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Verification of the Kinematic Analysis of the 6-DOF Stewart Platform Manipulator

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ABSTRACT

Virtual modelling and simulation of robot-based systems provides decisive advantages not only in design but also in planning and the normal operation of such systems. In addition, kinematic modelling provides a powerful framework to understand the structure, mobility and adapt the specific characteristics of both serial and parallel manipulators. In this work, the position and orientation of the Stewart Platform parallel manipulator are required in order to find the leg lengths which is the solution for the inverse kinematics problem. The lengths are automatically computed with respect to a given orientation and position and hence the end-effector performs a predefined trajectory in the task space. A virtual and mathematical verification of the position analysis of a 6-DOF Stewart platform was implemented by comparing different robot postures with the same position and orientation. SolidWorks and MATLAB are used to check the study and the robot motion simulation. The results are discussed and an agreement between the two programs is certainly obtained.

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Nomenclature

ϕ The angle of rolling around the z-axis.

θ The angle of pitching around the y-axis.

ψ The angle of yawing around the x-axis.

$R_z(\phi)$ The $3x3$ rotation matrix describing the rotation by an angle ϕ around the z-axis.

$R_y(\theta)$ The $3x3$ rotation matrix describing the rotation by an angle θ around the y-axis.

$R_x(\psi)$ The $3x3$ rotation matrix describing the rotation by an angle ψ around the x-axis.

$^{B}R_{P}$ The full rotation matrix of the platform coordinate frame relative to the base frame.

$\boldsymbol{p_i}$ The vector defining the coordinates of the anchor point p_i with respect to the platform frame.

$\boldsymbol{b_i}$ The vector defining the coordinates of the anchor point b_i with respect to the base framework.

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$\boldsymbol{l_i}$ The length of the i^{th} leg where $i = 1, 2, 3, \ldots, 6$.

$\boldsymbol{T}$ The translational position vector giving the origin of the platform frame with respect to the base coordinate frame.

1. Introduction

As being known for many, Robotics is a special branch of engineering that deals with the design, control, modelling and simulations of robots and many other systems. People nowadays have robots in industrial and everyday applications that includes: cleaning, pick and place tasks, milling, drilling, and different surgical and medical applications. Other examples are in toys and game simulators where many kids prefer driving a toy car or a motorbike instead of having difficulty in dealing with the real vehicle. Different robot structures with many closed loop kinematic chains' configurations have been implemented for the manipulation of the aforementioned tasks. Examples are spherical, revolute and prismatic. Unlike non-redundant robots, redundant manipulators play an important rule for having an infinite number of solutions for the manipulator to make a given task that is considered a perfect parameter for improving and optimizing the robot's operation.

A generalized parallel configuration, Stewart Platform with six degrees of freedom is well-suited for: a flight simulator for training pilots in an environment that best suited for a helicopter or as any vehicle which can be controlled by human being [1]. Although the final aim is real robotics which is often very useful to perform kinematic modelling and simulations prior to investigations with real robots[2]. This is because simulations have many advantages like they are faster, easier to setup, less expensive and more convenient to use [3]. A simulated robotics setup is less expensive than real robots and real world setups, thus allowing better design exploration. Modelling helps in understanding the robot's behavior. Simulations often run faster and easier than real robots while all the parameters are easily displayed on screen [4].

A manipulation task for any manipulator is usually given in terms of a desired pose or generally named as the end-effector trajectory [5]. Since the manipulator is controlled by joint servos, a mapping from the task space to the joint space is required. This is based on an inverse kinematic transformation approach which feeds the reference values corresponding to an assigned end-effector trajectory to the joint servos.

In recent years, the need for accurate and efficient manipulator kinematics has been extensively emphasized. The modelling and simulation of various robot systems by using many programs will facilitate the process of design, construction and inspection of robots in the real world [6]. A simulation is considered as an important tool for many robot programmers allowing them to evaluate and predict the robots behavior in addition to verifying and optimizing the path planning of the process [7]. Moreover, this will save time and money, help in solving many problems creatively and play an important role in the evaluation of manufacturing automation [8]. One can investigate, design, visualize and test an object before making it a reality.

In this work, a 6-DOF parallel robot system is being designed and developed using the SolidWorks program as shown in Fig. 1 and MATLAB program as shown in Fig. 5. A mathematical modelling for the calculations of the actual leg lengths of the manipulator was introduced by using the inverse kinematics approach.

