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Analysis of Reinforced Concrete D-Regions Using Strut-and-Tie Model

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A B S T R A C T

The developed Strut-and-Tie Model (STM) has no unique shape for each load case of a given structural problem as long as the selected idealized internal load-resisting truss is in equilibrium with boundary forces, and also stresses in its components "struts, ties, and nodes" are within acceptable limits. However, the optimal shapes are the well-designed with best ordinal weight number of conditional factors as the rebar amount, the load factor, and the structural concrete ductility. The current study investigates numerically based on FE method stress flow contours and micro truss techniques many alternatives with different shapes of struts and ties that transfer the flow of forces from top of the deep beam with opening to both right and left supports. Then, these alternatives with different concrete characteristics are analyzed by strut-and-tie computational tools using different code provisions for verifying its results accuracy with the numerical nonlinear finite element analysis results for studying the structure performance under applied service loads and over loading till failure. The chosen alternative produce load factor to reach capacity greater than 1, therefore the strut-and-tie method always give demand collapse load lower than the true capacity collapse load. This implies that the solution obtained from STM usually lies on the safe side with conservative sense for concrete structures subjected to service loads. That’s why the STM is emerging as an increasingly popular code-worthy methodology for the design and detailing of concrete structures D-Regions.

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1. Introduction

The St.Venant Disturbed or Discontinuity Regions (D-Region) are those parts extending a longitudinal distance equal to about the depth of the member of a structure, where there is a disturbance on stresses and strains distribution that occurs at changes in the structural elements geometry or where loads are applied and adjacent to supports.

Therefore the plane section doesn't remain plane where the normal stress along thickness direction cannot be neglected, and also the plane section doesn't remain normal to the neutral axis where the shear strain may become large.

B-Regions (Beam or Bernoulli) are those portions of a structure in which the assumption of plane section remaining plane is valid such that there is a linear distribution of strains over the depth or width of the member. So it may be designed by sectional theories and code provisions for shear, flexural, axial-load, and torsion.

The STM is applicable to both B-and D-Region problems, but it is not practical to be applied in B-Region problems where the conventional beam theory is recommended for these designs.

The STM design concept assumes that the designed D-Region is sufficiently ductile (to allow the force redistribute after concrete cracked) based on the lower bound plasticity theorem.

The lower bound theory of limit analysis states that: "A stress field that satisfies equilibrium and does not violate yield criteria at any point provides a lower-bound estimate of capacity of elastic-perfectly plastic materials; For this to be true, crushing of concrete (struts and nodes) does not occur prior to yielding of reinforcement (ties or stirrups)". So, lower bound plasticity theorem only satisfy "equilibrium and yield criteria", where the third requirement in solid mechanics framework "strain compatibility" does not have to be satisfied.

Hence, the designer must exercise greater care in selecting the appropriate idealized truss for each load case, and also in determining the STM design code provisions applicability, where the incorrect application of STM could lead to the formation of an undesirable brittle shear failure mechanism.

The concepts of the STM are originally referred to truss analogy proposed more than one century ago by Ritter [1] and Mörsch [2]. Then, Dilger [3], Nielsen [4], and Paulay [5] studied the shear stiffness on reinforced concrete beams with constant and variable analogous truss models.

Thürlimann [6], Ramirez et al. [7], Schlaich et al. [8], Adebar, Kuchma , and Collins [9], MacGregor and Alshegeir [10], Adebar and Zhou [11], Bergmeister et al. [12], Wollmann [13], Foster and Gilbert[14], Chen and Nielsen[15], Foster and Malik [16], Sahoo et al. [17], and Panjehpour M. et al. [18] worked on determining efficiency factor for maximum compressive stress in a strut.

Vecchio and Collins [19] developed the Compression Field Theory (CFT) constructed by Marti, Collins and Mitchell [20] for a new concrete softening model named as the Modified Compression Field Theory (MCFT) that take into consideration the average tension stiffening contribution effect of cracked concrete. Then Kaufmann and Marti [21] developed a Cracked Membrane Model (CMM) similar in nature to MCFT. The main difference between the two theories lies with the handling of stresses. MCFT relies on average stresses and strains in the cracked concrete, whereas, the CMM relies on local stress conditions at cracks.

Rogowsky et al. [22] examined the effects of a short longitudinal bar anchorage. Clark [23], Ferguson [24], and Watstein and Mathey [25] used welded steel plates onto the reinforcement to increase the anchorage of the bars. Cook and Mitchell [26], Breen et al. [27] conducted the effect of nodal regions detailing on the anchorage failure. Breen et al. [27], Yun and Ramirez [7] studied how the actual failure mechanisms of the beam could be determined based on the FEA of the nodal zones which indicated crushing of local zones. Breen et al. [27] examined the behavior of headed reinforcement in concrete structures.

Siao [28], Maxwell and Breen [29], Yun [30], Vollum et al. [31], Chen et al. [32], Hwang et al. [33], Brown [34], Ley et al. [35], Arabzadeh et al. [36], and To et al [37], worked on the use of STM for design and find capacity of different structural reinforced concrete members. Tan et al. [38] applied the STM for the design of pre-stressed deep beams. Carlos A. Flores [39] studied using STM if steel fiber reinforced concrete (SFRC) as a material is better than traditional reinforced concrete (RC).

Schlaich et al. [8] proposed principal stress trajectories and the load path method to obtain the STM shape. And also conducted that STM shape can be possibly drawn based on the cracked pattern.

Liang et al. [40] adopted the topology optimization or performance based optimization (PBO) for obtaining STM shapes. Park and Yun [41] adopted the minimum strain energy approach of structural grids for obtaining the optimal STM shape.

Hamed and M.Salem [42], and Nagarajan et al. [43] introduced the concept of micro truss model to predict the nonlinear response of concrete structures and it can be used to trace the crack pattern.

Over the past few decades, many code authorities world-wide have added STM specifications as a recommended rational approach to the traditional design specifications of reinforced concrete deep beams. These codes and standards are classified into two groups:

- The first group includes AASHTO LRFD-2012 [44], and AS3600 [45], which define the strut compressive strength as a function of the concrete tensile strain in the direction of a tie and the angle between the strut and the tie as in the Modified Compression Field Theory (MCFT) that take into consideration the average tension stiffening contribution effect of cracked concrete.
- While the second group comprises ECP 203-07 [46], ACI 318-14 [47], NZS 3101 [48], DIN 1045-1 [49], EC2 1992-1-1 [50], CEB-FIP Model Code 90 [51], and 1999 FIP Recommendations [52], which specify the strut compressive strength as a product of the concrete compressive strength and a reduction factor. These reduction factors take into account the strut shape, the concrete type, whether the strut is in a cracked or un-cracked region, and whether appropriate crack control reinforcement been used.

