Investigation of Rectangular Settling Tanks Performance in Drinking Water Treatment Plants

بحث أداء خزانات الترسيب المستطيلة في محطات معالجة مياه الشرب

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الملخص

السريان المنتظم عنصر أساسي لزيادة أداء خزانات الترسيب. وبناء عليه تتمكن الجسيمات العالقة من الترسيب بسرعة ثابتة في وقت أقل. وجود مناطق الدوامات يقلل من ترسيب الجسيمات والذي يعتبر ذو تأثير كبير على الخصائص الهيدروليكية لشكل السريان داخل خزانات الترسيب وعليه يقلل وجود مناطق دوامات كبيرة أداء خزان الترسيب. أحد الطرق المتبعة لتقليل مشاكل خزانات الترسيب هي استخدام حائل عند مدخل الخزان مما يعمل على تقليل تأثير بعض الظواهر الغير مرغوب فيها مثل مسارات القصر بين المدخل والمخرج في خزانات الترسيب. الغرض من هذه الدراسة هو بحث أداء خزانات الترسيب المستطيلة في محطات معالجة مياه الشرب نظريا باستخدام الطرق العددية وعمليا. تم بناء النموذج النظري ليتم حله باستخدام برنامج FLUENT®. تم تطبيق النموذج لمحاكاة نموذج ثلاثي الأبعاد لخزان ترسيب مستطيل باستخدام تشكيلات مختلفة لحوائل المدخل. أيضا تم استخدام نموذج معملي لخزان ترسيب مستطيل يعمل على أساس سريان مستمر وذلك لبحث أداء الخزان بقياس العكارة على طول الخزان في اتجاه السريان من المدخل للمخرج. اتضح من الدراسة تحقيق النتائج النظرية المعطاة من خلال النمذجة باستخدام الطرق العددية من خلال بحث التأثير على الأداء ممن خلال النموذج العملي حيث تم اثبات رفع أداء خزان الترسيب باستخدام حائل عند المدخل.

Abstract

A uniform flow field is essential to increase the efficient performance of settling tank. This enables particles to settle at a constant velocity and in less time. The existence of circulation regions reduces the sedimentation of particles and has major influence on the hydraulic condition of flow field inside the settling tanks. According to this, the existence of large circulation regions will reduce tank performance. A common approach for decreasing settling tanks problems is to use baffles at inlet which can reduce effects of the unfavorable phenomena such as short circuiting between inlet and outlet and density currents in settling tanks. The aim of this study is to investigate the performance of rectangular settling tanks in drinking water treatment plants numerically and experimentally. A computational model has been mapped to the commercial FLUENT® solver. Applied to simulate the flow within a 3D rectangular water settling tank with different inlet configurations where the optimum inlet design is selected. Also, a continuous flow bench scale settling tank is used to experimentally investigate the effect of inlet on the performance of the settling tank by measuring the water turbidity along the flow direction in the tank. The practical results validates the results given by CFD simulation where the performance of the tank is enhances using inlet baffles.

Keywords: settling tank, dead zones, CFD, FLUENT.

1 Settling and re-suspension velocities

Settling tanks are designed to reduce the velocity of water so as to permit suspended solids to settle out of the water by gravity. The velocity with which a particle in water will fall under the action of gravity depends upon the horizontal flow velocity of the water, the size, relative density and shape of the particle and the temperature of the water. The theoretical velocity of falling spherical particles in slowly moving water v_o (m/s), is given by Stokes' Law which is a simplified form of Newton's Law:

$$v_o = \frac{g}{18}\frac{\rho_o - \rho_w}{\mu} d^2 \qquad (1)$$

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where g = 9.81 m/s2, ρ0 is the density of the particles, ρw is the density of the fluid, d is the diameter of the particles in m and μ is the dynamic viscosity of water in pa.s, which varies with the temperature of the fluid. Apart from the settling rate in still water it is, of course, essential that once a particle has reached the base of the tank it shall not be re-suspended by the velocity of flow of water over the bed.

Camp (1946), gives the critical velocity v_C (m/s) required to start motion of particles of diameter d (mm) as:

$$v_C = \left(\frac{8\beta g(\rho_o - \rho_w)}{10^3 f\rho_w} d \right)^{1/2} \qquad (2)$$

where f is the friction factor in $(4flv^2/^2gd)$, β is in the range 0.04-0.06 for sticky flocculent materials, and 0.10-0.25 for sand and $g = 9.81$ m/s^2.