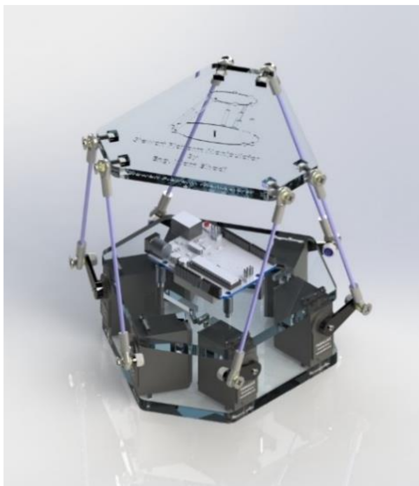

Fig. 1. The Stewart platform model in SolidWorks

To verify the obtained results in SolidWorks, modelling and simulation by using MATLAB will be carried out. The results of both will be presented and discussed. The kinematic equations for a Stewart platform were developed using basic algebra and trigonometric functions. The paper is organized as follows: in section 2 inverse kinematics modelling of the platform is introduced. Section 3 presents the simulation applications, different postures of the manipulator, visual and mathematical verification of the math. The total results of all robot postures are discussed in section 4. Section 5 shows all conclusions and further recommendations about the results and the geometry of the Stewart Platform manipulator.

2. Robot Inverse Kinematics

The Stewart Platform consists of two rigid frames connected by six variable leg length as in [9] and [10]. The base is considered to be the reference framework, with orthogonal axes x, y and z. The platform has its own coordinates $\acute{x}, \acute{y}$ and $\acute{z}$. The platform has 6 degrees of freedom with respect to the Base[11]. Different postures of the Platform coordinates can be defined by three translational displacements with respect to the Base, one for each axis and three angular displacements define the orientation of the platform with respect to the base [12].

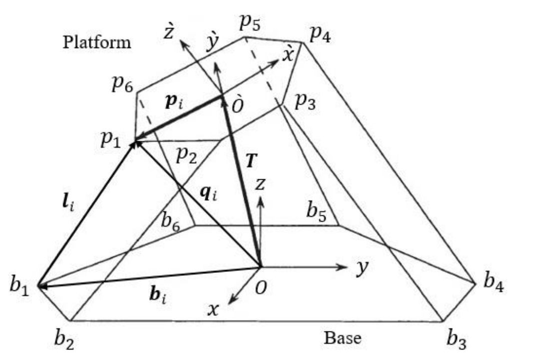

Fig. 2. Vector loop equation for actual leg length evaluation

The rotation of the platform relative to the base can be evaluated from the full rotation matrix [13]. The full rotation matrix of the platform with respect to the base can be given by the dot product of the rotation matrix around the z-axis, y-axis and x-axis respectively. This is introduced as follows:

$$^{B}R_{P} = R_z(\phi)\ .\ R_y(\theta)\ .\ R_x(\psi)$$

$$^{B}R_{P} =$$

$$\begin{bmatrix} \cos\phi\cos\theta & -\sin\phi\cos\psi+\cos\phi\sin\theta\sin\psi & \sin\phi\sin\psi+\cos\phi\sin\theta\cos\psi \\ \sin\phi\cos\theta & \cos\phi\cos\psi+\sin\phi\sin\theta\sin\psi & -\cos\phi\sin\psi+\sin\phi\sin\theta\cos\psi \\ -\sin\theta & \cos\theta\sin\psi & \cos\theta\cos\psi \end{bmatrix} \quad (1)$$

Now consider a Stewart Platform as in Fig. 2. The coordinates of the anchor point with respect to the Base reference framework are given by the equation

$$\boldsymbol{q_i} = \boldsymbol{T} + {}^{\mathbf{B}}\mathbf{R}_{\mathbf{P}}\ .\ \boldsymbol{p}_i - \boldsymbol{b_i} \qquad (2)$$

Where $\boldsymbol{T}$ is the translation vector, giving the positional linear displacement of the origin of the Platform frame with respect to the Base reference framework, and $\boldsymbol{p_i}$ is the vector defining the coordinates of the anchor point p_i with respect to the Platform framework. Similarly, the length of the i^{th} leg is given by

$$\boldsymbol{l_i} = \boldsymbol{T} + {}^{\mathbf{B}}\mathbf{R}_{\mathbf{P}}\ .\ \boldsymbol{p_i} - \boldsymbol{b_i} \qquad (3)$$

Defining the coordinates of the lower anchor point b_i. These six equations give the lengths of the six legs to achieve the desired position and orientation (attitude) of the platform [14].