The current study investigates numerically many alternatives with different shapes of struts and ties. Then, the alternatives with different concrete characteristics are analyzed by CAST [56] using different code provisions for verifying its results accuracy with ANSYS [54] results.

2. The Specimen details

The chosen beam specimen was the same specimens tested by (Breña and Morrison, 2007) [53] and (Carlos A. Flores, 2009) [39], with point load applied on a top of the beam. The beam has length (L) equals 74 in. (1880), depth (D) equals 47 in. (1194), and width (B) equals 4.4 in. (112). The square opening 15 in. (381) by 15 in. (381) has depth and span ratios (a/D, a/L) equal 0.326 and 0.203, respectively as shown in Fig. 1. The opening has depth and span eccentricities (e_D, e_L) equal 10.5 in. (267) and 24.5 in. (622), respectively, laying on a bottom of the beam, and interrupting the shortest load path between the load and the left support.

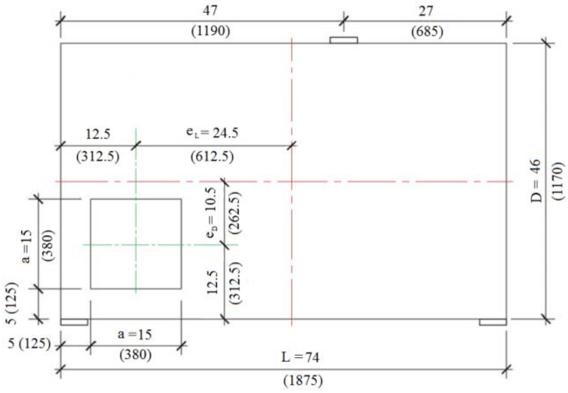
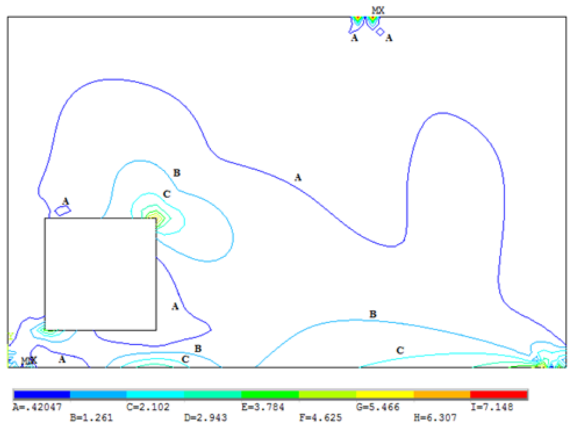


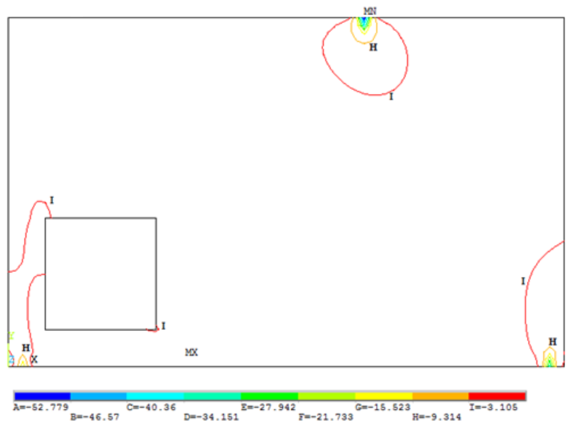
Fig. 1 The Overall Geometry in inches and (mm) Units.

3. Construction of STM shapes

Firstly, the nonlinear FE ANSYS [54] model performed at pre-conducted service load of 140 kN for drawing the principal stress contours using plain concrete membrane shell 41 meshing elements as shown in Fig. 2 and Fig. 3.



(a) Tension Principal Stress Contour.



(b) Compression Principal Stress Contour.

Fig. 2 The Principal Stress Contours of the P.C. FE Model.

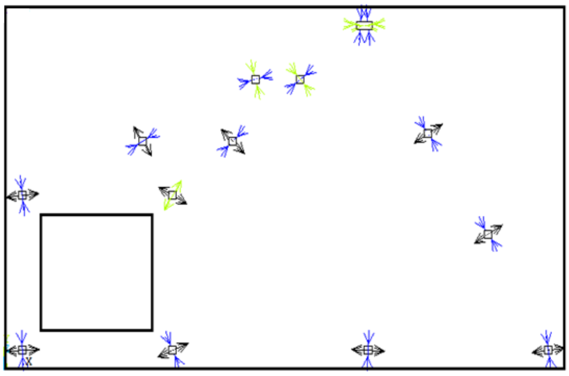


Fig. 3 The Flow of Principal Stresses Vectors for Chosen Key Elements (The Tensile Stresses Vector Directions Represented by Black Arrows and The Compressive Stresses Represented by Blue Arrows).

Secondly, the micro-truss technique as shown in Fig. 4 that may help to obtain the strut-and-tie model is used, where STM is a method that reduces complex stresses within structure to collection of simple stress paths.

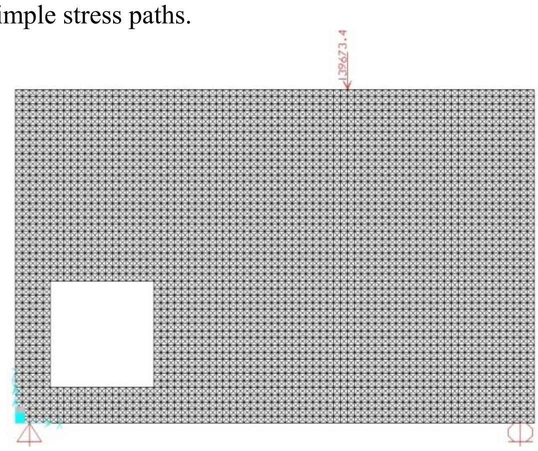


Fig. 4 The Micro-Truss Generating Using SAP2000.

This truss (as introduced by Hamed and Salem, 2004 [42]) is analyzed under the service applied load of 140 kN using commercial structural analysis software SAP2000 [55] with truss pattern parameter properties as following: k = 1, a = 25 mm, t = 112.5, A_h = 2104.38 mm^2, A_d = 2104.38 mm^2, A_v = 1491.55 mm^2. Where [42]:

$$A_h = \frac{3}{8}\left(3 - k^2\right)at \tag{1}$$

$$A_d = \frac{3}{16}\frac{\left(1 + k^2\right)^{3/2}}{k}at \tag{2}$$

$$A_v = \frac{3}{8}\frac{\left(3k^2 - 1\right)}{k}at \tag{3}$$

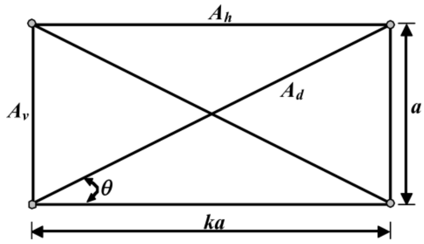


Fig. 5 Pattern of Truss Elements for Plane Stress Problems [42].