There is general agreement that this velocity should not be more than 0.3 m/s to allow sand grains to settle. This is, of course, too high a velocity for the settling of particles of light relative density (1.20 and less). At 0.2 m/s faecal matter, i.e. organic matter, will begin to settle (Twort et al 2000).

2 Ideal settling tanks

As shown in "Fig. 1" the ideal settling tank should have inlet zone, outlet zone which have the same importance of settling and sludge zones. There is certain critical settling velocity such that all particles settling faster than this value will be removed. The term Q/A_S is known as the surface loading rate or overflow rate and is equal to the critical settling velocity v_O where Q is the discharge rate and A_s is the surface area (Gregory 2006).

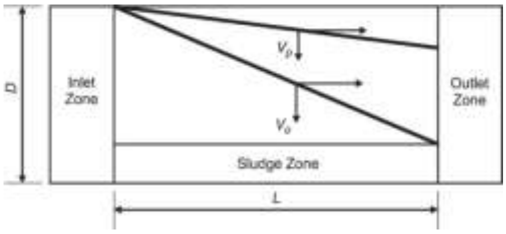


Figure 1 Typical Settling Tank Inlets

To approach this ideal as nearly as possible, the retention time τ for each particle of water is the same, equal to V/Q where V is the volume of the tank.

Also retention time τ is equal to the length of the tank divided by the horizontal velocity. This relation is valid for ideal settling tanks.

A terminal velocity v_o is selected for the sizing of a settling tank so that all particles having a velocity equal to or greater than v_o are removed. In the ideal tank; it is assumed that particles entering the tank are evenly distributed over the inlet cross section (Nalco 2008).

If all flow enters a settling basin at one point, the particles must be distributed across the entire basin width and depth before the flow velocity v_o is minimized. Depending on the efficiency of distribution, the velocity at some point could be many times faster than the velocity at other one (Alley 2007).

3 Inlet baffles

Typical settling tank inlets are shown in “Fig. 2”. According to the investigations of Camp (1946) and Swamee and Tyagi (1996) the investment costs of settling facilities contribute to a large portion (typically one-fourth to one-third) of the total cost of treatment plant construction. A uniform flow field is essential to increase the efficient performance of settling tank. This enables particles to settle at a constant velocity and in less time. The existence of circulation regions (dead zones) reduces the sedimentation of particles and has major influence on the hydraulic condition of flow field inside the settling basins. Circulation zones are named as dead zones in tanks because, in these regions, water is trapped and particulate fluid will have less volume for flow and sedimentation. According to this, the existence of large circulation regions will lower tank performance (Heydari 2013).

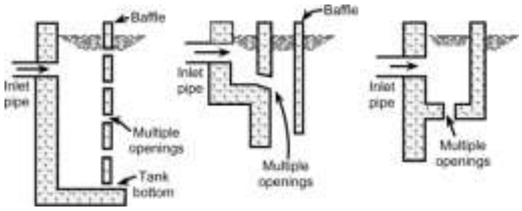


Figure 2. Typical Settling Tank Inlets

Zhang (2014) studied a different configuration of full width rectangular inlet baffle. He proves that re-circulating current is always existed in sedimentation tanks, circulation zones, dead zones occupy the effective sedimentation volume, so that the sedimentation tank will have less volume for settling, thereby the existence of re-circulating current or circulation zones will reduce tank efficiency.

Lutfy et al (2015) studied the effect of using half cylindrical inlet baffle of settling tank performance. They developed a numerical model for rectangular settling tank with length to width ratio 1:1. They concluded that the optimum half cylindrical inlet baffle hydraulic diameter to tank hydraulic diameter ranges from 18.8% to 25.2%.

Different methods are proposed for reducing the effects of these problems and increasing the tank performance. A common approach for decreasing settling tanks problems is to use baffles which can reduce effects of the unfavorable phenomena such as short circuiting between inlet and outlet and density currents in settling tanks. The baffles usually install at the bottom or surface of the rectangular settling tanks. Various studies have been done on the effects position and size of baffles on the flow and hydraulics of settling tanks. In settling tanks for increasing their sedimentation performance, baffles are usually placed in the front of inlet opening (Heydari 2013).