As seen in (2) and (3), the leg lengths are computed by using simple vector subtraction. The six vectors of the base plate are subtracted to the six vectors of the top plate. This yields a $1\ x\ 6$ vector where each element is the length of a leg. It is important that the top vectors correspond to the correct base vector. A visual verification has been done in MATLAB with a 3D plot, see Fig. 5. As shown in the plot, the end of the upper vectors is connected to form the top plate, the end of the base vectors forms the base plate and the top vectors are connected to their corresponding base vectors to form the legs. However, this program makes it easier to evaluate the leg lengths as a function of a position and thus, makes it possible to move, rotate and control a Stewart platform.

When considering the Forward Kinematics, this expression represents 18 simultaneous nonlinear equations in the 6 unknowns representing the position and attitude of the platform. Much work has been done on finding the solutions to these equations; in the general case there are 40 possible solutions, although in practice many of them would not be practical [15].

3. Simulation and Verification

3.1. Postures

For the evaluation of the inverse kinematics problem and the simulation of the whole model, two different postures in addition to the home position will be studied and simulated on both MATLAB and SolidWorks. First, the definition of the home position (default configuration) for the manipulator is introduced as follows:

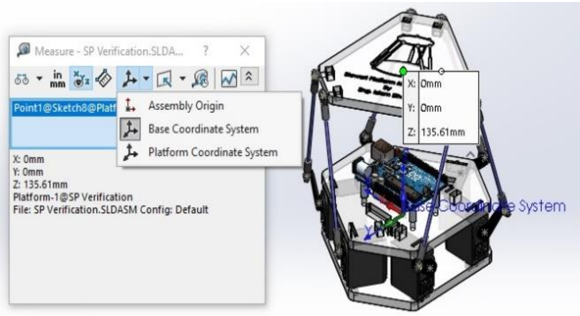

Fig. 3. Default configuration (home position) for the manipulator in SolidWorks

By default, without any applied rotations and only a translation in the positive z-axis, the platform stands only in a position relative to the base coordinate frame approximately of $x = 0$mm, $y = 0$mm and $z = 135.61$ mm. The positive value of the z-axis is the actual height between both the top and the bottom plates of the manipulator.

To verify the position of the default configuration, the platform in SolidWorks moved exactly to the position mentioned above resulting in the home position in Fig. 3. Actual leg lengths were measured as in Fig. 4 and The same configuration is evaluated and verified visually and mathematically in MATLAB by entering the actual position of the configuration measured from SolidWorks resulting in Fig. 5.

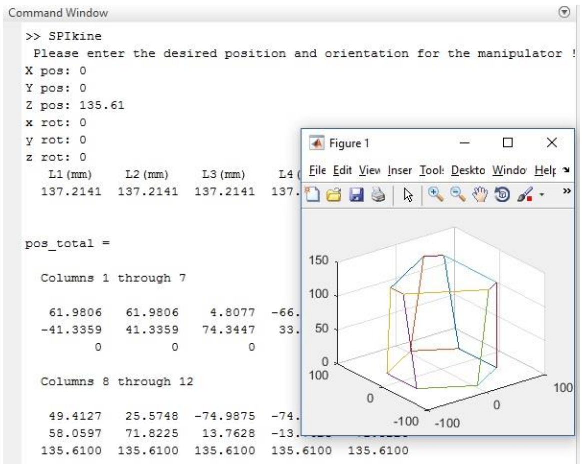

Fig. 5. Default configuration of the manipulator in MATLAB

In addition to the MATLAB plot, the coordinates of the twelve points forming the Stewart Platform Manipulator in space, i.e. six points that describe the position of the lower end of the legs (bottom plate) and the other six points that describe the position of the upper end of the legs have been evaluated relative to the base coordinate system as shown in Table 1 and 2.

Table 1&2. Coordinates of the six points of the bottom and upper plate respectively.