Then, some elements that are out of the proper selected range are omitted for reaching to the proper members that may help as indication for the selected shape. So, all members with force less than 10 % of maximum tension force values 11359.1 N, where 16.42 % of tension elements are left for drawing as shown in Fig. 6.

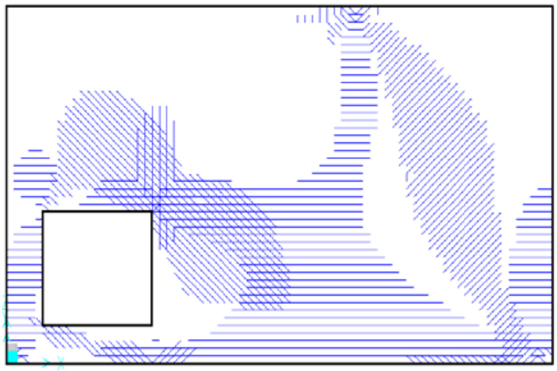


Fig. 6 The Micro-Truss Generated Using SAP2000 at 10 % of Maximum Tension Force.

Based on FE stress flow contours and micro truss techniques, many alternatives with different shapes of struts and ties could be formed, the current study chooses four alternatives of STMs as shown in Fig. 7.

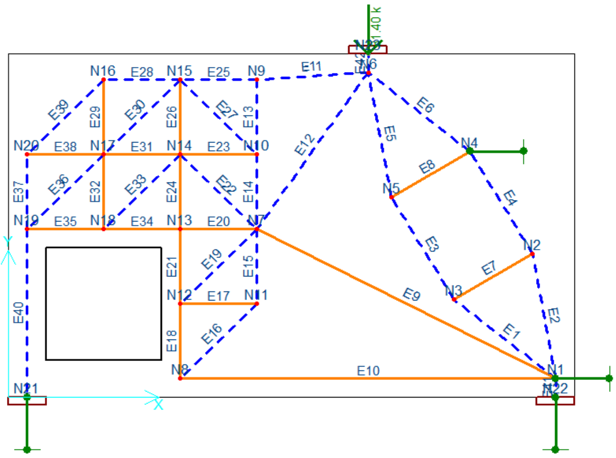
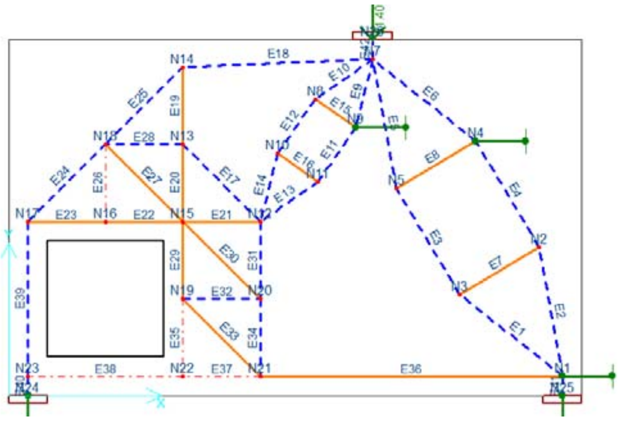
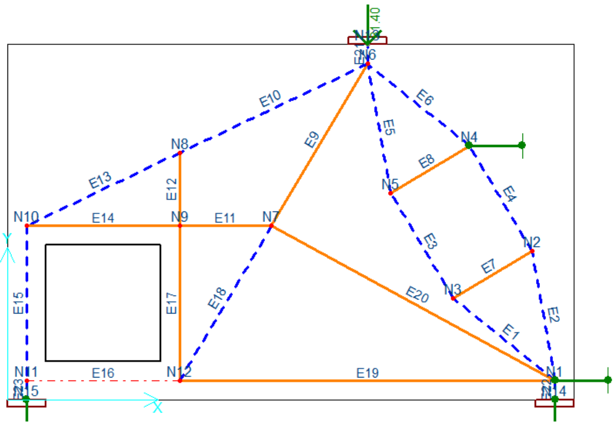


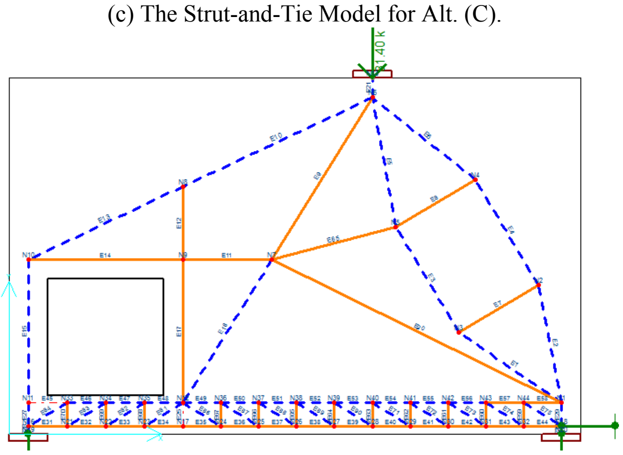
Fig. 7 (a) The Strut-and-Tie Model for Alt. (A).



(b) The Strut-and-Tie Model for Alt. (B).



(c) The Strut-and-Tie Model for Alt. (C).



(d) The Strut-and-Tie Model for Alt. (D).

Fig. 7 The Study Strut-and-Tie Models, (Solid Lines Indicate Tension Tie and Dashed Lines Indicate Compressive Strut).

4. The analysis scheme

Firstly, Computer Aided Strut-and-Tie (CAST) [56] analysis was done for calculating the capacity of STM with assumptions such as: tension in concrete is neglected, forces in struts and ties are uniaxial, external forces apply at nodes.

The descriptions of geometry, loading, and boundary conditions are performed to the selected alternatives to satisfy codes of practices provisions.

The concrete effective widths provided according to AASHTO recommendations for single and double tie [44] as (6 d_b) and (6 d_b+1"), respectively. For struts provided according to ACI318-14 as ($w_t \sin\theta + l_b \cos\theta$) = ((6 d_b) sin 0 + (12 d_b) cos 0). All the relative stiffness Leaved to 1 where the STM used in this design is statically determinate.