Shahrokhi et al (2011 a), investigated the effects of baffle location on the flow field. Using CFD and VOF methods, they developed a numerical simulation of flow in the tank through the FLOW-3D® software. Results show that the installation of a baffle improves tank efficiency in terms of sedimentation. The baffle acts as a barrier, effectively suppressing the horizontal velocities of the flow and reducing the size of the dead zones. A baffle also reduces turbulent kinetic energy and induces a decrease in maximum magnitude of the stream-wise velocity and upward inclination of the velocity field compared with the no-baffle tank. On the basis of these results, the baffle must be placed near the circulation region.

Shahrokhi et al (2011 b) studied the baffle effect on the flow in a rectangular primary sedimentation tank using numerical investigation. They conclude that circulation regions may appear with size sensitive to the position of the baffle when a baffle is used in the tank. The best position for the baffle is obtained when the volume of the circulation zone is minimized or the dead zone is divided into smaller parts. Thus, the best position for the baffle may lead to a more uniform distribution of velocity in the tank and minimize dead zones. Small recirculation zones, which are important to sedimentation, are also found near the entry and exit weir.

Razmi et al (2009) performed an experimental and numerical work to investigate the effects the baffle position on the flow field. In laboratory, a test rig was conducted to find the effect of the baffle position on the velocity profiles. Then, using CFD, a numerical simulation of flow in the tank was developed by fluent software. Using the experimental data, the numerical results were verified. Finally, the optimal location of the baffle was found numerically. Results show that this baffle can reduce the size of the dead zones and turbulent kinetic energy in comparison with the no-baffle condition.

Egyptian Housing & Building National Research Center (2008) considered the inlet and outlet baffles in the description of settling tanks for both water and wastewater treatment in the latest edition of Egyptian Code for the Design and Implementation of Water,

Sewage Treatment Plants, and Sewage Lift Stations.

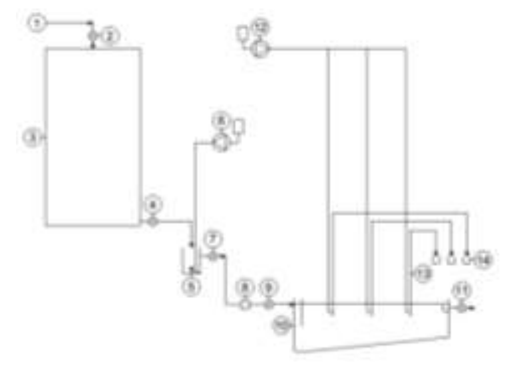
Goula (2007) studied the influence of a feed flow control baffle. The results show that an extended baffle forces the solids to move faster towards the bottom of the tank and decreases the inlet recirculation zone, thus yielding significantly enhanced sedimentation. Although the increase in the overall effectiveness by this baffle may show only a small change, this actually reflects a reduction of the effluent solids of estimated around 85%. He concluded that CFD can be a powerful tool for troubleshooting problems, particularly those associated with flow patterns in a sedimentation tank.

Water that by-pass the normal flow path through the basin and reaches the outlet in less than normal retention time occurs to some extent in every basin. It is a serious problem, causing floc to be carried out of the basin due to the shortened sedimentation time. The major cause of short-circuiting is poor inlet baffling (Adams, Jr. et al 2000).

4 Experimental apparatus

4.1 Apparatus Description

An apparatus is designed and operated to investigate the settling tank performance based with continuous feed flow. The schematic diagram of the apparatus is shown in "Fig. 3" and the real apparatus is shown in "Fig.4".



1	**Feed Line**	8	**Rotameter**
2	**Feed Line Valve**	9	**Settling Tank Feed Valve ¾”**
3	**Feed Tank 200 Lit**	10	**Settling Tank Model**
4	**Feed Tank Discharge Valve 1”**	11	**Settling Tank Discharge Valve ¾”**
5	**Flow Control Tank 4.2 Lit**	12	**Samples Collection Air Pumps 35 W**
6	**Mixing Air Pump 5 W**	13	**Samples Collection Air Lift**
7	**Flow Control Tank Discharge Valve ¾”**	14	**Samples Collection Cups**

Figure 3. Experimental Apparatus Schematic Diagram

Figure 4. Real Photo of Apparatus

The apparatus is consists of the following:

- Feed tank 200 liter (0.55 m diameter, 1.03 m height) net volume complete with feed line and discharge ball valve 1".

