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DETECTION OF DAMAGE LOCATION IN BEAMS USING DISCRETE WAVELET ANALYSIS

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ABSTRACT

Damage detection in structures using vibration analysis has been a subject of intensive investigation for the last two decades. In this paper, a method for damage detection using wavelet analysis is presented. MATLAB program is prepared to simulate a beam, calculate its mode shapes and analyze the first mode shape by discrete wavelet analysis. The first mode shapes of damaged and undamaged beams are transformed to the wavelet domain. The difference of the detail coefficients of damaged and undamaged mode shapes displays accurately the damage location. A detailed parametric study was conducted by changing the damage location, intensity as well as the boundary conditions and wavelet families. These studies have indicated that discrete wavelet transform accurately predicts the damage location (single or multi damages) from analyzing the first mode shape.

1. Introduction

Due to different actions such as earthquakes, overloads, thermal effects or corrosion, the structural systems accumulate damage during their time life. This is why a reliable procedure that allows the structural evaluation is needed. If damage is not detected correctly, it can lead to the deterioration of the structural elements and consequently to risk the stability of the structure. The classical model based on the vibration analysis such as the changes in the vibration frequencies and mode shapes of a structure could detect only the large damages. The need to detect small damages with numerical methods is under investigation. One of such methods is wavelet transform (WT).

The wavelet analysis (WA) became an efficient tool in the problems of structural damage detection due to high sensitivity to discontinuities in the signals caused by damages. So, the researchers started seeking for improvement of a sensitivity of wavelet-based - methods for damage identification problem and tried to develop novel methodology which estimate the damage presence and position with high precision.

Several studies using continuous wavelet transform (CWT) on the cracks detection in beams were presented by Douka et al. [1], Silva et al. [2], Khatam et al. [3], Ravanfar [4] and Sivasubramanian [5], [6]. Wang et al. [7] had made an experimental study of delamination detection. Rucka [8] used CWT on higher vibration modes. Masoumi et al. [9] used continuous wavelet transform and stationary wavelet transform. Also, Janeliukstis, et al. [10] studied damage identification in polymer composite beams based on spatial continuous wavelet transform.

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The B-spline wavelets for damage location in beams and the composite elements with non-linear geometry were studied by Katunin [11], [12].

Two-dimensional (2D) wavelet transform for detection of cracks in plates studied by Hadjileontiadis and Douka [13], Janeliukstis et al. [14], Katunin [15],[16]. Rucka et al. [17] used neuro-wavelet damage detection technique in beam, plate and shell structures with experimental validation. Nagarajaiah and Basu [18] presented short time Fourier transform (STFT), empirical mode decomposition (EMD), Hilbert transform (HT) and wavelet techniques for decomposition of free vibration response of MDOF systems into their modal components. Lima et al. [19] used two nonlinear frame models for simulation of the real condition of structures and restoring force response is calculated by Runge-Kutta method. Pnevmatikos [20] and Hongnan Li et al [21] studied damage detection of frame structures subjected to earthquake loading by continuous wavelet transform. Mandal et al. [22] presented a performance evaluation of damage detection algorithms for identification of debond in stiffened metallic plates using a scanning laser vibrometer. Other studies using discrete wavelet transform (DWT) on the cracks detection in beams were presented by Zabel [23] and Katunin [24].

Katunin et al. [25] proved that discrete wavelet transform (DWT) is the most effective wavelet transform in comparison with continuous wavelet transform (CWT), stationary wavelet transform (SWT) and lifting wavelet transform (LWT). As the DWT provides the lowest time-consuming algorithm in comparison with CWT, SWT and LWT, which is important when the large number of measurement points is considered in the analysis.

All the previous researches had been done on some cases with limited damage intensity using one or two wavelet families. Some researchers used higher modes in order to improve the damage detection accuracy. But the damage couldn't be detected at the nodal points of the mode shapes. So, in this paper, damage detection of single or multi-damages have been done accurately by analyzing the first mode only. Also, this technique could detect most boundary conditions cases such as (fixed-fixed, simply supported and cantilever) beams with damage percentage of (1% up to 90%) by the discrete wavelet transform (DWT) on the first mode shape. Four wavelet families (BiorSplines "bior2.4", Daubechies "db2", Coiflets "coif5" and Symlets "sym4") have been used also in this study.