The stress-strain curve for concrete and steel material in the format of [Strain (in./in.); Stress (psi)] defined as (0.002 ; 6185), and (0.0024 ; 60000), respectively. the STM components types is defined with a proper factors as shown in Table 1, The concrete strut types is (prismatic strut), the reinforcement tie are (main single and double ties), and the node are (CCC, CCT, and CTT).

Secondly, the FEA using ANSYS is performed with the following analysis assumptions: The bond between concrete and steel is assumed to be perfect, the Poisson's ratio (υ) is assumed to be constant throughout the loading, the non 45° inclined reinforcement can be simulated with horizontal and vertical components at its center.

The SOLID65 element type used for the 3-D reinforced concrete and the SOLID45 element type used for the steel plates at the supports and loading points. Each individual element contain different real constants that define embedded rebar details as a smeared crack approach where the effects of cracking are redistributed over a finite element (by appropriately modifying the material properties at the integration points).

The first material used for modeling concrete has (multi-linear elastic stress-strain curve) with: elastic modulus (E_c) as shown in Table 3, (υ) = 0.2, shear transfer coefficient (β_t) for an open crack = 0.2, for a closed crack = 0.8, uniaxial tensile cracking stress (f_t) = 4.348 MPa, uniaxial crashing stress = 43.48 MPa.

The second material used for modeling steel bars is idealized to be (elastic-perfectly plastic stress strain curve in tension and compression) with: elastic modulus (E_s) = 1.7575E+008 kPa, Poisson's ratio (υ) = 0.3, yield stress = 421.8 MPa.

Table 1 The STM Element Properties for Different Codes

Code		DIN 1045-1 (Alt.A-5)	CEB -FIP 90 (Alt.A-4)	FIP 1999 (Alt.A-3)	ECP 203 (Alt.A-2)	EC2 (Alt.A-1)	ACI 318 & NZS 3101 (Alt.A)
Prismatic Strut	Strut Equation	User-Defined/General	User-Defined/General	User-Defined/General	User-Defined/General	User-Defined/General	ACI
	Strength Reduction Factor φ	1	1	1	1	1	0.750
	Effectiveness Factor	$0.75\times1\times\frac{0.85}{1.5}$ = 0.425	$0.6\times\left(1-\frac{43.5}{250}\right)\times\frac{1}{1.5}$ = 0.33	$\left(1-\frac{43.5}{250}\right)\times\frac{0.85}{1.5}$ = 0.45	$0.67\times1\times\frac{0.8}{1.6}$ = 0.5234	$0.6\times\frac{\left(1-\frac{43.5}{250}\right)}{0.85}\times\frac{0.85}{1.5}$ = 0.33	0.85×1 = 0.850
Tie	Strength Reduction	1.15	1.15	1.15	1.15	1.15	1
Node	Node Equation	User-Defined/General	User-Defined/General	User-Defined/General	User-Defined/General	User-Defined/General	ACI
	Strength Reduction Factor φ	1	1	1	1	1	0.750
	Effectiveness Factor – CCC Node	$1.1\times1\times\frac{0.85}{1.5}$ = 0.623	$0.85\times\left(1-\frac{43.5}{250}\right)\times\frac{1}{1.5}$ = 0.468	$1.2\times\frac{0.85}{1.5}$ = 0.68	$0.67\times1\times\frac{0.8}{1.6}$ = 0.523	$1\times\frac{\left(1-\frac{43.5}{250}\right)}{0.85}\times\frac{0.85}{1.5}$ = 0.55	0.85×1 = 0.850
	Effectiveness Factor – CCT Node	$0.75\times1\times\frac{0.85}{1.5}$ = 0.425	$0.6\times\left(1-\frac{43.5}{250}\right)\times\frac{1}{1.5}$ = 0.33	$0.85\times\frac{0.85}{1.5}$ = 0.481	$0.67\times0.8\times\frac{0.8}{1.6}$ = 0.418	$0.85\times\frac{\left(1-\frac{43.5}{250}\right)}{0.85}\times\frac{0.85}{1.5}$ = 0.468	0.85×0.8 = 0.680
	Effectiveness Factor – CTT Node	$0.75\times1\times\frac{0.85}{1.5}$ = 0.425	$0.6\times\left(1-\frac{43.5}{250}\right)\times\frac{1}{1.5}$ = 0.33	$0.85\times\frac{0.85}{1.5}$ = 0.481	$0.67\times0.6\times\frac{0.8}{1.6}$ = 0.314	$0.75\times\frac{\left(1-\frac{43.5}{250}\right)}{0.85}\times\frac{0.85}{1.5}$ = 0.413	0.85×0.6 = 0.510

5. The load-deflection response

For illustrating the response of different alternatives that analyzed by FE method with respect to the experimental results performed by (Carlos A. Flores, 2009) [39], the load – deflection curve for a point at the lower tension chord of the specimen under the load point is presented in Fig. 8. And the ductility ratio can be conducted as shown in Table 2.

Cause of the CAST limitation in obtaining the displacement history, truss models analyzed with SAP software by adopting the same assumptions and element properties of STM with neglecting the concrete in zones between the STM components without any strength reduction factors.

Fig. 9 illustrate the STM SAP Results load – deflection curve for a point at the lower tension chord of the specimen under the load point.

Table 2 The FEA Displacement Ductility Ratio of the Different Alternatives.

Alt.	First Cracking Deflection (mm)	Failure Deflection (mm)	Displacement Ductility Ratio
Alt. (A)	0.123653	0.147768	1.195
Alt. (B)	0.111216	0.150249	1.351
Alt. (C)	0.116457	0.157607	1.353
Alt. (D)	0.116254	0.173684	1.494

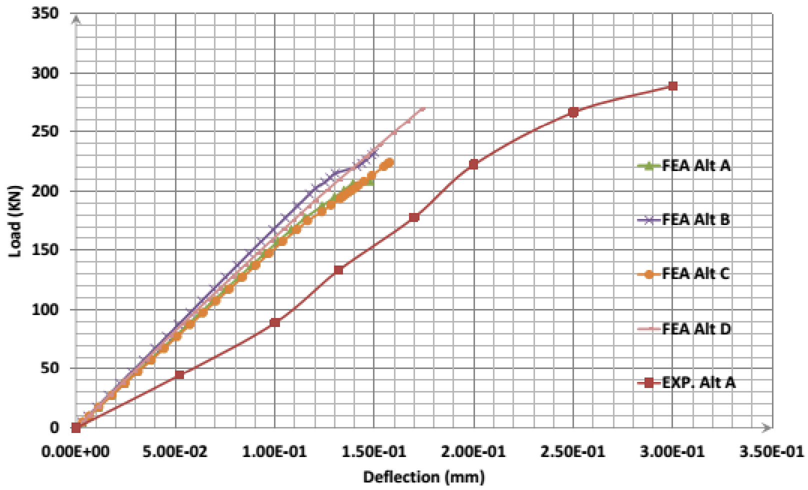


Fig. 8 The FEA Results of Load – Deflection Response for Different Alternatives, and the Gross Load – Deflection Response of the Experimental Results.

