductivity of the heat sink has a constant value.*
4) *The air flow velocity attached to all boundaries of the domain is zero.*

E. Mesh independence

Mesh independence test as shown in Table 2 was performed by varying the mesh elements from 100,071 to 721,420 hexahedral elements for the SPF heat sink and from 212,188 to 2,688,052 tetrahedral elements for the HPPF heat sink. The optimum grid obtained at 2.2 million of tetrahedral mesh elements. The suitable boundary conditions were attached to the domains shown in Fig. 2. Air and Aluminum were apportioned to the created fluid and solid domain respectively. Activated buoyancy in the Y direction, chosen viscous model as laminar, pressure-velocity coupling using the SIMPLE algorithm developing by Patankar [20].
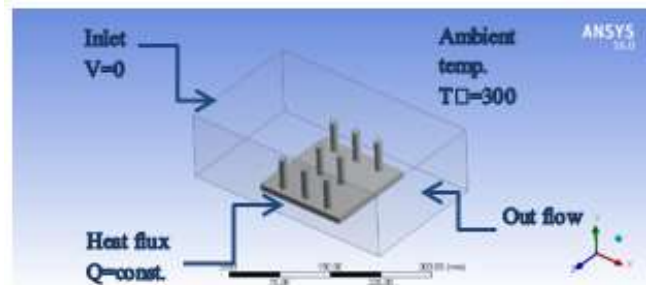

Fig. 2. The boundary conditions of pin fin heat sink.

TABLE 2
MESH INDEPENDENCE USING HEAT TRANSFER COEFFICIENT FOR THE SOLID AND HOLLOW/PERFORATED PIN FIN HEAT SINKS.

Solid Pin Fins

No. of Elements	100071	120171	137370	162113	212188	386180	500444	721420
$\overline{h}$	10.8748	10.93237	11.1781	11.28464	11.32015	11.3894	11.4157	11.4139

Hollow/Perforated Pin Fins

No. of Elements	212188	374681	899124	1118618	1643356	2001548	2217066	2688052
$\overline{h}$	11.0142	11.1548	11.3124	11.46141	11.59741	11.7284	11.8121	11.81521

III. RESULTS AND DISCUSSION

The numerical analysis has been done on the heat sink with solid and hollow with perforations pin fins and varying the hollow and perforation diameters. Three-dimensional pin fin array models are chosen and simulated by Fluent of ANSYS 16.0.For the heat sink and fluid domain, the temperature contours are explored. The effect of change of inner-to-outer diameter ratio on $\overline{Nu}$for the HPPF heat sink is studied. Finally, the effect of the change of the perforation diameter (d_p) on $\overline{Nu}$and the performance of pin fins related to the finning factor (Ψ), which is defined as the total-to-base plate area ratio. The range of Rayleigh number for laminar flow is$10^5 < Ra < 10^{10}$[21].

A. Model validation

Fig. 3 compares the variations of $\overline{Nu}$ with Ra for the SPF heat sink for the present work and the experimental work by Elshafei [2]. It shows that the maximum deviation in numerical results of $\overline{Nu}$ equal to 6.4 % at low Ra and the deviation decreased with increasing Ra until reach 0.9% at high Ra. Fig. 4 compares the variations of the temperature difference between the base plate and the ambient and shows the good matching between the numerical and experimental work.
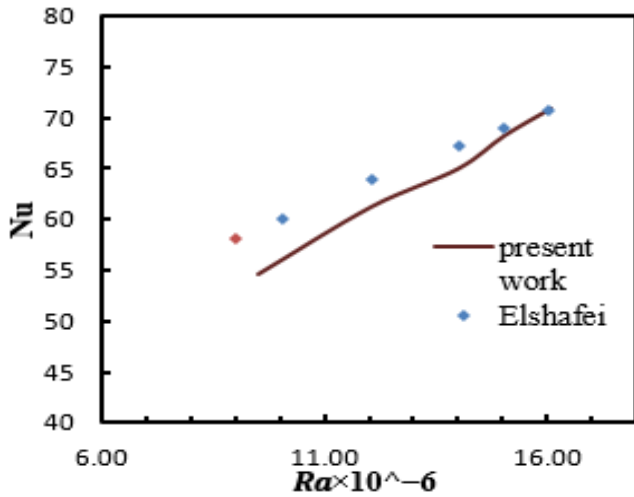
Fig. 3. Comparison of $\overline{Nu}$ variations with Ra for the present work and the data given by El-shafei [2] for the solid pin fin heat sink.
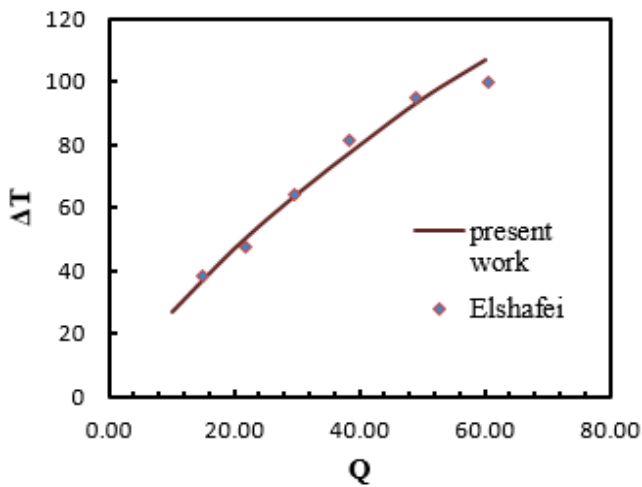