2- Discrete wavelet theory:

The discrete wavelet transform (DWT) was developed mostly by Daubechies in late 80 of the twentieth century basing on the multi-resolution signal representation proposed by Mallat [26]. This representation forms the descending sequence of closed functional spaces

$$V_j \subset L^2(R): \qquad (1)$$

$$\ldots V_2 \subset V_1 \subset V_0 \subset V_{-1} \subset V_{-2} \ldots \qquad (2)$$

with properties

$$\bigcup_{j \in z} V_j = L^2(R) \bigcap_{j \in z} V_j = \{0\} \qquad (3)$$

The orthogonal supplement in a space V_j for any j in a space V_{j-1} is a space with orthonormal base ψ_{jk} , $j,k \in Z$. Thus, there is a limitation for the wavelet and scaling function. They should be orthonormal or (semi-, bi-) orthogonal. During DWT the signal f (x) is decomposed to the set of approximation coefficients (A_j) and the set of detail coefficients (D_j) in each level of decomposition. The signal could be presented in the following form:

$$f(x) = \sum_{n=-\infty}^{\infty} f_n^0 \varphi(x-n) \qquad (4)$$

where φ (x-n) denotes translation procedure by the scaling function.

The DWT-based decomposition could be presented by the set of filters. In this case the resulted sets of approximation and detail coefficients could be presented as:

$$f_n^{(j)}(x) = \sum_l \tilde{h}_{2n-l} f_l^{(j-1)} \qquad (5)$$

$$d_n^{(j)}(x) = \sum_l \tilde{g}_{2n-l} f d_l^{(j-1)} \qquad (6)$$

where $\tilde{h}$ and $\tilde{g}$ are the impulse responses of the low-pass and high-pass filters, respectively. Due to the down sampling operation during DWT the resulted sets of coefficients have a half length with respect to the original signal in the case of single-level decomposition. Following to the dyadic rule, the resulted length of sets of obtained coefficients reduces twice with each next level of decomposition.

3. Procedure:

The proposed program deals with different models. The program constructs the model undamaged stiffness matrix (K), model damaged stiffness matrix (K_d) based on the assumed damage and the model mass matrix (M). In the modal analysis the mass matrix has been assumed unaffected by damage. These matrices can be constructed by assembling the element's stiffness and mass matrices which are defined as follow [27]:

$$K_e = \frac{EI}{L^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \quad (7)$$

$$K_d=\alpha * K_e \quad (8)$$

$$M_e = \frac{\rho AL}{420}\begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix} \quad (9)$$

where: K_e, K_d, M_e, E, I, ρ, A, l and α are the undamaged element stiffness matrix, the damaged element stiffness matrix, element consistent mass matrix, Young's Modulus, second moment of inertia, mass density, cross section area, element length and damage ratio respectively. Both the damaged and undamaged element's stiffness and mass matrices are assembled to get the overall stiffness and mass matrices of the beam structure.

From free vibration analysis, the mode shapes (Φ_i) and its frequencies (λ_i) for undamaged and damaged case are calculated. The program receives the model mode shapes and it's frequencies that are obtained directly from field using modern instruments like scanning laser vibrometer. The proposed program transforms the first mode shape of the undamaged and damaged beams to wavelet domain.

Analyzing the damaged beam only may detect the damage location but the analysis give high boundary distortion. So, in this research the damaged and undamaged beams have been used for analysis because of two benefits. First, if the original beam was designed to have a sudden change in the stiffness. Second, the difference between detail wavelet coefficients of damaged and undamaged cases would avoid the boundary distortion.