Coordinates	b_1	b_2	b_3	b_4	b_5	b_6
x-axis	61.9	61.9	4.8	-66.7	-66.7	4.8
y-axis	-41.3	41.3	74.3	33.0	-33.0	-74.3
z-axis	0	0	0	0	0	0

Coordinates	p_1	p_2	p_3	p_4	p_5	p_6
x-axis	49.4	49.4	25.5	-74.9	-74.9	25.5
y-axis	-58.0	58.0	71.8	13.7	-13.7	-71.8
z-axis	135.6	135.6	135.6	135.6	135.6	135.6

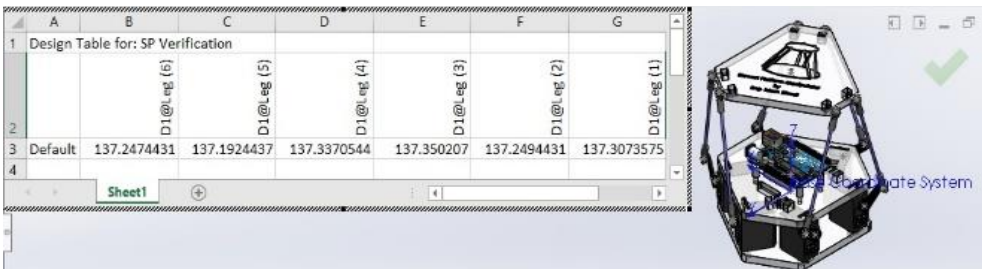

Fig. 4. Actual leg lengths for the home position in SolidWorks

3.1.1 Posture 1

For this posture, a translation for the manipulator's platform in the x, y, and z coordinates will be maintained relative to the base coordinate framework and without any rotations around the three axes. Fig. 6 declares the current position of the upper plate relative to the base.

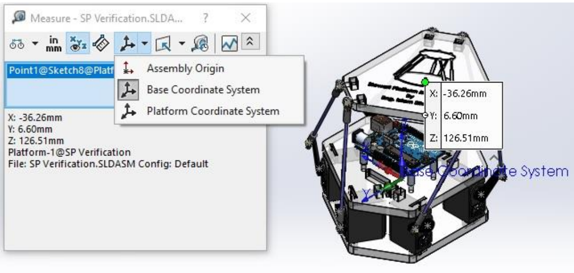

Fig. 6. Coordinates of the first posture for the manipulator's platform in SolidWorks

It seems that the platform coordinate frame stands at $x = -36.26$mm, $y = 6.60$mm and $z = 126.51$mm relative to the base coordinate frame. To measure the distance between the six points in the upper plate and the corresponding points at the bottom, which is the actual leg length for the manipulator, a design table was created in SolidWorks® to keep tracking the actual leg lengths while moving the platform to a predefined translation, rotation or both. The actual lengths for this posture are shown in Fig. 7.

To verify the math, the same position and orientation for posture 1 were entered as inputs to the MATLAB program and a further calculation of the leg lengths of the manipulator was computed by moving to the aforementioned position. Fig. 8 shows the calculated lengths for the last position in MATLAB.

3.1.2 Posture 2

Both a translation and a rotation movement will be applied to the robot's platform in this posture. The platform will translate in the x, y and z-axis and will rotate around the z axis. The actual translation is 0.03 mm in the x-axis which is near to zero, 0.6 mm in the y-axis and the height between the two plates will be reduced to 107.89 mm in the z-axis. Further rotation will be applied by 60 degrees around the z-axis. The platform moved to this position as shown in Fig. 9. After the platform has moved to the position shown in the above figure, the six leg lengths were measured and shown in Fig. 10.