Fig.4. Comparison of temperature differences variations with heat flux for the present work and the data given by El-shafei [2] for the solid pin fin heat sink.

B. Hollow Perforated Pin Fin heat sink

The present work was performed for the heat sink with hollow perforated pin fins with different inner-to-outer diameter ratio (D_i/D_o) and perforation diameter (d_p). with a staggered arrangement and the equal number and height of fins. Over a range of $9.3 \times 10^6 < Ra < 1.63 \times 10^7$, the variation of $\overline{Nu}$ with Ra is examined.

Fig. 5 shows the average $\overline{Nu}$ variations with Ra for the heat sink with varying the inner-to-outer diameter ratio (D_i/D_o) at the same perforation diameter (d_p=3.5 mm). It is observed that $\overline{Nu}$ increases with increasing D_i/D_o, which results from the increase of the coefficient of heat transfer by reason of the increase of the intensity of induced plumes with higher D_i/D_o. But at D_i/D_o is larger than 1/2, $\overline{Nu}$ decreases again. As, it is found that $\overline{Nu}$ for the HPPF with (D_i/D_o=1/3) is more than that for the SPF by 1.01% and for HPPF with(D_i/D_o =1/2) by 2.76%. But, $\overline{Nu}$ for (D_i/D_o =2/3) is less than that for SPF by 0.96%. It may be because of the reduction of the heat transfer

by conduction from the heat sink is more than the increase of heat transfer due to induced plumes effect.

The temperature contours of the HPPF heat sinks with constant d_p varying D_i/D_o (1/3, 1/2, 2/3) at constant heat flux Q=60W are shown in Fig. 6.
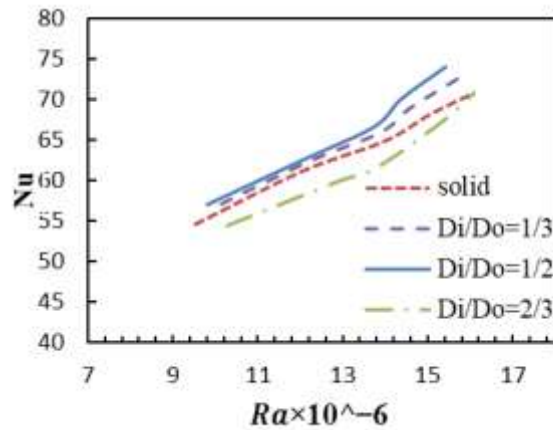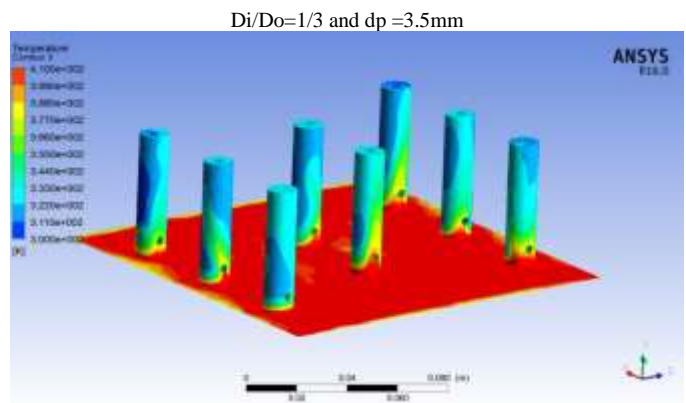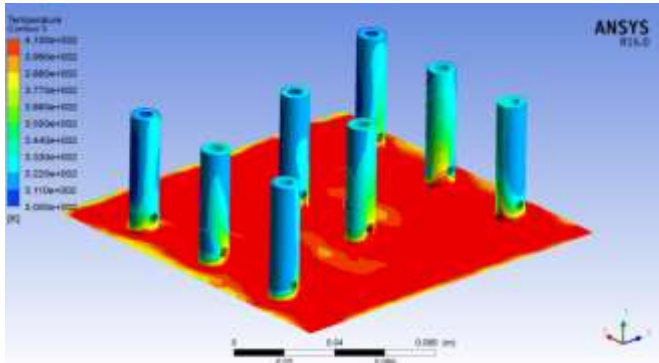