Three levels of discrete wavelet decomposition have been done by four wavelet families (BiorSplines "bior2.4", Symlets "sym4" "sym6", Daubechies "db2" and Coiflets "coif5"). The detail coefficients of the third level decomposition are calculated for damaged and undamaged beams. The location of damage can be calculated as the location of absolute maximum difference in the detail wavelet coefficients as in the following equations;

$$W_{Diff}=W_d-W_{un} \quad (10)$$

$$AMDWC=abs(W_{Diff}) \quad (11)$$

where: W_{un}, W_d are the detail wavelet coefficients of undamaged and damaged beams respectively, W_{Diff} is the difference in detail wavelet coefficients of damaged and undamaged beams, and AMDWC are the absolute maximum difference in wavelet coefficients. The locations of damages are indicated at the location of maximum AMDWC.

4. Finite element Models:

The proposed technique has been verified on three models; fixed- fixed, simply supported and cantilever steel beams. Single damage and multi damages are simulated and detected.

4.1The first FE model

The first model is a fixed-fixed steel beam with geometric and elements properties as shown in Table1. The beam has a length (L) equals 0.5m and is divided into 500 elements with equal lengths 1mm. Each element has four degrees of freedom (translation δi, δi+1 and rotation θi, θi+1) as shown in Fig. 1.

Table1: Steel beam geometric and elements properties

Properties	Symbol	Value
Breadth	B	4.00cm
Height	H	0.60 cm
Moment of Inertia	I	0.072 cm^4
Modulus of elasticity	E	200.00 Gpa
Density	ρ	7850.00 kg/m^3

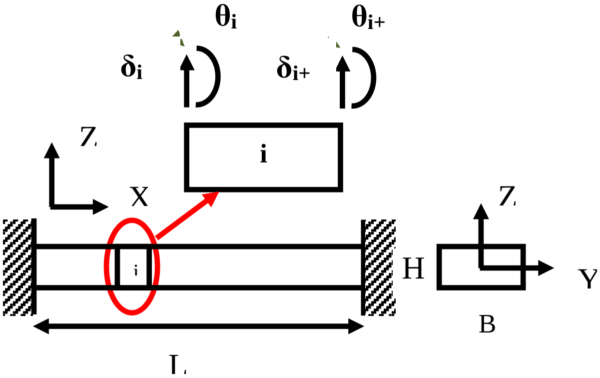


Fig. 1. Fixed-Fixed steel beam

4.2 The second FE model

The second FE model is a simply supported steel beam with dimensions and element's cross section as shown in Table1. and Fig. 2. The beam length (L) equals 0.6m and divided into 600 elements with equal lengths 1mm.

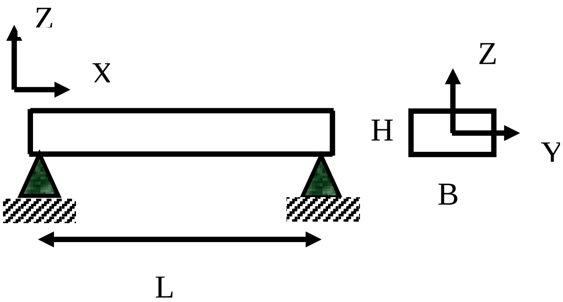


Fig. 2. Hinged-Hinged steel beam

4.3 The third FE model

The third finite element model is the cantilever steel beam with dimensions and element's cross section as shown in Table1 and Fig. 3. The beam length (L) equals 0.7m and divided into 700 elements with equal lengths 1mm.

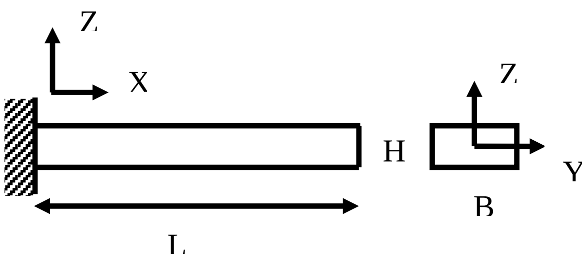


Fig. 3. Cantilever steel beam

5. Results of models:

For fixed-fixed beam (first model), the first mode shape of undamaged beam and three cases of damaged beams with damage percentages (5%, 10% and 30%) at element no. 150 are shown in Fig. 4.

