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EFFECTS OF EARLY SWITCHING OFF ANGLE ON STEPPING MOTOR PERFORMANCE*

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ABSTRACT

This paper analyzes the performance of the three-phase variable reluctance stepper motor (VRSM) in case of single-phase excitation mode. Equations of phase self inductances, instantaneous phase and supply currents, phase and total torques, total input and output power as well as the efficiency are all deduced for any rotor position.

Also the paper analyzes the effect of the early switching OFF of the supply on the performance characteristics of the motor. To avoid negative torque production, the switching OFF of the phase excitation should be controlled. In order to guarantee removal of the phase current before the phase inductance start to decrease, the instant of the switching OFF would take place before that of the teeth alignment

KEY WORDS: *Angle control of Stepping Motor*.

EFFETS DE L'ENFANCE ARRET D'ANGLE SUR LA PERFORMANCE MOTEUR PAS A PAS

RÉSUMÉ

Ce document analyse la performance des trois phases réluctance variable moteur pas à pas (VRSM) en cas de mode d'excitation monophasée. Equations de selfs de phase, la phase instantanée et les courants d'approvisionnement, de phase et des couples au total, entrée totale et la puissance de sortie ainsi que l'efficacité sont toutes déduites pour toute position du rotor.

Aussi le document analyse les effets de la coupure du début de l'offre sur les caractéristiques de performance du moteur. Afin d'éviter la production de couple négatif, la coupure de l'excitation de phase doit être contrôlée. Afin de garantir l'élimination de la phase actuelle avant l'inductance de phase commence à diminuer, l'instant de la commutation OFF aurait lieu avant celle de l'alignement des dents

MOTS CLÉS: CONTRÔLE D'ANGLE MOTEUR PAS A PAS.

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1. INTRODUCTION

A stepper motor is an electromechanical device, which converts electrical pulses to discrete mechanical movements. The shaft of the stepper motor rotates in discrete step increments when electrical pulses are applied to it in the proper sequence. The motor rotation has several direct relationships with these applied input pulses. The sequence of the applied pulses is directly related to the direction of motor shaft rotation. The speed of the motor shaft rotation is directly related to the frequency of the input pulses and the length «step» of rotation is directly related to the number of input pulses [6].

The stepper motor can be classified into several types according to machine structure and principle of operation. There are three types of stepper motors according to their complexity: variable reluctance, permanent magnet and hybrid. The variable reluctance stepper motor has a solid soft steel rotor with salient poles. The permanent- magnet stepper motor has a cylindrical permanent- magnet rotor. The hybrid stepper motor has soft steel teeth added to the permanent magnet rotor for a smaller step angle [3].

2. PERFORMANCE ANALYSIS

2.1 Motor Equivalent Circuit

The equivalent circuit of the variable reluctance stepper motor (VRSM) is similar to that of switched reluctance motor (SRM). The induced emf appears across the rotor terminal (e) is due to the rotor rotation through the stator magnetic field. Therefore the equivalent circuit could be represented as shown in Fig. (1-(a))

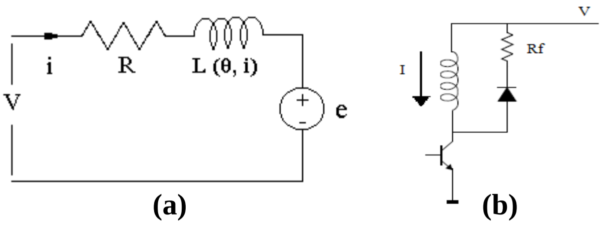

Fig. (1): a) Phase Equivalent Circuit of VRSM, b) Basic Circuit for Driving VRSM

R: stator phase resistance
R_f: freewheeling resistance
L: stator phase inductance

2.2 Voltage Equations

According to Kirchoff's voltage law, the applied phase voltage equals to sum of the resistive voltage drop and the rate of change of the flux linkages i.e.

$$V = R * i + \frac{d\lambda(\theta,i)}{dt} \quad (1)$$

Where:

λ: The flux linkage per phase given by:

$$\lambda = L(\theta, i) * i \quad (2)$$

The phase voltage equation is given by following equation:

$$V = R * i + L(\theta, i) * \frac{di}{dt} + \frac{dL(\theta,i)}{d\theta} * \omega_m * i \quad (3)$$