- Flow control tank (0.3 m length, 0.1 m width, and 0.15 m height) which is used to control and calibrate the feed discharge to the system based on the head over the outlet orifice and it is complete with discharge ball valve 3/4". This tank is aerated via air pump 5 W to provide a mixing inside the tank to prevent settling.
- Settling tank made of glass with the dimensions in mm shown in "Fig. 5".
- Three different baffles; 1/3, 2/3, and 3/3 of tank width "Fig. 6".
- Air lift system for samples collection. The system complete with required hoses for water and air phases, pneumatic fittings, and air pump 35 W with capacity 50 L/min at 0.028 Mpa differential pressure.
- Samples cups each with 120 mL capacity.

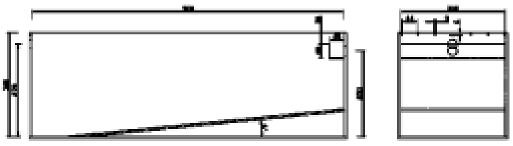


Figure 5. Experimental Settling Tank

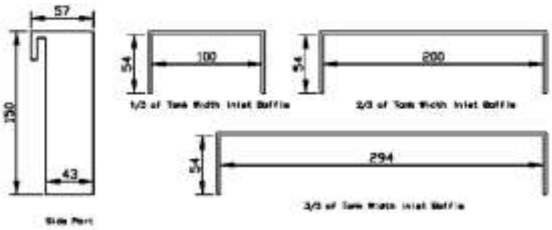


Figure 6. Inlet Baffles Dimensions

4.2 Turbidity Meter

The main issue in this study is to determine the effect of baffle on the homogeneity of the tank performance over its length by measuring the turbidity. The used turbidity meter is LaMotte 2020we with the specification shown in table 1.

Table 1. Turbidity Meter Specifications

Item	Specifications
Unit of Measure:	**2020we: NTU, AU, ASBC, EBC**
Range:	**0-4000 NTU/FNU, 0-10,500 ASBC, 0-150 EBC**
Resolution:	**0.01 NTU/FNU 10.00-10.99 0.1 NTU/FNU 11.00-109.9 1 NTU/FNU 110-4000**
Accuracy:	**From 0-2.5 NTU the accuracy is ±0.05 NTU From 2.5-100 NTU the accuracy is ±2% From 100 NTU the ccuracy is ±3%**
Detection Limit:	**0.05 NTU/FNU**
Range Selection:	**Automatic**
Reproducibility:	**0.02 NTU/FNU or 1%**
Light Source:	**Tungsten (EPA) complies with EPA 180.1 Standard 860 LED (ISO) complies with ISO 7027**

4.3 Experiment Procedures

The experiment is based on the following procedures:

1. A sample with volume 4.2 liter is used to determine the particles settling velocity comparing to the turbidity. Table 2 shows the results of this experiment and "Fig. 7" shows the turbidity values with time during one hour and half and particle diameters in µm. The settling depth is 14.0 cm which is used to determine the settling velocity.
2. Settling tank is 900 mm in length while it should be considered 700 mm to exclude the inlet and outlet zones. At surface loading equal to 1.0 m/hr; the feed flow should be 210 lit/hr.
3. The flow is calibrated by volumetric method, and the level of flow control tank is the parameter of measure.
4. The feed tank and settling is filled with clear (city) water.
5. In the flow control tank the silt is added and mixed. The silt should be

added gradually based measuring the turbidity until the targeted turbidity value is reached (16.0 NTU).

6. The valves then should be opened to allow continuous flow in the tank.
7. After 30 minutes; the samples should be collected. Samples points is shown in "Fig. 6" where sample 01 and 08 is collected manually and other samples via air lift system which is collect the samples instantly.
8. Turbidity of samples should be measured.
9. Results to be recorded and discussed.
10. Procedures from 2 to 9 should be repeated based on the following cases: tank without baffle at inlet, tank with baffle 1/3 of tank width, tank with baffle 2/3 of tank width, and tank with baffle 3/3 of tank width.