Fig. 5. $\overline{Nu}$ versus Ra of solid and hollow/perforated pin fin array with constant dp=3.5mm and varying Di/Do (1/3, 1/2, 2/3).

The variations $\overline{Nu}$ with Ra for the heat sink when varying d_p (3.5, 4, 5 and 6 mm) and fixed D_i/D_o equal to 1/2 to study the effect of increasing the perforation diameter on the heat sink performance is shown in Fig. 7. It is found that with increasing d_p, the average $\overline{Nu}$ slightly increases. That may be due to the circulation of air through the HPPF heat sink which increases $\overline{h}$ so increases $\overline{Nu}$. From the present data, it is concluded that the HPPF heat sink with (D_i/D_o =1/2 and d_p= 6 mm) give the maximum system performance of the other samples. The temperature contours of the (HPPF) heat sinks with constant D_i/D_o varying d_p (3.5,4,5,6 mm) at constant heat flux Q=60W are shown in Fig. 8. So, it is preferable to produce the hollow/perforated fins than the solid ones to reduce the amount of material using in it.
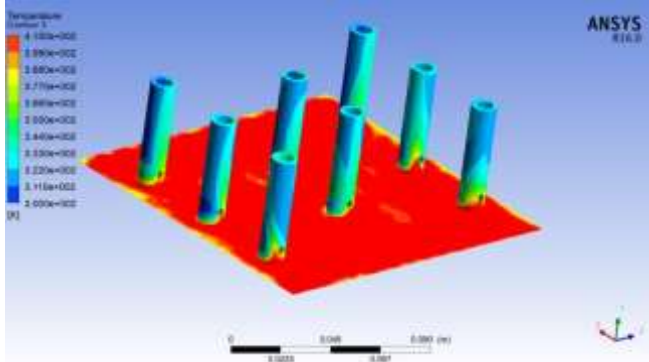
Di/Do=1/3 and dp =3.5mm

Di/Do=1/2 and dp =3.5mm
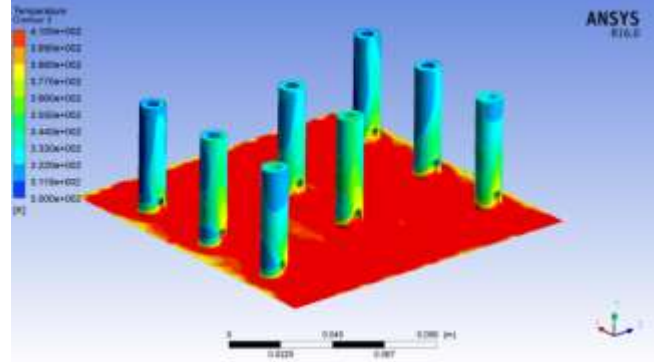

D_i/D_o=1/2 and d_p =3.5mm
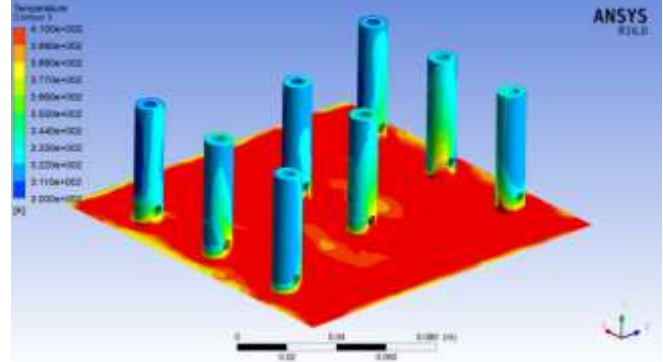
Di/Do=2/3 and dp =3.5mm

Fig. 6. Temperature contours of hollow/perforated pin fin array with constant dp=3.5mm and varying Di/Do (1/3, 1/2, 2/3) at constant heat flux Q=60W.