The three terms on the right-hand side represent the resistive voltage drop, inductive voltage drop, and induced emf, respectively. The result is similar to the series excited dc motor voltage equation. The induced emf, *e*, is obtained as:

$$e = \frac{dL(\theta,i)}{d\theta} * \omega_m * i \qquad (4)$$

2.3 Phases Inductance

The inductance of each stator phase winding varies with rotor position such that the inductance has a maximum value when the rotor axis is aligned with the magnetic axis of that phase and a minimum value when the two axes are in perpendicular. The self-inductances are expressed in the following form:

$$\left.\begin{aligned} L_A &= L_1 + L_2 {}^* Cos\,(N_r\,\theta) \\ L_B &= L_1 + L_2 {}^* Cos\,(N_r\,\theta - \theta_s) \\ L_C &= L_1 + L_2 {}^* Cos\,(N_r\,\theta - 2\,\theta_s) \end{aligned}\right\} \qquad (5)$$

Where:

$L_1 = L_u + [(L_a - L_u) / 2] = [(L_a + L_u)/2$

$L_2 = [(L_a - L_u) / 2]$

L_1: *The average inductance,*

L_2: *The amplitude of the inductance waveform,*

L_a: *The stator phase inductance at the alignment position,*

L_u: *The stator phase inductance at the un-alignment position,*

θ: *The rotor position angle with respect to the axis of phase excitation.*

θ_s: *The angle between each two sequential stator phase in space, which is equal to 120°.*

The stator winding inductance varies sinusoidaly with rotor position and has an angular period given by

$$\theta_{cy} = \frac{360}{N_r} \qquad (6)$$

Where N_r is the number of rotor poles.

2.4 Phase Currents

When, a DC voltage is connected to the winding of the motor, the current increases exponentially according to equation (7).

The rotor is rotated by an angle of 6°, for the motor under study. The rotor angle is measured from phase-A, when, the rotor is aligned with it. In order to rotate the rotor from this position, phase-B must be supplied from the supply, and thus the current of phase-B starts to increase.

The instantaneous value of this current I_{Bi} during the period from $\theta = 0^o$ (aligned position) to $\theta = 6°$ is determined from the following equation:

$$I_{Bi} = I_{max} *[1 - e^{-(t_{Bi} / \tau_{Bi})}] \qquad (7)$$

Where:

$$I_{max} = V / R \qquad (8)$$

V: The voltage of a dc supply,

t_{Bi}: *The time of starting from the moment of phase-B connection to the supply, sec.*

τ_{Bi}: *The electrical time constant of phase-B, sec.*

The electrical time constant of phase-B, phase -C, and phase-A, are determined after evaluating the phases inductance from the following equation:

$$\left.\begin{aligned} \tau_{Bi} &= L_B / R \\ \tau_{Ci} &= L_C / R \\ \tau_{Ai} &= L_A / R \end{aligned}\right\} \qquad (9)$$

It will be noticed that, L_B, L_C and L_A are not constant but are varied with rotor position θ as shown in Fig. (2- (a)) and are determined from equation (5).

There is a synchronizing variation of rotor rotational angle with rotational speed. By supposing the value of rotational speed N and the value of rotational angle θ from zero position, the respective time t_{Bi} to this angle is determined from the following equation:

$$t_{Bi} = [\theta / (6\,N)] \qquad (10)$$

where:

N: is the rotational speed in r. p. s.

The current of phase-B is increased during the conduction period as shown in Fig. (2-(b)). The current is reached a maximum value I_{max}, only if the time is enough at low speeds. The higher value of current I_{Bi} is less than I_{max}, where I_{max} is determined from equation (8).

In Fig. (2- (b)), when θ equal 6 degrees, the rotor reaches the alignment position with phase-B, then disconnecting phase-B from the supply because the torque is reached zero. And thus the current of phase-B starts to decrease through freewheeling circuit.

The decreased currents I_{Bd} is determined during this period from the following equation:

$$\left.\begin{aligned} I_{Bd} &= I_{fi} * e^{-(t_{Bd} / \tau_{Bd})} \\ I_{Cd} &= I_{fi} * e^{-(t_{Cd} / \tau_{Cd})} \\ I_{Ad} &= I_{fi} * e^{-(t_{Ad} / \tau_{Ad})} \end{aligned}\right\} \qquad (11)$$

Where:

I_{fi} is the higher value of current I_{Bi} at θ = 6 degrees.