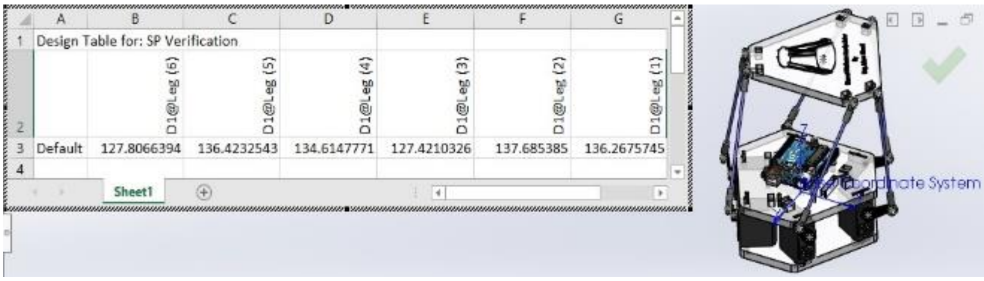

Fig. 7. Actual leg lengths for posture 1 in SolidWorks

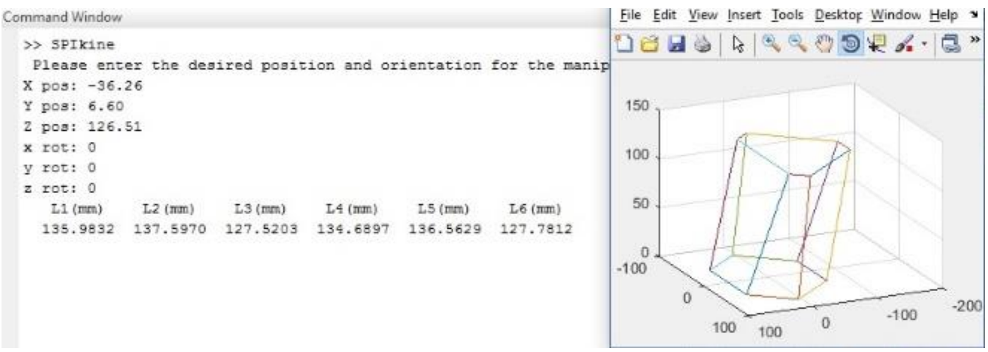

Fig. 8. Evaluated leg lengths for posture 1 in MATLAB

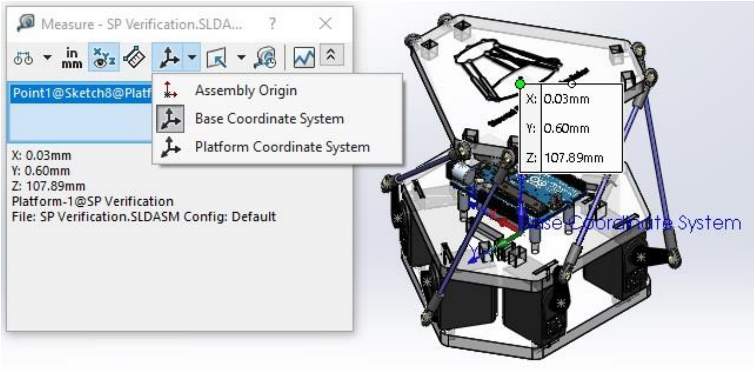

Fig. 9. Coordinates of the second posture for the manipulator in SolidWorks

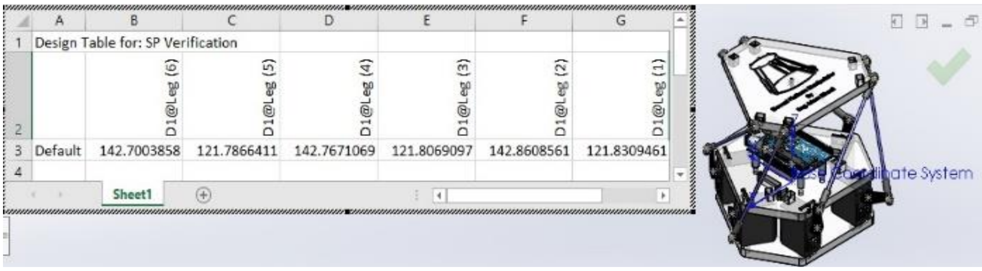

Fig. 10. Actual leg lengths for posture 2 in SolidWorks

Another substitution in the MATLAB program for the coordinates of the current posture gives the following lengths of the manipulators' legs.