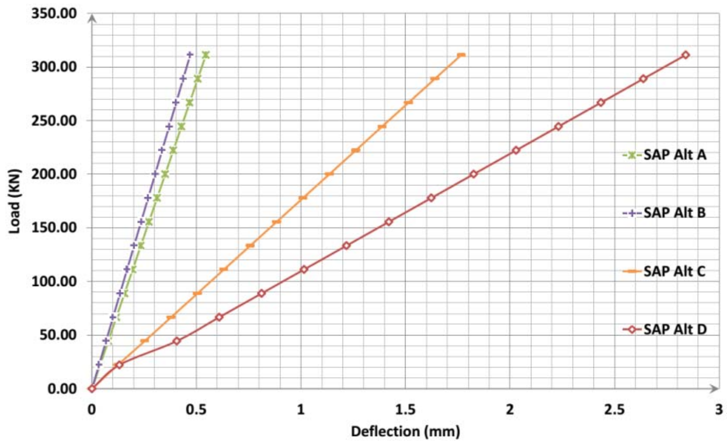


Fig. 9 The STM SAP Results of Load – Deflection Response for Different Alternatives.

Table 3 The Comparison of the Different Alternatives CAST and ANSYS Results with Taking the Effect of Concrete Compressive Strength.

Alternatives	Concrete Compressive Strength MPa (psi)	Elastic Modulus (MPa)	Reinforcement Weight (Ib)	CAST		ANSYS			
				Governing STM Component	Load Factor to Reach Capacity	First Cracking Load (KN)	First Cracking Location	Failure Load (KN)	Load Factor To Reach Failure
Alt. (A)	43.48 (6185)	36680	7.73	E9 - Alt.A (Inclined Lower Tie)	1.374	187.61	Opening Lower Left Corner	208.47	1.489
Alt. (A')	32 (4552)	24890		E9 - Alt.A (Inclined Lower Tie)	1.374	142.61	Opening Lower Left Corner	161.5	1.13
Alt. (A'')	40 (5690)	27828		E9 - Alt.A (Inclined Lower Tie)	1.374	167.61	Opening Lower Left Corner	211.8	1.26
Alt. (B)	43.48 (6185)	36680	5.58	E20 - Alt.B (Opening Upper Right Corner)	1.501	187.61	Under the Load Point	232.69	1.662
Alt. (B')	32 (4552)	24890		E20 - Alt.B (Opening Upper Right Corner)	1.501	117.61	Mid Height Over Opening Centre	207.7	1.77
Alt. (B'')	40 (5690)	27828		E20 - Alt.B (Opening Upper Right Corner)	1.501	147.61	Mid Height Over Opening Centre	265.02	1.79
Alt. (C)	43.48 (6185)	36680	13	E7 - Alt.C (Right Bottle Shape Strut)	2.025	175.11	Above the Right Support	195.80	1.398
Alt. (C')	32 (4552)	24890		N7 - Alt.C (Opening Upper Reinf. End)	1.741	127.61	Above the Right Support	162.74	1.28
Alt. (C'')	40 (5690)	27828		E7 - Alt.C (Right Bottle Shape Strut)	2.025	157.61	Above the Right Support	178.86	1.13
Alt. (D)	43.48 (6185)	36680	13.27	E35 - Alt.D (Lower Tension Tie)	1.38	186.36	Under the Load Point	224.39	1.602
Alt. (D')	32 (4552)	24890		N23 - Alt.D (Lower Tension Tie)	1.068	117.61	Above the Right Support	210.36	1.79
Alt. (D'')	40 (5690)	27828		N23 - Alt.D (Lower Tension Tie)	1.335	157.61	Above the Right Support	246.11	1.56

6. The overall comparison of results

The two univariate analysis of variance statistical hypothesis test (factorial ANOVA using SPSS software [57]) is used to make the comparison between two independent random factors (shape alternatives & analysis methods), within the dependent variable data set (estimated marginal means of the load factor). The significant differences results for hypothesis of confidence level equal 0.01 are shown in Table 4.

Table 4 The SPSS Hypothesis Significant Differences.

Null Hypothesis Source	Sig.	Significant Differences
Shape Alternatives	.804	No Significant Differences (Between Each Other)
Analysis Methods	.911	No Significant Differences (Between Each Other)
Shape Alternative * Analysis Method	.000	There are Significant Differences (Fig. 10)

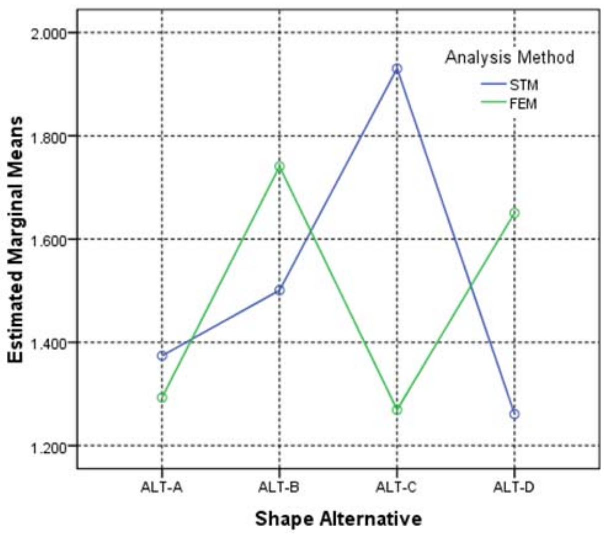


Fig. 10 The Estimated Marginal Means of Load Factor.

Also, for studying the difference between codes of practice in predicting the load factor to reach capacity and the governing STM component, the Alt. (A) is analyzed by CAST as shown in Table 5.

Table 5 The Difference Between Different Codes of Practice.