All phases have the same value of higher current I_{fi}, due to symmetrical operation of all phases.

Also the decreased currents I_{Cd}, I_{Ad} at θ = 12, θ = 18 degrees respectively are determined from equation (11).

The time t_{Bd} of decreasing current is determined from the following equation:

$$t_{Bd} = [(\theta - 6) / (6\,N)] \qquad (12)$$

where:

the angle θ is assumed for the required current.

The values of τ_{Bd}, τ_{Cd}, and τ_{Ad} are differed from the values of τ_{Bi}, τ_{Ci}, and τ_{Ai} and are calculated by the following equations:

$$\left.\begin{aligned} \tau_{Bd} &= L_B / (R + R_f) \\ \tau_{Cd} &= L_C / (R + R_f) \\ \tau_{Ad} &= L_A / (R + R_f) \end{aligned}\right\} \qquad (13)$$

This difference is due to existence of the freewheeling circuit resistance Rf as shown in Fig. (1-(b)). It is noticed that, the changing of LB with an angle θ during this period has a negative slop and differed from its changing during the period of increasing current.

When $\theta = 6^o$, disconnecting phase-B from the supply, and at the same time, connect phase-C to the supply. So, the current of phase-C is started to increase according to the following equation:

$$I_{Ci} = I_{max} \; [1 - e^{-(t_{Ci} / \tau_{Ci})}] \qquad (14)$$

The time t_{Ci} is determined from the following equation:

$$t_{Ci} = [(\theta - 6) / (6\,N)] \qquad (15)$$

Where, the angle θ is assumed for the required current. The time constant τ_{Ci} is determined from equation (9)

When, the angle θ increases to 12 degree, disconnecting phase- C from the supply. So, the current I_{Cd} is started to decrease according to equation (11).

The decreasing time t_{Cd} is determined from the following equation:

$$t_{Cd} = [(\theta - 12) / (6\,N)] \qquad (16)$$

Also, the time constant τ_{Cd} during this period is determined from equation (13).

When, the angle arrives to 12 degree, disconnecting phase-C from the supply, and connect phase- A to the supply, so the current I_{Ai} is started to increase according to the following equation,

$$I_{Ai} = I_{max} [1 - e^{-(t_{Ai}/\tau_{Ai})}] \quad (17)$$

Time t_{Ai} of increasing current is determined from the following equation:

$$t_{Ai} = [(\theta - 12) / (6 N) \quad (18)$$

Where, the angle θ is assumed for the required current. The time constant τ_{Ai} is determined from equation (9)

When the angle arrives to 18^o, phase-B is repeated to connect and its current I_{Bi} is increased according to equation (7).

At the same time, the current of phase- A is started to decrease. The decreased current I_{Ad} is determined from equation (11), where the time t_{Ad} is determined from the following equation:

$$t_{Ad} = [(\theta - 18) / (6 N)] \quad (19)$$

Also, the time constant τ_{Ad} during this period is determined from equation (13).

So, connecting and disconnecting phases are repeated continually and sequentially. The currents of phases are still changed by the same rate, as shown in Fig. (2-(b)).

During the increasing periods of these currents, the current of the supply I is equal to the summation of the all phase's currents. But during the decreasing periods, the current of phases is not supplied from the supply because some phases are disconnected. Therefore, the supply current I is determined from the following equation, (Fig. (2-c)):

$$\left.\begin{array}{lll} I = I_{Bi} & at & 0 < \theta < 6 \\ I = I_{Ci} & at & 6 < \theta < 12 \\ I = I_{Ai} & at & 12 < \theta < 18 \end{array}\right\} \quad (20)$$

2.5 Power and Efficiency

Substituting for the flux linkages of equation (2) in the voltage equation (1) and multiplying with the current results in instantaneous input power P_{in} given by:

$$P_{in} = V * i = R * i^2 + i^2 * \frac{dL(\theta,i)}{dt}$$
$$+ L(\theta, i) * i * \frac{di}{dt} \quad (21)$$