Time [Minutes]	Turbidity[NTU]	Settling Velocity [m/hr]	Particle Diameter [µm]
0	**62.9**	**-**	**-**
6	**40.2**	**1.4**	**21.811**
12	**36.2**	**0.7**	**15.422**
20	**34.4**	**0.42**	**11.946**
24	**33.1**	**0.35**	**10.905**
30	**32.4**	**0.28**	**9.754**
50	**30.6**	**0.168**	**7.555**
60	**29.1**	**0.14**	**6.897**
90	**25.5**	**0.093**	**5.631**

Table 2. Turbidity, Settling Velocity, and Particle Diameter over Time

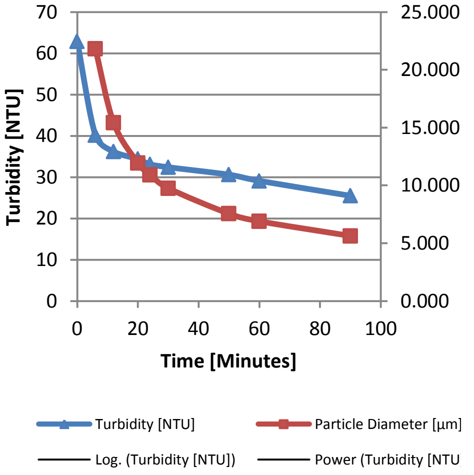


Figure 7. Turbidity and Settled Particles Diameter over Time

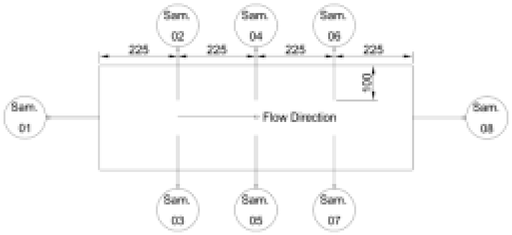


Figure 6. Samples Locations

5 Numerical model

5.1 Model and Boundary Conditions

Solving the model is based on steady state viscous k- epsilon model. As the flow is considered balanced; the steady state model shall be valid. The boundary conditions are assigned as shown in "Fig. 8" and table 3.

The turbulence intensity shall calculated using the following equations:

$$\mathrm{Re} = \frac{vD}{\gamma} \quad (3)$$

$$TI = 0.16\,\mathrm{Re}^{-1/8} \quad (4)$$

where Re is the Reynolds Number, D is the diameter m, γ is Kinematic Viscosity m^2/s, and TI is the Turbulent Intensity.

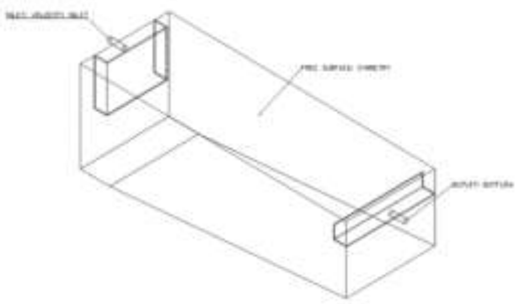


Figure 8. Model Boundary Conditions

Table 3. Boundary Condition Types

Inlet	Velocity Inlet
Outlet	Outflow
Free Surface	Symmetry
Walls	Walls

5.2 Inlet flow

Table 4 shows the specifications of velocity inlet boundary condition based on feed flow 210 lit/hr and inlet diameter 12 mm.

Table 4. Velocity Inlet Boundary Conditions

Feed Flow [lit/hr]	Surface Loading [m/hr]	Inlet Velocity [m/sec]	Re	TI %
210	1.0	0.52	6727.56	5.32

5.3 Grid Dependence Test

Grid dependence test conducted for three grids; first one is coarser mesh with 29512 nodes and 147986 elements, second one is the used mesh with 49652 nodes and 253799 elements, and third one finer mesh with 77205 nodes and 397180 elements. Used mesh has the same trend of the coarser and finer mesh as shown in "Fig. 9".