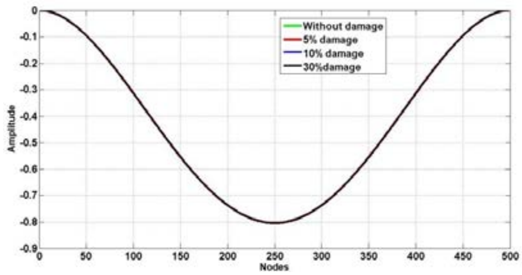


Fig. 4. First mode shape of undamaged and damaged beams with percentages (5%, 10% and 30%) at element no. 150

The first mode shape for hinged-hinged beam (second model) of undamaged beam and three cases of damaged beams with damage percentages (5%, 25%and 50%) at element no. 200 are shown in Fig. 5.

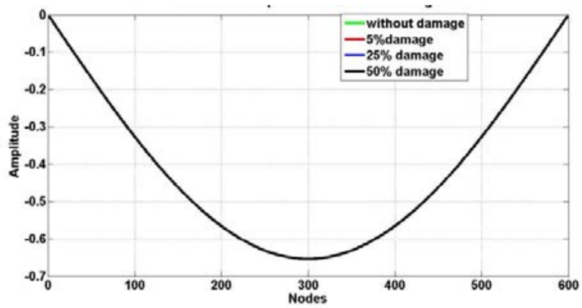


Fig. 5. First mode shape of undamaged and damaged beams with percentages (5%, 25% and 50%) at element no. 200

Also, the first mode shape of cantilever beam (third model) for undamaged and damaged beams with damage percentages (5%, 15%, 35%) at element no. 400 are shown in Fig. 6.

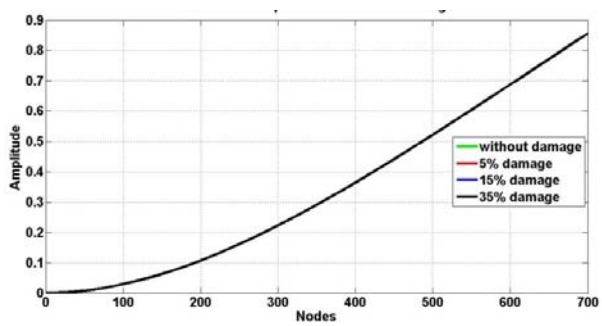


Fig. 6 First mode shape of undamaged and damaged beams with percentages (5%, 15% and 35%) at element no. 400

From previous mode shapes figures, it is shown that the first mode shapes of the different cases of damaged beams coincide with that of the undamaged beams. Also, the first mode shapes for all cases of damage percentage of the three models don't show clearly the damage location as the difference is very small. So, the first mode shapes of the three models of undamaged and damaged cases are transformed to wavelet domain.

5.1 Single damage scenarios

When the damage in element 150 is assumed in the first model with damage percentage 30%, and the wavelet family "db2" is used for the decomposition, the detail wavelet coefficients at the third level of decomposition are calculated. It is found that at the damage location, a sudden change in wavelet coefficients are detected as shown in Fig. 7. a, b.