Here, the last term is physically uninterpretable. To draw a meaningful inference, it may be cast in terms of known variables as in the following:

$$\frac{d}{dt}\left[\frac{1}{2} L(\theta, i) * i^2\right] = L(\theta, i) * i * \frac{di}{dt} + \frac{1}{2} * i^2 * \frac{dL(\theta,i)}{dt}$$

Substituting the above equation into Equation (21) results the instantaneous input power given by the following equation:

$$P_{in} = R * i^2 + \frac{d}{dt}\left[\frac{1}{2} * L(\theta, i) * i^2\right]$$
$$+ \frac{1}{2} * i^2 * \frac{dL(\theta,i)}{dt} \quad (22)$$

This equation is in the familiar form found in introductory electro mechanics texts, implying that the input power is the sum of the winding resistive losses, the rate of change of the field energy, and the output power (air gap power).

Also, the instantaneous power losses P_{Loss} is determined from the following equation (acceptance the iron losses),

$$P_{Loss} = P_{in} - P_o = P_{in} - (T * \omega_m) \quad (23)$$

The instantaneous output power (air gap power) P_o is determined form the following equation:

$$P_o = P_{in} - P_{Loss} = \frac{1}{2} * i^2 * \frac{dL(\theta, i)}{dt}$$

$$= \frac{1}{2} * i^2 * \frac{dL(\theta, i)}{d\theta} * \frac{d\theta}{dt} \quad (24)$$

Substituting for time in terms of the rotor position and speed, with

$\omega_m = \frac{d\theta}{dt}$ Rotor angular speed

In the output power results in:

$$P_o = \frac{1}{2} * i^2 * \frac{dL(\theta, i)}{d\theta} * \omega_m \quad (25)$$

Also, the instantaneous output power P_o (air gap power) is determined by knowing the total electromagnetic torque and rotor speed N and can be given by:

$$P_o = \omega_m * T_e \quad (26)$$

So, the instantaneous value of motor efficiency η is determined from the following equation:

$$\eta = P_o / P_{in} \quad (27)$$

The variation of instantaneous value for input power, output power and efficiency is shown in Fig. (2- (d) and (2- (e)).

2.6 Computed Motor Torque

The motor torque is obtained from the effect of direct current passing through stator phases and the changing of rotor reluctance. The instantaneous value of torque is proportional with square of instantaneous current also with rate of phase inductance variation with respect to rotor angle ($dL/d\theta$) or the inductance curve deviation with rotor angle θ. In order to obtain motoring torque, phase current is switched on during the rising period of phase inductance. Generating operation or braking torque can be obtained by switching phase current during the decreasing period of phase inductance.

Assuming that mutual coupling between the phases does not exist and that the phase inductance is linear, the torque is obtained by equating equations (25) and (26), and gives the following torque equation:

$$T_e = \frac{1}{2} * i^2 * \frac{dL(\theta, i)}{d\theta} \quad (28)$$

So, the instantaneous value of motor torque, resulting from each phase is determined as follows:

$$\left.\begin{aligned} T_A &= 0.5\, I_A^2 (dL_A/d\theta) \\ T_B &= 0.5\, I_B^2 (dL_B/d\theta) \\ T_C &= 0.5\, I_C^2 (dL_C/d\theta) \end{aligned}\right\} \quad (29)$$

Where, the currents I_A, I_B and I_C are the instantaneous values, whether, in the periods of increasing or decreasing.

The derivation of the phases inductance will be as follow:

$$\left.\begin{aligned} dL_A/d\theta &= -N_r * L_2 * \sin(2\theta) \\ dL_B/d\theta &= -N_r * L_2 * \sin(N_r\theta - \theta_s) \\ dL_C/d\theta &= -N_r * L_2 * \sin(N_r\theta - 2\theta_s) \end{aligned}\right\} \quad (30)$$

The instantaneous value of the total torque T_t is equal to the sum of the instantaneous phases torque resulting from the motor, and is determined as follows:

$$T_t = T_A + T_B + T_C \quad (31)$$

The variation of this torque is shown in Figure (2-(f)).