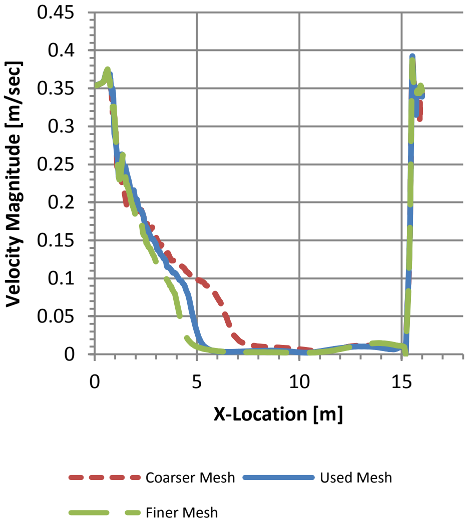


Figure 9. Grid Dependency Test

5.4 Scope of Study

The simulation should be developed based on the following cases: tank without baffle at inlet, tank with baffle 1/3 of tank width, tank with baffle 2/3 of tank width, and tank with baffle 3/3 of tank width.

6 Results

The results shall be demonstrated in the following forms:

- First form is the velocity contours at z plane =0.
- Second form is the velocity contours at y plane =2.7.
- Third form is plot shows the y-velocity over the tank length at y plane =2.0 intersection with z plane=0.5 and y plane =2.0 intersection with z plane=-0.5. Also, measured turbidity values over tank length are shown on the same figure.

6.1 Inlet Zone without Baffles

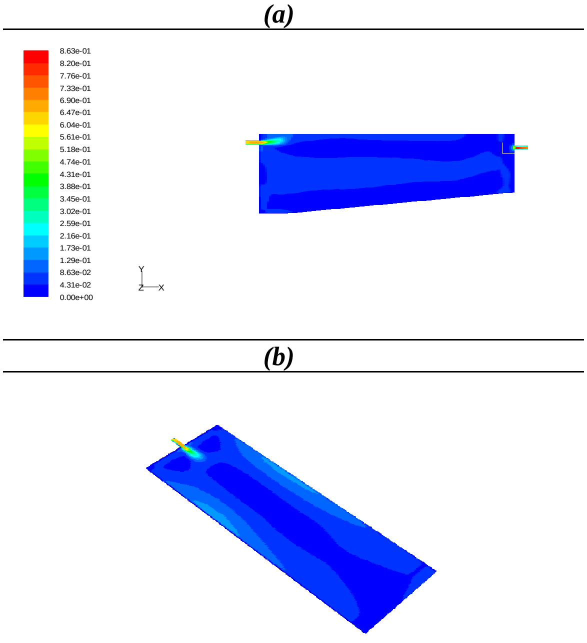


Figure 10. Velocity contours at (a) z plane=0, (b) y plane=2.0 based on no baffle at inlet

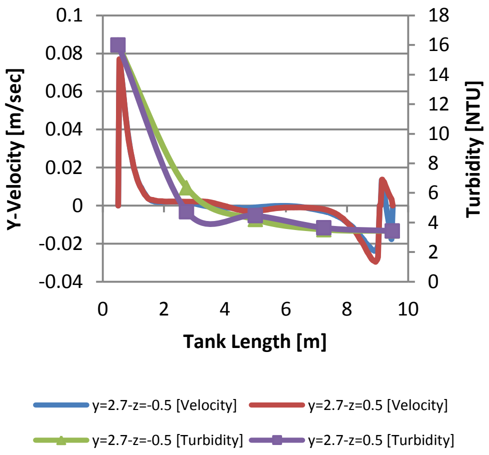
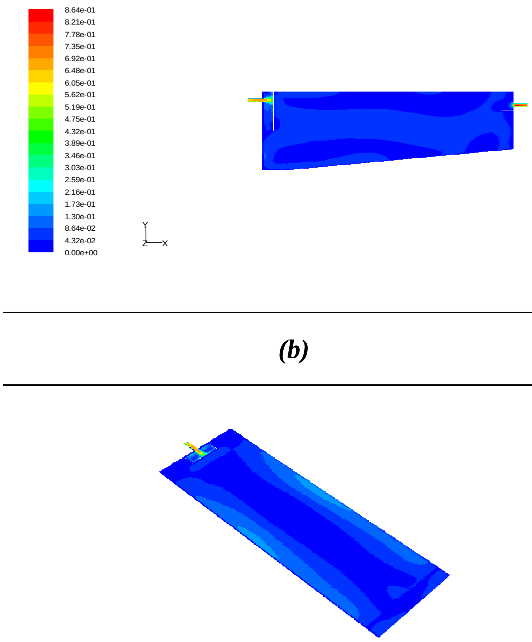