Codes of Practice	The Load Factor to Reach Capacity	The Governing STM Component
ACI 318 & NZS 3101 (Alt.A)	1.374	E9 - Alt.A (Inclined Lower Tension Tie)
EC2 (Alt.A-1)	1.287	E42 - Alt.A (Under the Load Point)
ECP 203 (Alt.A-2)	1.374	E9 - Alt.A (Inclined Lower Tension Tie)
FIP 1999 (Alt.A-3)	1.374	E9 - Alt.A (Inclined Lower Tension Tie)
CEB -FIP 90 (Alt.A-4)	1.287	E42 - Alt.A (Under the Load Point)
DIN 1045-1 (Alt.A-5)	1.374	E9 - Alt.A (Inclined Lower Tension Tie)

Finally, Fig. 11, Fig. 12, and Table 6 are presented to illustrate the STM stress ratio results conducted by different codes of practice comparing to the experimental tie element stress ratio results.

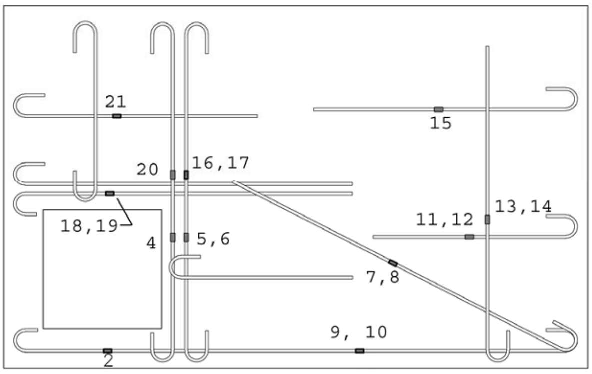


Fig. 11 Location of Strain Gages (Carlos A. Flores, 2009).

Table 6 The Tie Elements Beta Ratio Comparison of the Different Codes of Practice Conducted by CAST and Experimental Results.

		Location	1	2	3	4	5	6	7	8
		Element ID	E7-H	E7-V	E8-H	E10	E21	E24	E31	E34
Experimental		Strain Gage ID	11,12	13,14	15,-	9,10	4,-,5,6	20,-,16,17	21	-,-,18,19
		Force (Kip)	-0.01	1.71	3.78	0.35	0.36	2.79	0.43	0.34
		Yield Force (Kip)	9.9	9.9	9.9	9.9	19.8	19..8	9.9	19.8
		Beta Ratio	-0.001	0.172	0.352	0.035	0.018	0.141	0.043	0.017
Cast		Force (Kip)	5.38Cos 30.86 = 4.57	5.38Sin 30.86 = -2.84	5.38Cos 30.86 = 4.57	6.17	12.35	12.35	4.41	11.18
	ACI 318 & NZS 3101 (Alt.A)	Yield Force (Kip)	9.9	9.9	9.9	9.9	19.8	19..8	9.9	19.8
		Beta Ratio	0.138Cos 30.86 = 0.117	0.138Sin 30.86 = -0.073	0.138Cos 30.86 = 0.117	0.158	0.219	0.219	0.113	0.198
	EC2, ECP 203, FIP 1999, CEB -FIP 90, DIN 1045-1 (Alt.A-1:5)	Yield Force (Kip)	15.18	15.18	15.18	15.18	30.36	30.36	15.18	30.36
		Beta Ratio	0.103Cos 30.86 = 0.087	0.103Sin 30.86 = -0.054	0.103Cos 30.86 = 0.087	0.119	0.164	0.164	0.085	0.149

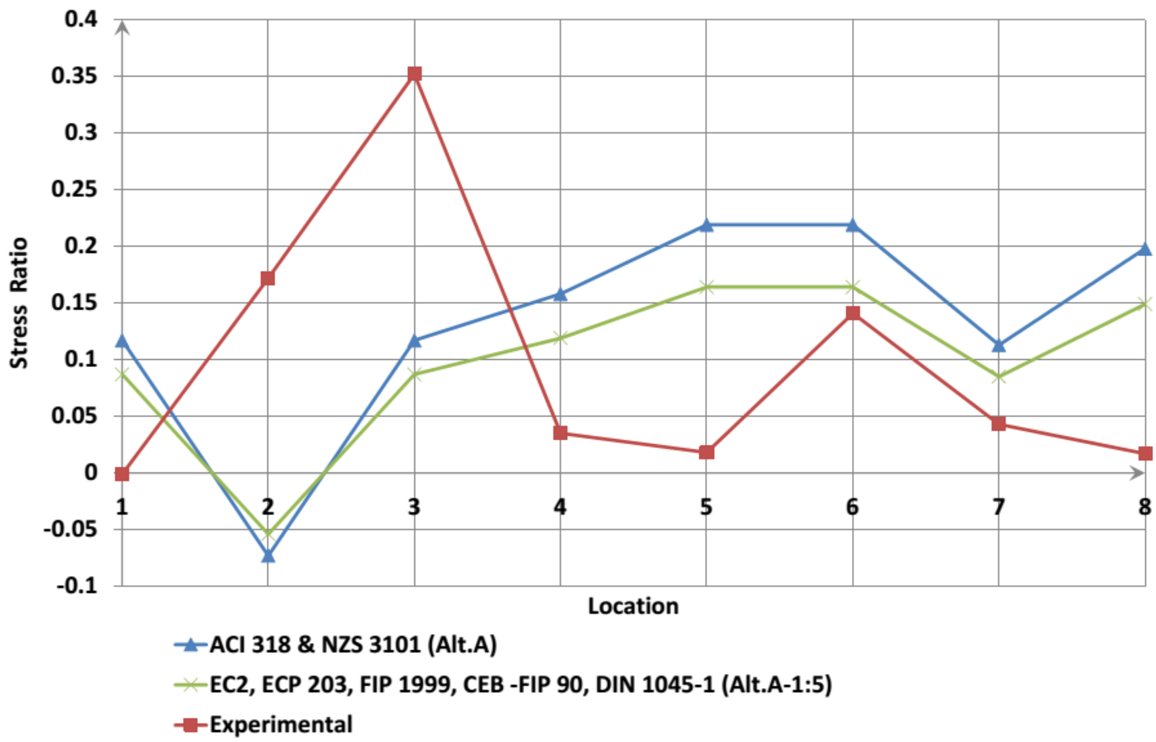


Fig. 12 The Tie Elements Beta Ratio of Different Codes of Practice and the Experimental Test Results.

7. Conclusions

Since different shapes of struts and ties models yield different results, the shape model with best ordinal weight number of conditional factors should be adopted as the optimal shape.

With assuming that all of conditional factors as the rebar amount, the load factor, and the structural concrete ductility have the same priority. Table 7 illustrates the better alternative from the different research alternatives.

The priority of these conditional factors could be changed according to the designer logic for each structural case, but at all the STM components of any model should satisfying the acceptable stress ratio limits, then adopt the strut-and-tie model shape with the best ordinal weight number of conditional factors.

Table 7 The Better Alternative from the Different research Alternatives.