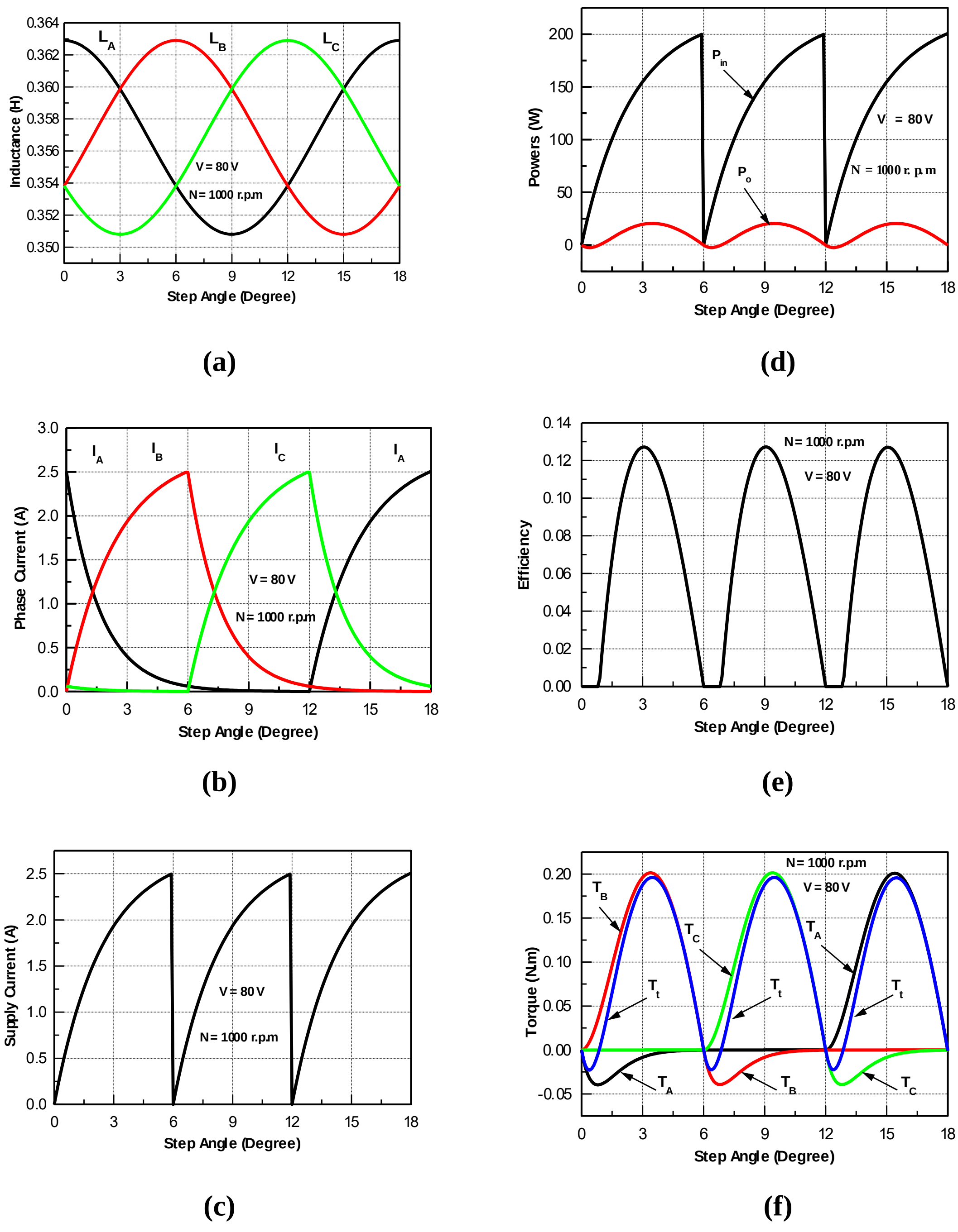

Fig (2): Variation of (a) Inductance, (b) Phases Currents, (c) Supply Current, (d) Motor Input Power and output Power, (e) Motor Efficiency, and (f) Torque with Rotor Position.

3. ANGLE CONTROL

The early switching OFF angle **β** means disconnecting the phase winding from the supply before connecting the next phase to the supply. The connection instant of each phase to the supply does not change, but the conduction period of each phase to the supply becomes less than 6 degree by the early switching OFF angle **β**.

3.1 Instantaneous Motor Characteristics

Fig. (3-(a)) illustrates the variation of the instantaneous phases currents with rotor angle at different values of early switching OFF angle **β**. With increasing the angle **β**, the phase current is decreased. The current does not reach to its maximum values, because of the conduction period of each phase to the supply is less than 6 degrees. Also with increasing the angle **β**, the supply current is decreased, as shown in Fig. (3-(b)).