Figure 11. Y-Velocity and turbidity at y=2.0 at z plane=0.5 and z plane=-0.5 based on no baffle at in

6.2 Inlet Zone with Rectangular Baffle 1/3 of Tank Width

(a)



(b)

Figure 12. Velocity contours at (a) z plane=0, (b) y plane=2.0 based on baffle 1/3 of tank width at inlet

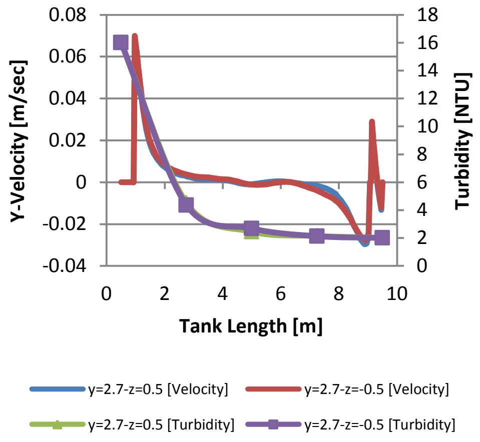
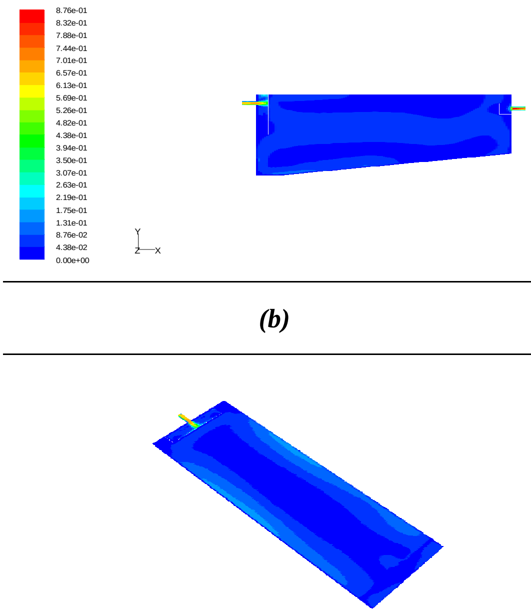


Figure 13. Y-Velocity and turbidity at y=2.0 at z plane=0.5 and z plane=-0.5 based on baffle 1/3 of tank width at inlet

6.3 Inlet Zone with Rectangular Baffle 2/3 of Tank Width

(a)



(b)

Figure 14. Velocity contours at (a) z plane=0, (b) y plane=2.0 based on baffle 2/3 of tank width at inlet

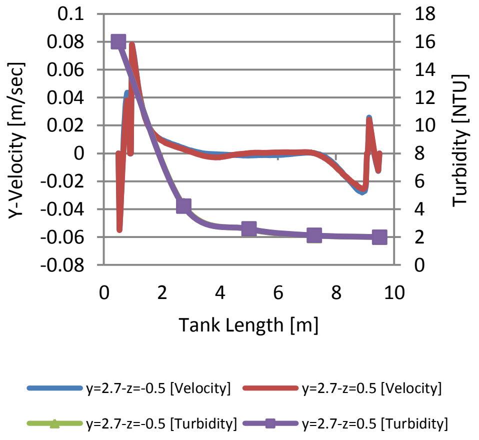


Figure 15. Y-Velocity and turbidity at y=2.0 at z plane=0.5 and z plane=-0.5 based on baffle 2/3 of tank width at inlet