The Conditional Factors	The Better is The	Ordinal Number Alt. (A)	Alt. (B)	Alt. (C)	Alt. (D)
Reinforcement Weight	Smaller	2	1	3	4
FEA Displacement Ductility Ratio	Bigger	4	3	2	1
FEA Load Factor of Estimated Marginal Means	Bigger	3	1	4	2
STM Load Factor of Estimated Marginal Means	Bigger	3	2	1	4
Difference between STM and FEA Load Factor of Estimated Marginal Means	Smaller	1	2	4	3
All Its Elements and Nodes Satisfying the Stress Ratio Limits	-	Yes	Yes	No	No
Ordinal Weight Number	-	13	9	14	14
Order Levelling	-	2nd	1st	3rd-R	3rd

Finally, the following conclusions could be conducted:

- The research Alt. (B) has the better order leveling that all its elements and nodes satisfying the stress ratio limits with bigger FEA load factor of different concrete strength properties estimated marginal means and smaller rebar amount.
- There is no statistical significant difference in the mean of load factor to reach capacity between the two different analysis methods STM and FEA with confidence level of 99%. And the magnitude of difference in the estimated marginal means between the two analysis methods is not equal across different shape alternatives where the Alt. A has lower difference than other different alternatives.
- The stress ratio of STM components may be changed according to the design code of practice as the N14 - Alt.A triangulation stress ratio exceeds 1.0 with ECP 203 and CEB -FIP 90, on other side all elements and nodes that designed according to other codes of practice satisfying the stress ratio limits.
- The concrete material strength can affect in both the first crack pattern and the magnitude of load factor for the same STM shape alternative.
- The ACI 318-08 and the NZS 3101 have more conservative ties safety factor than other codes. And the EC2 and the CEB -FIP 90 have less load factor to reach capacity than other codes which specify the strut and node compressive strength as a function of the product of the concrete compressive strength and a reduction factor.

8. References

[1] Ritter, W., “The Hennebique Design Method” (Zürich) (in German), Vol. 33, No. 7, pp. 59-61 (Feb 1899).

[2] Mörcsh, E., "Reinforced Concrete Construction-Theory and Application", 5th Ed., Wittwer, Stutgart, V.1, Part 1, Part 2, (1920, 1922).

[3] Dilger, W. “Veränderlichkeit der Biege-und Schubsteifigkeit bei Stahlbetontragwerken und ihr Einflußauf Schnittkraftverteilung und Traglast bei statisch unbestimmter Lagerung.’’ Deutscher Ausschuss für Stahlbeton, Heft 179, Berlin, Germany (1966).

[4] Nielsen, M. P. "Limit Analysis and Concrete Plasticity" ,Prentice-Hall, Englewood Cliffs, New Jersey (1984).

[5] Paulay, T. “Coupling Beams of Reinforced Concrete Shear Walls.’’ Journal of the Structural Division, ASCE, 97(ST3), 843–862 (1971).

[6] Thürlimann, B., Marti, P., Pralong, J., Ritz, P., and Zimmerli, B. "Application of the theory of plasticity to reinforced concrete" Institute of Structrual Engineering, ETH Zürich (1983).

[7] Yun, Y. M., and Ramirez, J. A. “Strength of Struts and Nodes in Strut-and-Tie Model.” Journal of Structural Engineering. V. 122, No. 1, pp 20-29 (January, 1996).

[8] Schlaich, J. and Schäfer, K. “Design and Detailing of Structural Concrete Using Strut-and-Tie Models.” The Structural Engineer. V. 69, No. 6, pp 113-125 (May-June, 1991).

[9] Adebar, P., Kuchma, D., and Collins, M.P., “Strut-and-Tie Models for the Design of Pile Caps: An Experimental Study,”ACI Structural Journal, Vol. 87, No. 1, pg. 81-92 (1990).

[10] Alshegeir, A., J.A. Analysis of Disturbed Regions with Strut-and-Tie Models. Dissertation, Department of Civil Engineering, Purdue University, 274 pp (December 1992).

[11] Adebar, P., and Zhou, Z., "Design of Deep Pile Caps Using Strut-and-Tie Models," ACI Structural Journal, Vol. 93, pp. 437-48 (July/Aug 1996).

[12] Bergmeister, K., Breen, J.E., Jirsa, J.O., and Kreger, M.E., “Detailing for Structural Concrete,” Center for Transportation Research Report CTR 0-1127-3F, Austin, Texas (1993).

[13] Breen, J.E., Burdet, O., Roberts, C., Sanders, D., and Wollmann, G., “Anchorage Zone Reinforcement for Post-Tensioned Concrete Girders,” National Cooperative Highway Research Program Report No. 356, Washington, DC (1994).

[14] Stephen J. Foster and Adnan R. Malik,”Evaluation of Efficiency Factor Models used in Strut-and-Tie Modeling of Non-flexural Members.” J. Str. Engineering, 128(5), p. 569–577 (2002).

[15] Chen, W.F, and Han, D.J. Plasticity for Structural Engineers, Springer-Verlag New York Inc., 606pp (1988).

[16] Stephen J. Foster and Adnan R. Malik ”Evaluation of Efficiency Factor Models used in Strut-and-Tie Modeling of Nonflexural Members.” J. Struct. Eng., 128(5), 569–577 (2002).

[17] D. K. Sahoo; B. Singh; P. Bhargava, "Effect of Inclination on the Strength of Struts", Magazine of Concrete Research, Volume 63, Issue 2, p. 111 –117 (Dec. 2010).

[18] Panjehpour M, Chai HK, Voo YL, "Refinement of Strut-and-Tie Model for Reinforced Concrete Deep Beams", PLOS ONE 10(6): e0130734 (2015).

[19] Vecchio, F. J., and Collins, M. P. "The modified compression-field theory for reinforced concrete elements subjected to shear." Journal of the American Concrete Institute, 83(2), 219-231 (1986).

[20] Collins, M.P., and Mitchell, D., “Rational Approach to Shear Design – The 1984 Canadian Code Provisions,” ACI Journal, Proceedings, Vol. 83, No. 6, pp. 925-933 (Nov-Dec 1986).

[21] Marti, P. "How to treat shear in structural concrete." ACI Structural Journal, 96(3), 408-414 (1999).

[22] Rogowsky, D.M., MacGregor, J.G., Ong, Y.S. Tests of Reinforced Concrete Deep Beams ACI Journal, Vol. 83, Issue 4, p 614-623 (July-August 1986).

[23] Clark, A. P., “Diagonal Tension in Reinforced Concrete Beams,” ACI Journal, V. 48, No. 10, pp. 145 – 156 (1951).