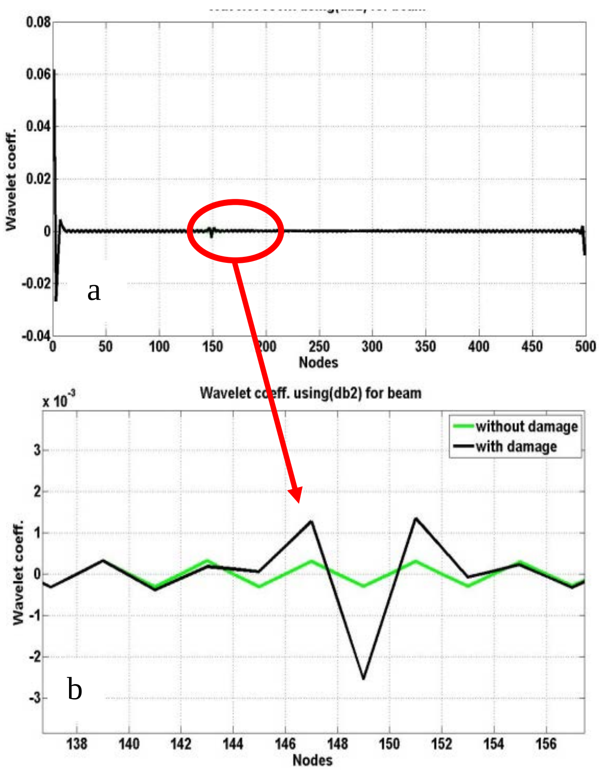


Fig. 7. a. Detail wavelet coefficients at level 3 using "db2" for damage percentage 30%, b. Detail of damage location

Then, the proposed technique calculates AMDWC where the maximum AMDWC indicates clearly the damage location as shown in Fig. 8.

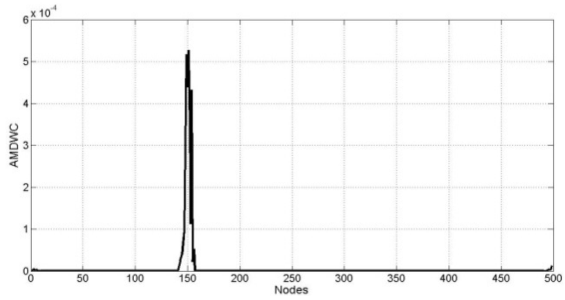


Fig. 8 Damage detection of element no. 150 in first model with damage percentage 30% using "db2"

In single damaged element scenarios, damage presents in one element only, while the rest of the elements are healthy. The proposed technique is applied to the three models (fixed-fixed, hinged-hinged, cantilever). The damage is assumed to be varied from 5% up to 90%.

The errors in the damage detection of element 150 when using four wavelet families (BiorSplines "bior2.4", Symlets "sym4", Daubechies "db2" and Coiflets "coif5") are shown in Fig. 9. The error is zero for using BiorSplines "bior2.4" at all damage percentages and for using Daubechies "db2" in damage percentage 25% and 30%. The maximum error in the estimated damage location is approximately 0.004% i.e. 2mm which are very small. This means that the error of the proposed technique in detecting the damage is very small. And the proposed technique is accurate in detecting the damage in beams with fixed-fixed boundary condition in all percentages of damage.

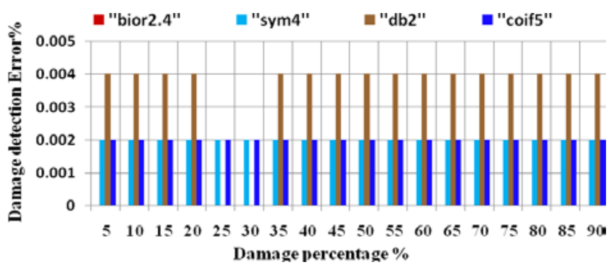


Fig. 9. Damage detection of element no. 150 in the first model with damage percentage up to 90% using different wavelet functions

The damage detection of element no. 200 in the second model using different wavelet families such as "db2", "bior2.4", "coif5" and "sym4" with different damage percentages (5% up to 90%) have been shown in Fig. 10.

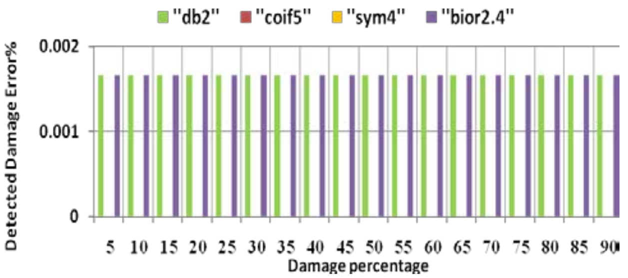


Fig. 10 Damage detection of element no. 200 in second model with damage percentage up to 90% using different wavelet functions

The proposed technique that is used in detecting element 200 in the second model shows that the error is zero when using Symlets "sym4" and Coiflets "coif5" in all damage percentages. The max errors using BiorSplines "bior2.4" and Daubechies "db2" are approximately 0.00167% i.e are very small. This means that the proposed technique is accurate in detecting the damage in beams with hinged - hinged boundary condition in all percentages of damage.