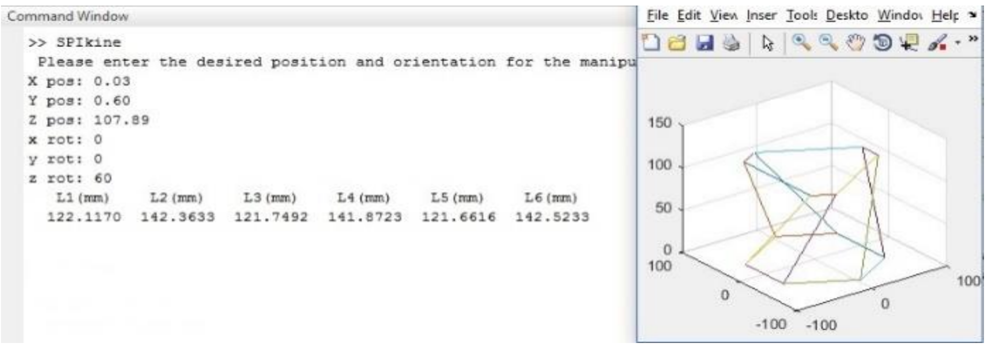

Fig. 11. Evaluated leg lengths for posture 2 in MATLAB

4. Results and discussion

From the above section, all the results of the verified mathematics of the 6-DOF Gough-Stewart platform for the last two postures with the manipulator's home position can be completely rearranged and summarized in the following three tables, one for each posture.

Table 3 Actual leg lengths' measurements and calculations for each posture respectively

Home position	x pos.	y pos.	z pos.	x rot.	y rot.	z rot.
Coordinates	0	0	135.61	0	0	0
Leg lengths, mm	L_1	L_2	L_3	L_4	L_5	L_6
SolidWorks	137.3073	137.2494	137.3502	137.3370	137.1924	137.2474
MATLAB	137.2141	137.2141	137.2141	137.2141	137.2141	137.2141

Posture 1	x pos.	y pos.	z pos.	x rot.	y rot.	z rot.
Coordinates	-36.26	6.60	126.51	0	0	0
Leg lengths, mm	L_1	L_2	L_3	L_4	L_5	L_6
SolidWorks	136.2675	137.6853	127.4210	134.6147	136.4232	127.8066
MATLAB	135.9832	137.5970	127.5203	134.6897	136.5629	127.7812

Posture 2	x pos.	y pos.	z pos.	x rot.	y rot.	z rot.
Coordinates	0.03	0.60	107.89	0	0	60°
Leg lengths, mm	L_1	L_2	L_3	L_4	L_5	L_6
SolidWorks	121.8309	142.8608	121.8069	142.7671	121.7866	142.7003
MATLAB	122.1170	142.3633	121.7492	141.8723	121.6616	142.5233

Analyzing the results above, it seems that the six leg lengths of the manipulator are approximately the same for all latter configurations. The process of moving the manipulator to a predefined posture, measuring the leg lengths then entering the coordinates of the same position in MATLAB for the evaluation of the correct lengths of the legs and comparing the results in both programs is a pure verification stating that all mathematics written is correct.

When considering the home position for example, the lengths are approximately identical for all legs with a maximum iteration of 0.13 mm for leg 3 and a minimum of 0.02 mm for leg 5. Looking at table 4 and by comparing the results of both programs for posture 1, it's clearly obvious that the maximum difference in length is 0.28 mm for leg 1 and leg 6 has the minimum difference between all legs which is actually 0.02 mm. Applying the same comparison for posture 2 which is the last in this paper study, the maximum difference is in leg 4 by 0.89 mm and the minimum is in leg 3 by 0.05mm.

For the studied postures, both maximum and minimum differences are considered the least in the home position and that is mainly due to the symmetry of the manipulator. Because of it, the platform is exactly above the base plate and all legs are in the same length.

Moving to the next posture, the minimum difference seems to be the same as the one in the home position but the maximum difference of posture 1 increased a little to be approximately the double of its equivalent of the default structure of the manipulator. This change in length returns to the translation made by the robot while moving from a position to another.

While moving to the last posture, the maximum difference increased to reach around 0.9 mm because of the added rotation to the upper plate.

Looking at the maximum and minimum deviations above, accuracy is considered best for the last pre studied postures.

5. Conclusion

In this paper, we used both programs SolidWorks and MATLAB for the verification of the theory and the simulation of the parallel robot. Checking the results obtained by both helped in highlighting the relevance of the studied model.

In our study, we can conclude (based on the analysis of the home position and posture 1) that the accuracy of the calculations is considered perfect and it has not been affected by adding more translations to the movement of the platform. On the contrary, good accuracy is achieved in posture 2 by applying both rotations and translations to the movement of the parallel robot. Further improvements shall to be carried out to the geometry of the manipulator in SolidWorks to maintain higher accuracies in all cases of movement.

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