Fig. (3-(c)) illustrates the variation of the instantaneous phase's torques with rotor angle at different values of early switching OFF angle **β**. With increasing the angle **β**, the negative torque is decreased; this due to the phase current is early switched OFF before the aligning position.

Fig. (3-(d)) illustrates the variation of the total torque with rotor angle at different values of early switching OFF angle **β**. Also with increasing the angle **β**, the negative phase torque is decreased.

Fig. (3-(e)) illustrates the variation of the instantaneous input power with rotor angle at different values of early switching OFF angle **β**. The instantaneous input power is varied with rotor angle in the same shape of the supply current variation, as long as the supply voltage is constant at 80 V. With increasing the angle **β**, the input power is decreased due to the reduction of the conduction periods for each phase to the supply.

The instantaneous output power is varied by the same shape as the total torque variation, because the rotational speed of the motor is constant at 1000 r. p. m, as shown in Figure (3-(f)). With increasing the angle **β**, the negative output power is decreased.

Variation of the instantaneous efficiency with rotor angle at different values of early switching OFF angle **β** is illustrated in Fig. (3-(g)). The efficiency is equal to zero for the periods of the negative output power; also the efficiency is more affected by increasing the angle **β**, due to the reduction of the conduction periods for each phase of the supply.

3.2 Average Characteristics

By calculating the average values of the instantaneous performance characteristics at different speeds and at different values of early switching OFF angle **β**, the average motor characteristics are obtained in the following figures. Fig. (4-(a)) illustrates the variation of the average phase current with speed at different values of early switching OFF angle **β.** With increasing the angle **β**, the phase current is decreased for all speeds range. In addition, the phase current is decreased by increasing the speed. The rate of decreasing the phase current at low speeds higher than that at high speeds, as shown in Fig. (4-a). Also with increasing this angle **β**, the supply current is decreased, as shown in Fig. (4-(b)).