The damage detection of element no. 400 in the third model using different wavelet families with different damage percentages (5% up to 90%) have been shown in Fig. 11

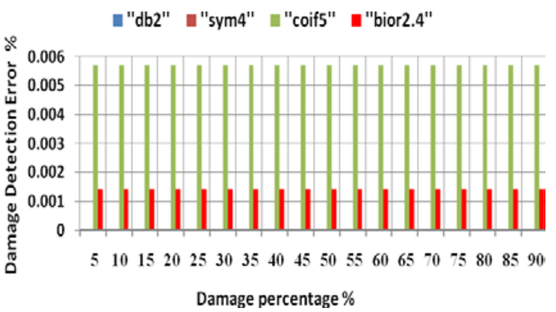


Fig. (11) Damage detection of element no. 400 in the third model with damage percentage up to 90% using different wavelet functions

It is noticed that the damage detection errors using wavelet families Symlets "sym4" and Daubechies "db2" are zero. The max error in the damage detection of element 400 is approximately 0.0057%. This means that the error of the proposed technique in detecting the damage is very small and the proposed technique is accurate in detecting the damage in beams with fixed–free boundary conditions in all percentages of damage.

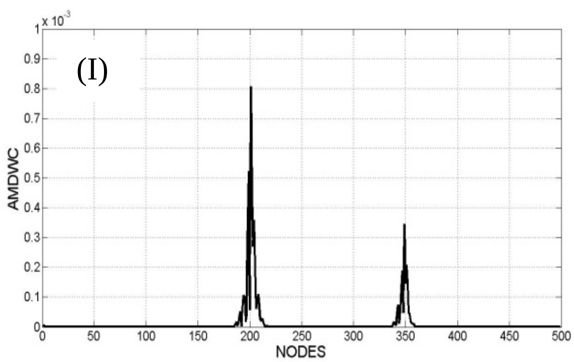
5.2 Multi – damages scenarios

Four cases of damages in the three models are assumed with damage percentage 20% for all cases. The wavelet family Symlets "sym6' is used in the analysis. The proposed technique calculates the absolute maximum difference in the detail wavelet coefficients at the third level of decomposition (AMDWC).

For fixed-fixed steel beam (first model) the estimated damage location using "sym6" for the four cases of damages are indicated at Table2. When damage is assumed in elements 200 and 350 with damage percentages 20% and the proposed technique calculates (AMDWC), the AMDWC obtained at the elements 201 and 349 as shown in Fig.(12-I) with difference equals 1mm. Also, for Fig. ((13) II,III, IV) this note is found in estimating the damage in elements 100,200,300,350 and 400 with difference equals 1mm. This result shows that the proposed technique detects accurately the damage location with maximum difference equals 1mm (error = 0.002%).

Table2 Cases of damage detection location with damage ratio 20% for first model

Damage case	Damage assumed	Damage estimated using "sym6"
I	200,350	201, 349
II	100,300,400	101 , 301 , 401
III	100,200,300 , 400	101 , 201 , 301 , 401
IV	100,200,300, 350,400	101 , 201 , 301 , 349 , 401