[24] Ferguson, P. M., “Some Implication of Recent Diagonal Tension Tests,” ACI Journal, V. 53, No. 1, pp. 157-172 (1955).

[25] Watstein, D., and Mathey, R. G., “Strains in Beams Having Diagonal Cracks,” ACI Journal, V. 55, No. 12, pp. 717-728 (1958).

[26] Cook W.D and Mitchell, D. Studies of Disturbed Regions near Discontinuities in Reinforced Concrete Members ACI Structural Journal, V. 85, No. 2, p 206-216 (Mar-Apr 1988).

[27] James O. Jirsa, Konrad Bergmeister, Robert Anderson, John E. Breen, David Barton, and Hakim Bouadi, “Experimental Studies of Nodes in Strut-and-Tie Models," Structural Concrete IABSE Colloquium Stuttgart, 1991, Report, International Association for Bridge and Structural Engineering, Zurich, pp. 525–532 (1991).

[28] Siao, W.B. Strut-and-Tie Model for Shear Behavior in Deep Beams and Pile Caps Failing in Diagonal Splitting ACI Structural Journal, Vol. 90, No. 4, p 356-363 (Jul-Aug 1993).

[29] Maxwell, B. S. and Breen, J. E., “Experimental Evaluation of Strut-and-Tie Model Applied to Deep Beam with Opening”, ACI Structural J.V. 97, No. 1, p. 142-148 (Jan.-Feb. 2000).

[30] Yun, Y. M., "Nonlinear Strut-Tie Model Approach for Structural Concrete", ACI Structural Journal, 97(4), p. 581-590 (2000).

[31] Sagaseta, J. & Vollum, R. L., "Shear Design of Short-Span Beams", Magazine of Concrete Research 62 (4), p. 267-282 (2010).

[32] Chen, B.S., Hagenberger, M.J., and Breen, J.E., “Evaluation of Strut-and-Tie Modeling Approach to Dapped Beam with Opening”, ACI Structural Journal, Vol. 99, No. 4, p. 445-450 (2002).

[33] Huang, F.C., Lee, I.S., and Mo, Y.L., “Designing Pier Caps with Strut-and-Tie Models,” Concrete International, Vol. 20, pp. 43-7 (1998).

[34] Brown, M. D., Bayrak, O., “Design Of Deep Beams Using Strut And Tie Models – Part I, II: Evaluating U.S. Provisions, Design Recommendations”, ACI Structural J. V. 105, No. 4, p. 395-413 (July-Aug., 2008).

[35] Ley, M.T., Riding, K.A., Widianto, Bae S., and Breen, J. “Experimental Verification of the Strut and Tie Model Design Method”, ACI Str. J., V. 104, No. 6, p. 749-755 (2007).

[36] A. Arabzadeh, A. R. Rahaie, R. Aghayari,"A Simple Strut-and-Tie Model for Prediction of Ultimate Shear Strength of RC Deep Beams", International J. of Civil Engineering. Vol. 7, No. 3, (Sep. 2009).

[37] To, N. H. T., Ingham, J. M., and Sritharan, S. "Strut-and-tie computer modeling of reinforced concrete bridge portal frames." Bulletin of the New Zealand Society for Earthquake Engineering, 35(3), 165-189 (2002).

[38] Zhang N., Tan K.-H. "Direct strut-and-tie for Single Span and Continuous Deep Beams", Engineering Str. 29(11):2987-3001 (2007).

[39] Carlos Flores, "Performance of Large Scale Steel Fiber Reinforced Concrete Deep Beam with Single Opening under Monotonic Loading", Master Thesis, University of Texas, (2009).

[40] Liang, Q. Q, Uy, B., and Steven, G.P “Performance-Based Optimization for Strut-Tie Modeling of Structural Concrete,” J. of Str. Eng. Vol. 128: p. 815-823 (June 2002).

[41] Yun, Y. M., "Nonlinear Strut-Tie Model Approach for Structural Concrete", ACI Structural Journal, 97(4), p. 581-590 (2000).

[42] Hamed and M.Salem. "The Micro Truss Model: An Innovative Rational Design Approach for Reinforced Concrete", Journal of Advanced Concrete Technology. 2(1), p. 77-87 (2004).

[43] Nagarajan p., U.B. Jayadeep, and Pillai T.M.M. "Application of Micro Truss and Strut and Tie Model for Analysis and Design of Reinforced Concrete Structural Elements", Songklanakarin J. Sci. Technol.31 (6), p. 647-653, (Nov. - Dec. 2009).

[44] AASHTO LRFD 1996 - 2012 Bridge Design Specifications, American Association of State Highway and Transportation Officials, first edition, Washington, DC.

[45] AS3600 (1994) also AS3600 (2001, 2009), Australian Standard for Concrete Structures, North Sydney.

[46] ECP203-07, Egyptian Code of Practice; Ministry of Housing, Cairo, (in Arabic).

[47] ACI 318-08 and Commentary (318R-08) also ACI 318-14 revision, American Concrete Institute, Farmington Hills, Detroit, Michigan, USA.

[48] NZS 3101-1, Part 1: The Design of Concrete Structures, NZS Concrete Design Committee P 3101 for the Standards Council. Concrete Structures Standard, Wellington, 2006.
[49] DIN 1045-1, Part 1: Design and Construction. Berlin, Germany: Deutsches Institut für Normung (DIN-Normen); 2001.
[50] Eurocode 2 EN 1992-1-1 (2004): Design of Concrete Structures, Part 1-1: General Rules and Rules for Buildings.
[51] CEB-FIP MC 90, Design of Concrete Structures, Comité Euro-International du Béton. London: Thomas Telford Services; 1993.
[52] FIP Commission 3 (1999), Practical Design of Structural Concrete, Federation International de la Precontrainte.
[53] Breña, S. F.; and Morrison, M. C., "Factors Affecting Strength of Elements Designed Using Strut-and-Tie Models," ACI Str. Jo., V. 104, No. 3, (May-June 2007).
[54] ANSYS 13.0, Help Manual Inc.
[55] SAP2000 TM. Advanced 14.0.0, Computer and Structures, Help Manual Inc, Berkeley, CA.
[56] Tjhin, T.N. and Kuchma, D.A. " Computer-Aided Analysis of Reinforced Concrete using a Refined Nonlinear Strut and Tie Model Approach", ACI Structural Journal, Vol. 99, No. 5, p. 586-594, (Sept.-Oct. 2002). [http://dankuchma.com/stm/sitemap.htm].
[57] SPSS, Statistical Package for Social Science 17.0, Help Manual Inc, Polar Engineering and Consulting.