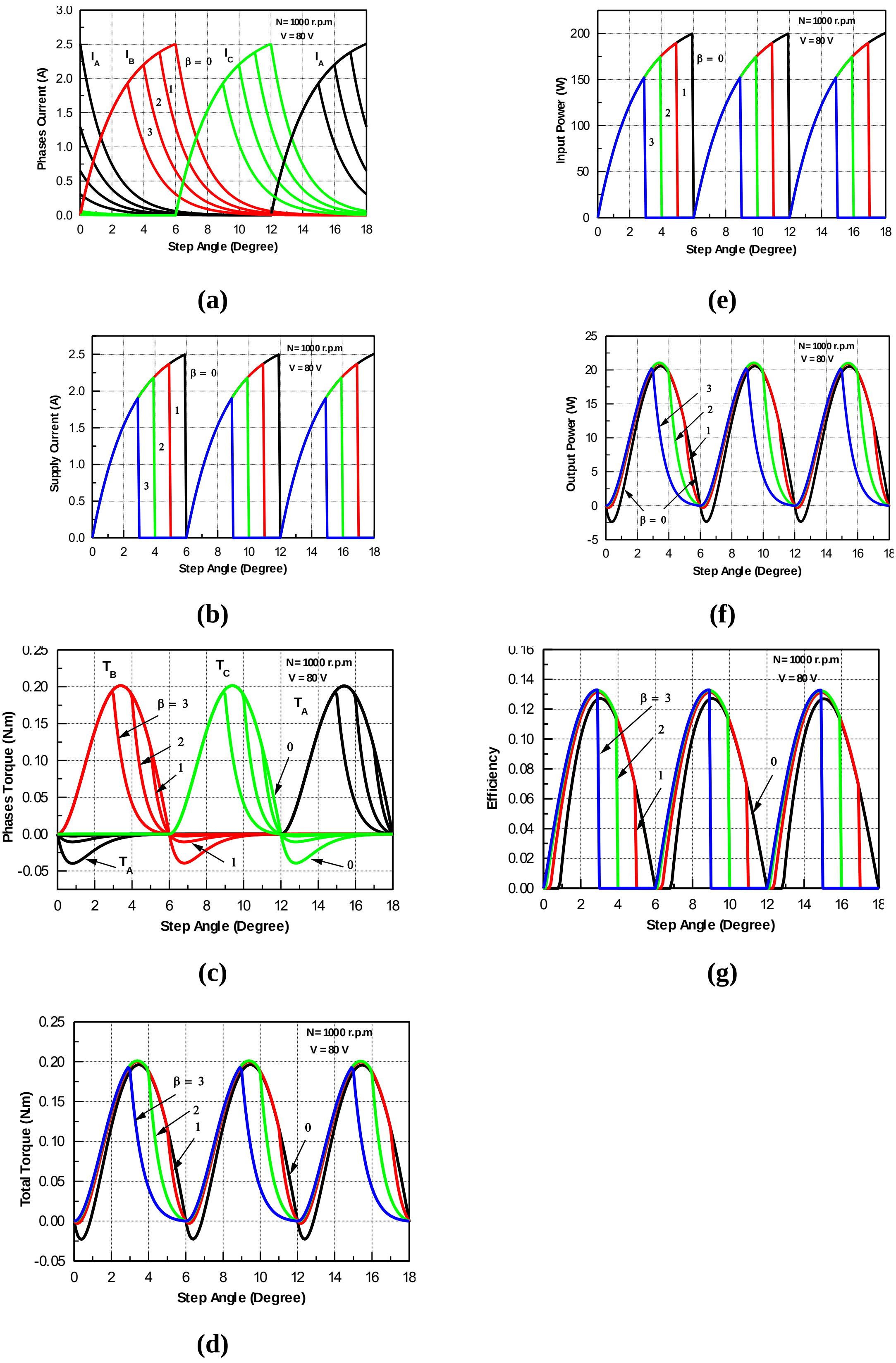

Fig (3): Variation of the instantaneous (a) phases currents, (b) Supply Current, (c) Phases Torques, (d) Total torque, (e) Input Power, (f) Output Power, and (g) Efficiency with rotor position at different values of early switching OFF angle β.

Fig. (4-(c)) illustrates the variation of the torque with speed at different values of early switching OFF angle **β**. With more increasing in the angle **β**, the torque is decreased for the low range of the speeds, on the contrary, the torque is increased for the higher range of the speeds. When, the applied voltage equals 80-V and the early switching OFF angle **β** equal zero, the motor torque is decreased with increasing the speed and reaches zero at approximately 3000 r. p. m. With increasing the angle **β**, (at the same voltage), the torque is increased. Therefore, the motor availability to rotate with load by speed higher than 3000 r. p. m, and which will be achieved by increasing the angle **β**. So at (**β =** 1) the motor torque reaches zero at 4650 r. p. m, but at (**β =** 2) the motor torque reaches zero at 7250 r. p. m.

Fig. (4-(d)) illustrates the variation of the input power with motor speed at different values of early switching OFF angle β. With increasing the angle β, the input power is decreased for all speeds. The input power is varied with speed in the same shape as the supply current variation with speed. This is because the supply voltage is constant at 80 V for all speeds; also the input power is decreased with increasing the speed.

Fig. (4-(e)) illustrates the variation of the output power with motor speed at different values of early switching OFF angle β. By increasing the angle β, the output power is decreased in the range of the low speeds. But the angle β leads into increasing the output power for the range of the high speeds. It is noticed that, the output power becomes negative after 3000 r. p. m at β = 0, and after 4650 r. p. m at β = 1 degree, while it is negative after 7250 r. p. m for β = 2 degree. In addition also the maximum output power occurs in the low speeds.

Fig. (4-(f)) illustrates the variation of the efficiency with motor speed at different values of early switching OFF angle β. The increasing of the angle β, increases the efficiency. The motor efficiency has high values at low speeds, and is decreased by increasing the speed. For angle (β = 0), the maximum efficiency occurs within the range of speeds (950: 1350 r. p. m) and reaches zero after speed of 3000 r. p. m and stayed zero after this speed because the output power becomes negative. Also when (β = 1) the maximum efficiency occurs within the range of speeds (1450: 1750 r. p. m) and reaches zero after speed of 4650 r. p. m, at (β = 2) the maximum efficiency occurs within the range of speeds (1900: 2300 r. p. m) and reaches zero after speed of 7250 r. p. m.