6.4 Inlet Zone with Rectangular Baffle 3/3 of Tank Width

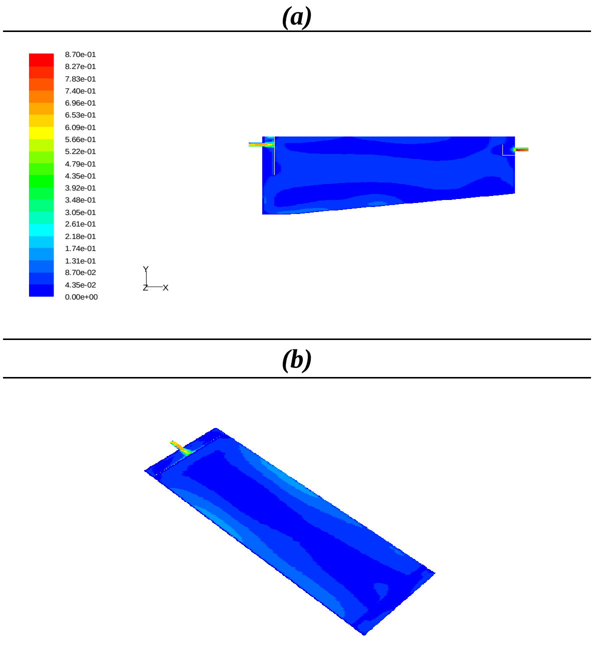


Figure 16. Velocity contours at (a) z plane=0, (b) y plane=2.0 based on baffle 3/3 of tank width at inlet

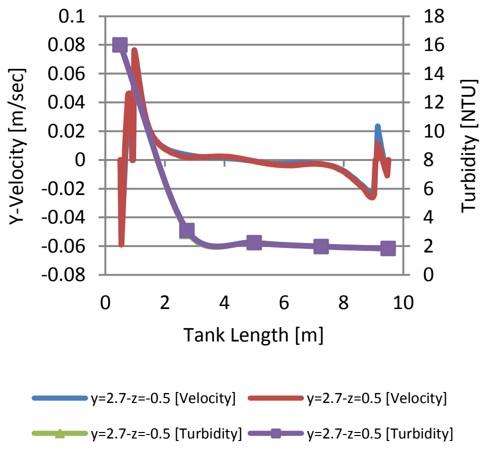


Figure 17. Y-Velocity and turbidity at y=2.0 at z plane=0.5 and z plane=-0.5 based on baffle 3/3 of tank width at inlet

7 Comparison With Previous Works

The study proves Zhang conclusions where re-circulating current is always existed in settling tanks. This study proves the importance of inlet baffle, tank length to width ratio as a vital and maim design criteria.

Also, this study proves that the inlet baffle helps to improve the tank efficiency which comply with Shahrokhi et al and also comply with Goula as baffle forces the solids to move faster towards the bottom of the tank and decreases the inlet recirculation zone.

8 Discussion

Comparing the CFD results based on velocity magnitude and Y-velocity with turbidity values. Turbidity values have the same trend of Y-velocity which validate the relation between vertical velocity and turbidity value regarding to particle size.

The simulation and visual investigation show that in absence of baffle in experimental conditions; waves in generated on the free surface which shall be duplicated in real life with the existence of wind effect. Furthermore, the velocity distribution along the tank is very fluctuated.

Using baffle 1/3 of tank width reduces the waves in the free surface and provides partial homogenous flow distribution and load. The turbidity in the last third of the tank shows very close values but the velocity in y-direction over the tank length shows higher velocities at the inlet.

In case of inlet baffle 2/3 of tank width the waves are not visually observed at the free surface. The velocity vectors shows uniform distribution but the velocity y-direction shows that there is no uniformity along tank width.

Finally, with inlet baffle 3/3 of tank width the waves are not visually observed at the free surface. The velocity vectors shows uniform distribution and the velocity in y-direction shows that there is uniformity along tank width. Due to the baffle is full width, the zone after inlet zone has some circulation. An odd values of turbidity is obtained in the first third of the tank and the main reason is the circulation in this zone which may cause odd values comparing with the second and third part of tank.

9 Conclusions

This study validates the importance of inlet baffle and its direct effect on providing good distribution inside the tank. Based on experimental and numerical results of this study; the rectangular tank can provide a partial homogenous flow distribution and load based on the flow path length without inlet baffle. Inlet baffles provides homogenous distribution for flow and loads and enhance the settling tank performance.

Using inlet baffle reduces the inlet zone volume which increases the actual settling zone. In settling tanks with length to width ratio 3:1, the performance is enhanced with ratio 70.0%, 71.5%, and 87.4% for baffles 1/3, 2/3, and 3/3 inlet baffles respectively.

The 3D modeling creates an easy way to investigate the velocity contours and vectors in the entire tank.

10 Recommendations

The ratio between turbidity and suspended solids should be determined which gives overall view of tank performance in case of discrete settling.

It is recommended not to use turbidity parameters when testing the distribution inside the tank as it very difficult to prepare the solution at a certain value of turbidity; Total Dissolved Solids may be better parameters to be tested.

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