Di/Do=1/2 and dp =4mm

Di/Do=1/2 and dp =5mm

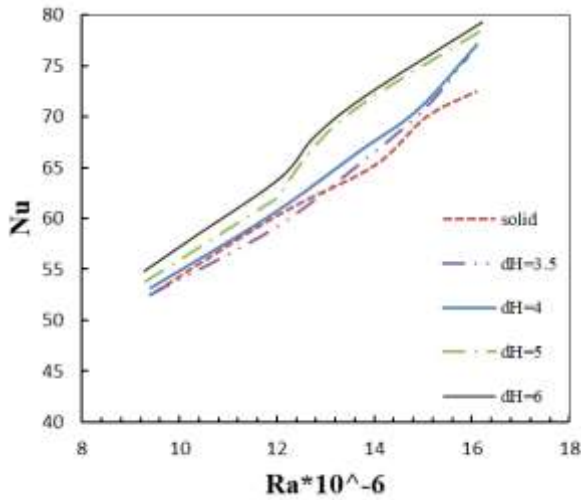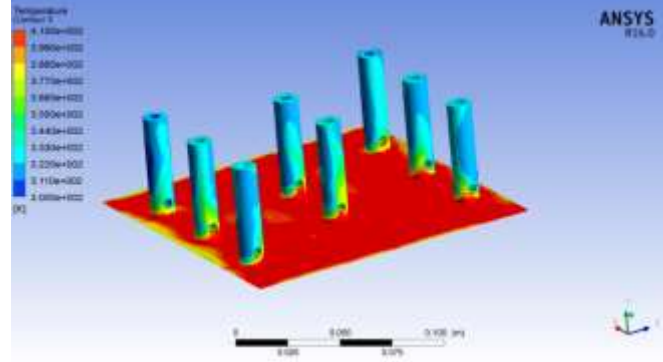Fig. 7. $\overline{Nu}$ versus Ra of solid and hollow/perforated pin fin array with constant Di/Do=1/2 and varying dp (3.5, 4, 5, 6 mm).
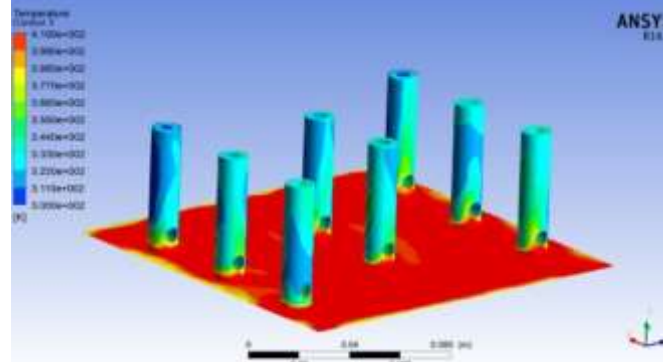
Di/Do=1/2 and dp =6mm

Fig. 8. Temperature contours of hollow/perforated pin fin array with constant Di/Do=1/2 and varying dp (3.5, 4, 5, 6 mm) at constant heat flux Q=60W.

IV. Conclusion

In this paper, a numerical study of 3-dimensional laminar fluid flow and free convection heat transfer over an array of solid and hollow/perforated fins are made. Over a range of Rayleigh number from 9.3×10^6 to 1.63×10^7, the following conclusions have been reached:

1. For all cases of heat sink studied, $\overline{Nu}$ increases with increasing Ra.

2. $\overline{Nu}$ for hollow/perforated pin fin heat sink is higher than that for solid pin fin heat sink.

3. Maximum system performance may be achieved with hollow/perforated pin fins when the inner-to-outer diameter ratio and perforation diameter are optimized. With the present configuration of pin fins, a sample of fins with $D_i/D_o = \frac{1}{2}$ and $d_p = 6$ mm is the best sample with high thermal performance.

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