CONCLUSIONS

This paper describes the effect of angle variation of a stepping motors performance; the conclusions can be summarized as follows:

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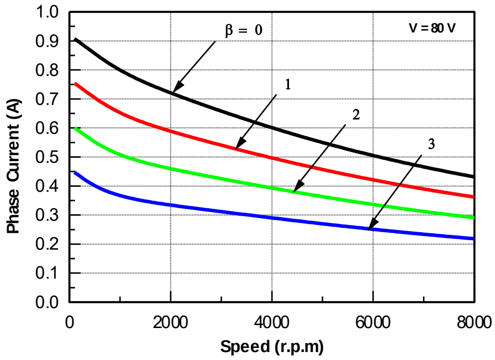
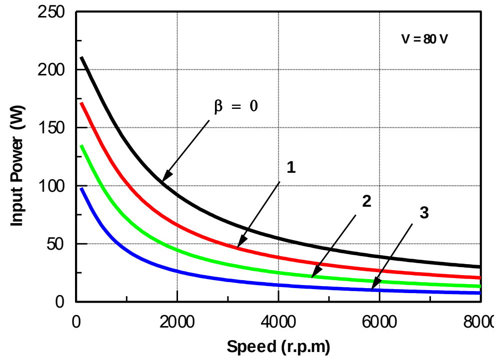


(a)



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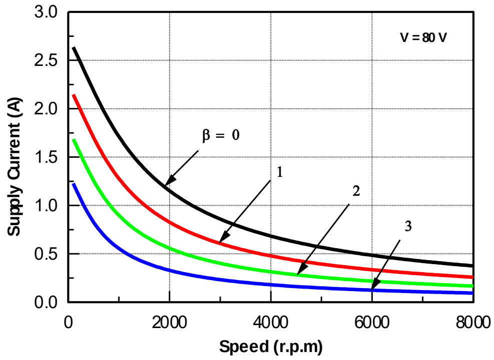

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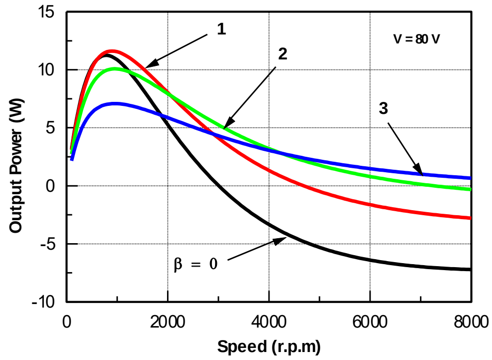
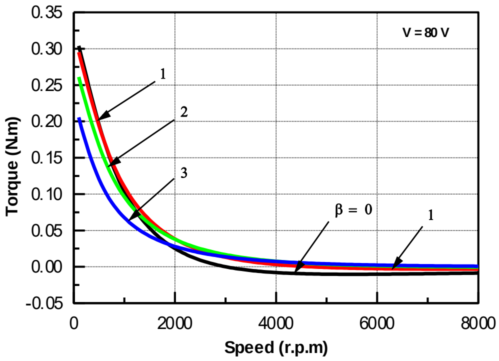
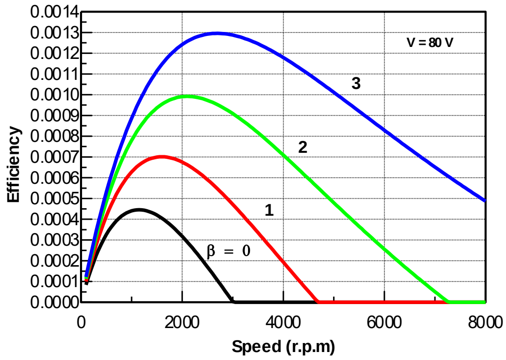


(e)



(c)



(f)

Fig (4): Variation of average (a) phase current, (b) Supply current, (c) Torque, (d) Input power, (e) Output power, and (f) Efficiency with speed at different values of early switching off angle (β).

(i) At any constant speed, with increasing the early switching OFF angle. This leads to decreasing the phase current, negative torque, and conduction period of each phase of the supply. For more increase in this angle, the positive torque is decreased

(ii) To guarantee avoiding the decrease of the torque, the early switching OFF should not be used for low speeds.

(iii) To guarantee increasing the torque, output power, and efficiency, the early switching OFF should be used for high speeds.

(iv) Angle variation predictions show good results, and indicate the improvements in performance.

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