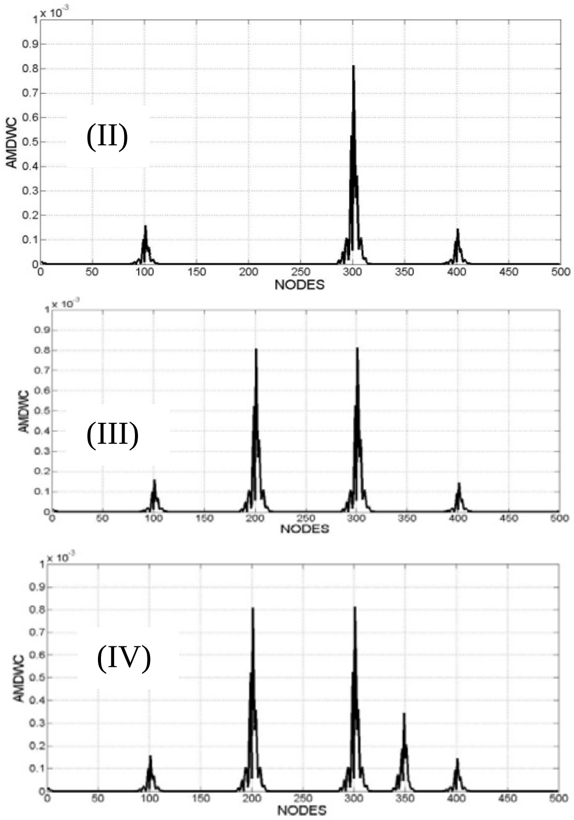


Fig. 12. (I, II, III, IV) Damage detection locations cases using "sym6" in the first model

For the hinged-hinged steel beam (second model), the estimated damage locations using "sym6" for the cases of damages are shown in Table3. The maximum AMDWC at the estimated damage location is shown in Fig. 13 for these cases. When damage is assumed in elements 200 and 350, the damages locations are estimated at elements 201 and 349 with difference 1mm. This means that the error is 0.002 i.e. very small. Also, for all cases in Table 3, the maximum difference in estimation the damage location is 1mm. This small difference in the damage location estimation indicates that the proposed methodology is accurate in the damage location detection in simply supported beam.

Table3 Cases of damage detection location with damage ratio 20% for second model

Damage case	Damage assumed	Damage estimated using "sym6"
I	200 , 350	201,349
II	120 , 320, 440	121,321,441
III	100, 150, 300, 400	101,149,301 , 401
IV	80, 150, 200, 250,400	81,149, 201, 249,401

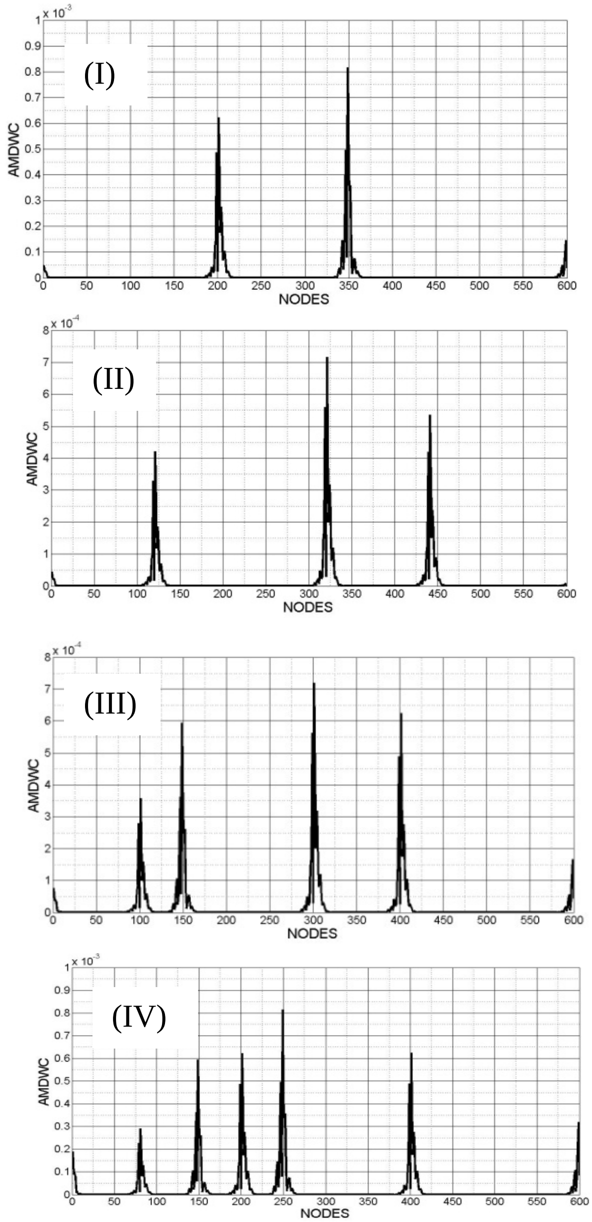


Fig. 13. (I, II, III, IV) Damage detection locations cases using "sym6" in second model

For the cantilever steel beam (third model), when damage is assumed in elements 160 and 250 with damage percentages 20%, the damage location is estimated at elements 161 and 249 with difference 1mm as shown in Fig.14. This means that the error is 0.002 i.e. very small. Also for all cases in Table (4), the maximum differences in estimation the damage location is 1mm. This small difference in the damage location estimation ensures that the proposed methodology is accurate in the damage location detection in cantilever beam.

Table 4 Cases of damage location with damage ratio 20% for third model

Damage case	Damage assumed	Damage estimated using "sym6"
I	160 , 250	161,249
II	140 , 310, 400	141,309,401
III	150, 190, 250, 450	149,189,249,449
IV	80, 120, 200, 280,500	81,121,201,281,501

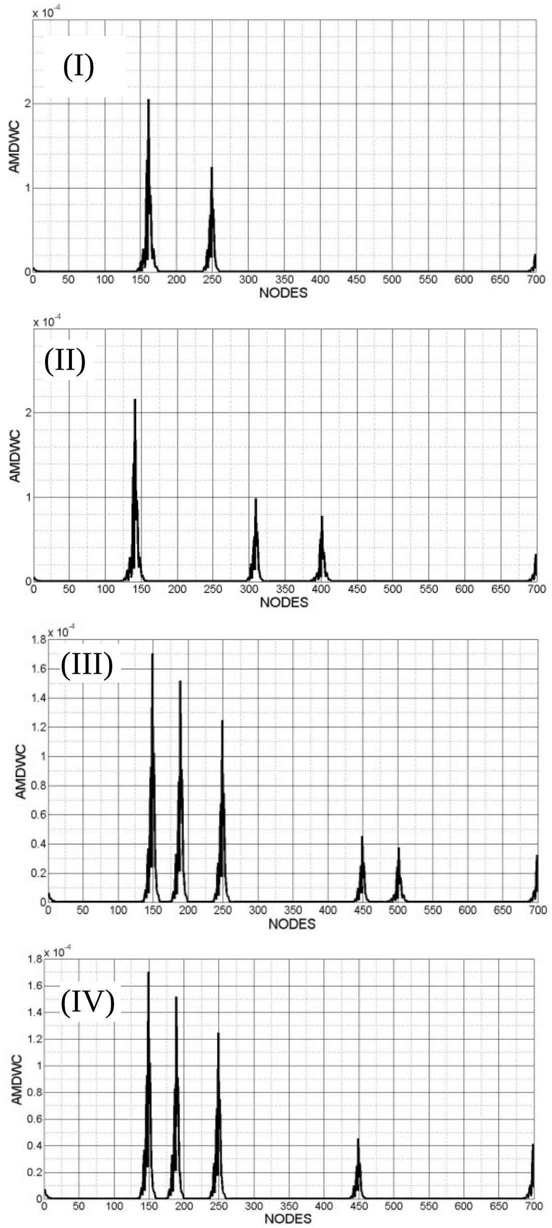


Fig. 14 (I, II, III, IV) Damage detection locations cases using "sym6" in third model

6. Conclusion:

In this paper, damage detection location in beams using the discrete wavelet transform is presented. The proposed technique uses a MATLAB program to simulate a beam, calculate its mode shapes and analyze the first mode shape by discrete wavelet analysis. Four wavelet families have been used for analysis. Three models of beams with different boundary condition are discussed.
From this study, it is concluded that:

1. The first mode shape couldn't detect the damage location as the differences of mode shapes are very small.
2. Analyzing the first mode shape of damaged and undamaged beams using any discrete wavelet family detects accurately the damage location.
3. The proposed program deals with beam structure with different properties (boundary conditions, material properties, beam length, beam cross section and damages percentages up to 90%).
4. This methodology detects accurately the location of single or multi- damages with small, moderate and severe damages.

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